Exchange Rates, Optimal Debt Composition and Hedging in Small Open Economies

Jose Berrospide*
University of Michigan

Job Market Paper

January 2007

Abstract

This paper develops a model of the choice between local and foreign currency debt by firms facing exchange rate risk and hedging possibilities in small open economies. The model shows that the currency composition of debt and the optimal level of hedging are both endogenously determined as optimal firms’ responses to a tradeoff between the lower cost of borrowing in foreign debt and the higher risk involved due to exchange rate uncertainty. Both debt composition and hedging depend on common factors such as foreign exchange risk and financial default, interest rates, the size of net worth and costs of exchange rate risk management. Results of the model are broadly consistent with lending and hedging behavior of the corporate sector in small open economies recently hit by a currency crisis. In particular, the model is able to explain why, unlike predictions of previous work in the literature of currency crisis, the collapse of the fixed exchange rate regime in Brazil in early 1999 did not cause a major change in the currency composition of debt and the hedging behavior of the corporate sector.

JEL Classification: F31, F34, F41, G18, G32

Keywords: Exchange rate regime, Debt composition and Hedging

* Email: jberros@umich.edu.

I would like to thank Linda Tesar, Uday Rajan, Jing Zhang, Kathryn Dominguez, Chris House and participants of the International Macro Lunch and the International Seminar at the University of Michigan for valuable advice. I am also grateful to the Center for International Business Education (CIBE) for financial support. All remaining errors are mine.
1. **Introduction**

Recent currency crises in East Asia and Latin America have been mainly characterized by the presence of currency mismatches between assets and liabilities and inadequate hedging in the balance sheets of the corporate sector.\(^1\) This mismatch between foreign currency liabilities and domestic currency denominated assets in firm balance sheets has been argued to be the root cause of the large output collapses following currency depreciations.\(^2\) Under a fixed exchange rate regime, firms understand fixed exchange rates to be a guarantee and fail to insure their foreign exposure.\(^3\) A direct implication of this line of reasoning is that once the exchange rate is allowed to float, firms will recognize their exposure, and foreign currency loans will be viewed as more costly so that firms reduce their foreign currency borrowing. Firms that still opted to borrow internationally would have incentives to hedge their foreign exposure. Unlike these predictions, firm level evidence from Brazil over the period of 1996-2001, suggests that the collapse of the fixed exchange rate regime in Brazil, in early 1999, did not cause a major change in the currency composition of debt and the hedging behavior of the corporate sector. This paper attempts to offer an explanation to this apparently surprising result.

To study this phenomenon the paper develops a theoretical framework that examines the firm’s choice of local and foreign currency debt in the presence of exchange rate risk and hedging possibilities. The model shows that the currency composition of debt and the optimal level of hedging are joint decisions and depend on common factors such as exchange rate risk and financial default, interest rates, the size of net worth and costs of foreign currency risk management. The key element driving the model results is the tradeoff that firms face between the lower cost of borrowing in foreign currency and the higher risk involved due to exchange rate uncertainty. When affordable, hedging complements the

\(^1\) A currency mismatch occurs when a large fraction of firms’ debt is denominated in foreign currency while income and assets are denominated in domestic currency.

\(^2\) To see more on the balance sheet explanation of currency crises see for example Krugman (1999), Aghion, Bacchetta and Banerjee (2001) and Schneider and Tornell (2000).

\(^3\) This implication is also consistent with another strand in the literature that emphasizes a moral hazard problem introduced by implicit bailout guarantees provided by government, or free exchange rate risk management also provided by government when fixing the exchange rate. These guarantees bias the composition of debt toward foreign currency debt and eliminate incentives to hedge risk. See for example Burnside et al (2001) and Dooley (2000).
minimum net worth required by banks and expands the firms’ debt capacity. Furthermore, firms heavily indebted in foreign currency are not necessarily exposed to exchange rate risk if they have enough net worth or if they are able to hedge their currency risk through forward markets.

The model predictions are broadly consistent with lending and hedging behavior of the corporate sector in small open economies that recently faced a currency crisis. The theory suggests that, when the economy moves from fixed to floating exchange rates, some firms change financing policies and the population of firms exposed to foreign exchange risk is altered. Firms with insufficient net worth and those unable to afford buying a hedge lose access to capital markets. Firms with high enough net worth and those able to hedge increase their foreign debt. Firms with intermediate net worth but unable to hedge borrow less in foreign currency, turn to domestic banks and are monitored in order to maintain their access to foreign capital markets. With a macroeconomic environment characterized by a moderate probability of currency depreciation and inexpensive hedging, these changes in the population of firms can offset each other so that the average currency composition of debt and hedging activities do not vary significantly across regimes.

