Homework Set 5

Due February 13

NOTE: Please hand in the book and proof problems separately.

Section 3.1
Problems 2, 14, 26, 38
Section 3.2
Problems 8, 20, 26, 40
Section 3.3
Problems 10, 14, 18, 32

The Proof Problems:

PROBLEM 5.1: Find all values of $k \in \mathbb{R}$ for which the following matrix is invertible:

$$A = \begin{pmatrix} k^2 & 2 & 5 & k & 1 \\ k & 3 & 4 & 1 & 1 \\ -k & 25 & 1 & -1 & 10 \\ k^2 & 4 & -9 & k & 2 \\ k^3 & 4 & 8 & k^2 & 6 \end{pmatrix}.$$

PROBLEM 5.2: Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a horizontal shear by 2 units, that is

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + 2y \\ y \end{pmatrix},$$

and denote by C the standard matrix of T.

a) Show that C^{146} is invertible.

b) Find $\det((C^{146})^{-1})$.

c) Intepret your findings geometrically in terms of the area of the parallelogram whose sides are $T_2(\overrightarrow{e_1})$ and $T_2(\overrightarrow{e_2})$, where $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ is the transformation whose standard matrix is $(C^{146})^{-1}$.

PROBLEM 5.3: Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the rotation by $\pi/3$ radians counterclockwise, and let A be the associated standard matrix. (Note that T is linear.)

a) Find $|\det A|$ (without any explicit computation).

b) Find all values of $n \geq 1$ for which

$$A^n = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

PROBLEM 5.4: Let D_n be the $n \times n$ matrix defined by $D_n = \{d_{ij}\}_{1 \leq i,j \leq n}$ with

$$d_{ij} = \begin{cases} 1 & \text{if } |i-j| \leq 1 \\ 0 & \text{otherwise,} \end{cases}$$

that is

$$D_n = \begin{pmatrix} 1 & 1 & & & & & \\ 1 & 1 & 1 & & & & \\ & 1 & 1 & 1 & & & \\ & & 1 & 1 & 1 & & \\ & & & & \ddots & & \\ & & & & 1 & 1 & 1 \\ & & & & & 1 & 1 \end{pmatrix},$$

and write $P_n = \det(D_n)$.

a) Find P_2, P_3 and P_4.

b) For $n \geq 2$, find a formula for P_{n+2} in terms of P_{n+1} and P_n. Do this using row expansion.

c) Find all values of n for which D_n is not invertible.