Homework Set 10

Due April 10

Section 5.5
Problems 4, 12, 14
Section 4.9
Problems 1, 11
Section 6.1
Problems 10, 16, 24
Section 6.2
Problems 17, 26, 28, 32

The Proof Problems:

PROBLEM 10.1.

(a) Let A be a $n \times n$ matrix and let $a'_1 \dots a'_n$ be the *rows* of A. Suppose $y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$ is such that $y_1 a'_1 + \dots + y_n a'_n = 0$. Prove that, $\forall x \in \mathbb{R}^n$, $Ax \cdot y = 0$.

(b) For $n \geq 2$, find a nonzero $n \times n$ matrix A such that $\forall x \in \mathbb{R}^n$, $Ax \cdot x = 0$.

PROBLEM 10.2.

(a) Let A be an $m \times n$ matrix, and $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$. Prove that $Ax \cdot y = x \cdot A^{\mathrm{T}} y$.

(b) Let A be an $n \times n$ real matrix such that $A^{\mathrm{T}} = A$. We call such matrices "symmetric." Prove that the eigenvalues of a real symmetric matrix are real (i.e. if λ is an eigenvalue of A, show that $\lambda = \bar{\lambda}$).

(c) Let A be a real symmetric $n \times n$ matrix, and suppose A has n real, distinct eigenvalues, $\lambda_1, \dots, \lambda_n$ with corresponding eigenvectors $\phi_1, \dots, \phi_n$.
Prove that $\Phi = \{\phi_1, \dots, \phi_n\}$ is an orthogonal basis of $\mathbb{R}^n$.