The paper builds on the model developed by Holmstrom and Tirole (1997) adapted to the context of a small open economy. The paper with the analytical framework most closely related to this paper is Martinez and Werner (2002). They also extend the Holmstrom and Tirole model to the small open economy case and find that before the Mexican crisis in 1994, the decision of borrowing in pesos or dollars depended on the exchange rate regime to the extent that an implicit guarantee was provided by the government by fixing the exchange rate. However, these authors treat the exchange rate as a deterministic variable so that no hedging strategies on the firm side are discussed. Arguably, their model captures only part of the story; there is no discussion on how the interaction between domestic and dollar debt changes and how the population of firms is altered when the economy moves to a floating regime and firms are able to hedge. In their paper, as in most of the previous literature, it is implicit that large amounts of foreign currency debt represent a high degree of exposure of firms to exchange rate risk.
As mentioned above, prior theoretical predictions suggest a negative impact of a large devaluation on companies’ foreign currency borrowing and a reduction in currency mismatches. However, empirical findings on the relationship between exchange rate regimes and currency mismatches in the balance sheet of corporate sectors are mixed and still subject to debate. Some studies find that the large devaluation that followed the collapse of the exchange rate regime reduced currency mismatches by reducing the foreign currency borrowing and increasing the levels of hedging while others find that firms were able to borrow even more after the currency crisis.\footnote{Empirical support of currency mismatches and exchange rate exposure during fixed exchange rate regimes is found by Burnside et al (2001) in the Asian crisis, Tesar and Dominguez (2001) in 8 non-industrialized and emerging markets and Bonomo et al (2003) using financial data from Brazilian. On a different study with Brazilian firms Rossi (2004) finds that the adoption of the floating regime reduced the foreign vulnerability of the corporate sector by having a negative impact on firms’ foreign borrowing and a positive impact on hedging. However, Martinez and Werner (2002) find that firms in Mexico were able to increase their dollar debt borrowing after the crisis in 1995 and Arteta (2002) in a cross-country study finds no evidence that a floating exchange rate regime reduces bank currency mismatches. Similarly, Allayannis et al (2002) find that non-financial firms in Asian countries were able to maintain substantial levels of foreign currency debt even after the currency crisis. It is important to say however, that except for only few studies such as Rossi (2004) and Allayannis et al (2002), most studies in the empirical literature only used data on the currency composition of debt at the firm and bank level since data on hedging are often not available or hard to collect.} The model presented in this paper contributes to explain these apparently contradicting results between theory and empirical evidence by providing an analytical framework in which currency mismatches in the balance sheet of the corporate sector are reduced after the currency crisis without having firms borrowing less in foreign debt. High enough net worth as well as hedging operations allow corporations, on aggregate, to maintain their access to international capital markets and have their currency composition of credit almost unchanged.

The reminder of the paper is organized as follows. Section II presents the main empirical facts and the motivation for the analysis using firm-level data from Brazil. Section III describes the model of optimal debt allocation and hedging at the firm level. Section IV offers the main results of the model under flexible and fixed exchange rates. Section V concludes.
2. **Empirical evidence on foreign currency exposure and hedging: Brazil 1996-2001**

In this section firm-level data on Brazilian firms are examined before and after the currency crisis in 1999. Like other Latin American countries, Brazil suffered from unexpected reversals in capital flows after subsequent crises in Mexico (1994), East Asia (1997) and Russia (1998). The macroeconomic adjustment forced by substantial capital outflows at the end of 1998 brought large and persistent swings in the exchange rate that finally led to the collapse of the crawling peg and a sharp devaluation of the real in January 1999.

Firm-level data correspond to financial information for 350 companies publicly traded in the Sao Paulo Stock Exchange Market over the period of 1996-2001.\(^5\) This information was taken directly from companies’ annual financial reports. Data contain information on balance sheet variables such as the book value of assets, the currency composition of debt and shareholder’s equity. Data on hedging transactions was hand-collected and consists of the year-end notional value of currency derivatives (forwards, futures, swaps and options) obtained from the explanatory notes to financial statements. Financial and state-owned firms are excluded because of their different motivation for using currency derivatives.

