Math 217
Homework 7 Problems
W 2013

Book 4.4-4.6
4.4 : 15,18,25
4.5 : 19,29,30
4.6 : 17,18,27

Proof Problems

Problem 8.1. Prove the following:

(1) Let V and W be vector spaces. Let $\{v_1, \ldots, v_p\}$ be a basis of V and let $\{w_1, \ldots, w_p\}$ be an arbitrary set of vectors. Show that there is a unique linear transformation such that $T(v_i) = w_i$ for all i.
(2) Let V and W be vector spaces. Let $\{v_1, \ldots, v_p\}$ be a basis of V and $\{w_1, \ldots, w_p\}$ be a basis of W. Show that a linear transformation $T : V \to W$ such that $T(v_i) = w_i$ for all i is an isomorphism.
(3) Let T be an isomorphism between vector spaces V and W. Show that $\{v_1, \ldots, v_p\}$ is a basis if and only if $\{T(v_1), \ldots, T(v_p)\}$ is a basis.

Problem 8.2.

(1) Let A be an n-by-n matrix. Show that
$$Nul(A) \subseteq Nul(A^2) \subseteq Nul(A^3) \subseteq \cdots$$
(2) Let A be an n-by-n matrix. Show that
$$Col(A) \supseteq Col(A^2) \supseteq Col(A^3) \supseteq \cdots$$
(3) Let A be a m-by-n matrix and let B be a n-by-p matrix. Show that
$$rank(AB) \leq rank(A), rank(AB) \leq rank(B).$$

Problem 8.3 Let U, V, W be vector spaces and $T : U \to V$ and $S : V \to W$ are isomorphisms. Show that $S \circ T$ is also an isomorphism.

Problem 8.4 Prove the following:

(1) Let U and W be subspaces of a vector space V. Show that $U + V$, which is defined as the set $\{u + v | u \in U, v \in V\}$ is also a vector space.
(2) Show that $dim(U) + dim(V) = dim(U \cap V) + dim(U + V)$. (Hint : Start from a basis of $U \cap V$)