Problem Set 12

Do not turn in

Problem 1. Prove or disprove: the product of two skew Schur functions is a skew Schur function.

Problem 2. Let $\ell(\lambda) \leq m$ and $\lambda_1 \leq n$. Define

$$\tilde{\lambda} = (n - \lambda_m, n - \lambda_{m-1}, \ldots, n - \lambda_1).$$

Prove that

$$(x_1 x_2 \cdots x_m)^n s_\lambda(x_1^{-1}, \ldots, x_m^{-1}) = s_{\tilde{\lambda}}(x_1, \ldots, x_m).$$

Problem 3. Let $\delta = (n-1, n-2, \ldots, 1)$ be the staircase shape. Express a_δ^2 as the determinant of a matrix with entries belonging to $\{p_0, p_1, p_2, \ldots, p_{2n-2}\}$, where the p_i are power sum symmetric functions with the temporary convention that $p_0 = n$.

Problem 4. It is known that $s_\lambda(1, q, \ldots, q^n)$ is a symmetric and unimodal polynomial in the variable q. Use this to prove that the q-binomial coefficient $\begin{bmatrix} n \\ k \end{bmatrix}_q$ is symmetric and unimodal.

Problem 5. Define *decreasing subsequence* similarly to increasing subsequence. Given $\pi \in \mathcal{S}_n$, let λ be the shape of $P(\pi)$. Show that $\ell(\lambda)$ is equal to the length of the longest decreasing subsequence of π.