Problem Set 2

Due: Friday, January 20

Problem 1. Given a permutation $\pi = \pi_1 \dots \pi_n$, let $\pi^\dagger$ be the permutation $\pi_n \pi_{n-1} \dots \pi_1$. Determine $i(\pi^\dagger)$, $I(\pi^\dagger)$, $d(\pi^\dagger)$, $D(\pi^\dagger)$, and $\mathrm{maj}(\pi^\dagger)$ in terms of $i(\pi)$, $I(\pi)$, $d(\pi)$, $D(\pi)$, and $\mathrm{maj}(\pi)$, respectively. Here $I(\pi)$ is the inversion table of π (see Proposition 1.3.9 and nearby of Stanley for notation).

Problem 2. Determine (with proof) the number of sequences of 0's and 1's $(\varepsilon_1, \dots, \varepsilon_n)$ that satisfy

$$\varepsilon_1 \leq \varepsilon_2 \geq \varepsilon_3 \leq \varepsilon_4 \geq \varepsilon_5 \leq \cdots .$$

Problem 3. Evaluate the sum

$$\sum_{i=0}^{k} (-1)^i \binom{n}{k-i}.$$

(Suggestion: use generating functions.)

Problem 4. Recall that the Eulerian number $A(n,k)$ is the number of permutations of $[n]$ with $k-1$ descents. Prove that $\sum_{k=1}^{n} kA(n,k) = \frac{1}{2}(n+1)!$.

Problem 5. Solve the recurrence:

$$\begin{aligned} g_0 &= 1, \\ g_n &= g_{n-1} + 2g_{n-2} + \cdots + ng_0. \end{aligned}$$