Table I illustrates the currency composition of debt and hedging activities of firms across exchange rate regimes. Over the sample, the share of dollar-denominated debt in total debt remains stable around 46 percent. On average the dollar debt ratio slightly increased from 45 percent during the fixed exchange rate period to 48 percent during the float period. Across different groups of firms the dollar debt ratio seems to exhibit a similar pattern, except for small firms that on average slightly decrease their dollar debt ratio. On the other hand, hedging operations, approximated by the fraction of the notional value of currency derivatives to dollar debt, increased from 1 percent in 1996 to 18 percent in 2001. Interestingly, data show that hedging operations increased during the floating exchange rate regime but were already observed during the fixed exchange rate period, especially for medium and large

---

\(^5\) This period is chosen because it provides valuable information to a comparative analysis on firm’s behavior regarding financing and risk management policies before and after the currency crisis in 1999, and also because disclosure of information on derivatives was mandatory only after 1995.
corporations. Large companies increased their hedging ratio on average from 7 to 18 percent. This latter observation indicates that even before the currency crisis, and most likely after crisis episodes in East Asia and Russia, some firms indebted in foreign currency were taking care of the devaluation risk by hedging their positions using currency derivatives. As will be seen later, this behavior is consistent with the model predictions highlighting the hedging side of financing policies in small open economies subject to currency risk. While highly suggestive, this table also seems to show some evidence of costly hedging since the increase in the hedging ratio mostly occurred in large and medium size companies while small firms appear to hedge a small fraction of their dollar debt even during the floating regime.

Table 1 - Brazil: Currency Composition of Debt and Hedging Operations: (in means)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollar Debt/Total Debt</td>
<td>39.1</td>
<td>45.1</td>
<td>48.0</td>
<td>47.5</td>
<td>48.1</td>
<td>48.0</td>
<td>44.9</td>
<td>47.9</td>
<td>47.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>25.8</td>
<td>35.6</td>
<td>27.4</td>
<td>27.1</td>
<td>29.1</td>
<td>28.0</td>
<td>29.4</td>
<td>28.1</td>
<td>28.1</td>
</tr>
<tr>
<td>Medium</td>
<td>40.0</td>
<td>43.1</td>
<td>51.1</td>
<td>49.1</td>
<td>49.7</td>
<td>48.6</td>
<td>45.7</td>
<td>49.1</td>
<td>49.1</td>
</tr>
<tr>
<td>Large</td>
<td>57.3</td>
<td>61.6</td>
<td>64.7</td>
<td>63.5</td>
<td>62.4</td>
<td>64.4</td>
<td>62.0</td>
<td>63.4</td>
<td>63.4</td>
</tr>
<tr>
<td>Dollar Derivatives/Dollar Debt 1/</td>
<td>1.0</td>
<td>5.3</td>
<td>5.5</td>
<td>4.9</td>
<td>8.1</td>
<td>18.3</td>
<td>4.3</td>
<td>10.5</td>
<td>10.5</td>
</tr>
<tr>
<td>Small</td>
<td>0.0</td>
<td>0.0</td>
<td>1.9</td>
<td>1.7</td>
<td>1.9</td>
<td>4.1</td>
<td>0.7</td>
<td>2.6</td>
<td>2.6</td>
</tr>
<tr>
<td>Medium</td>
<td>0.0</td>
<td>6.4</td>
<td>6.0</td>
<td>5.1</td>
<td>6.2</td>
<td>15.5</td>
<td>4.7</td>
<td>9.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Large</td>
<td>4.0</td>
<td>8.1</td>
<td>7.1</td>
<td>6.4</td>
<td>15.2</td>
<td>31.4</td>
<td>6.7</td>
<td>17.8</td>
<td>17.8</td>
</tr>
</tbody>
</table>

1/ Dollar derivatives correspond to the notional value in dollars of currency derivatives (forwards, futures, swaps and options).
Source: Brazil Securities and Exchange Commission (CVM). Financial annual reports and explanatory notes to financial statements of companies publicly traded at the Sao Paulo Stock Exchange Market.

An interesting feature regarding the hedging behavior of the corporate sector in Brazil is that most financial hedging transactions correspond to currency swap contracts. Unlike evidence for U.S. large non-financial corporations, which report currency forwards and options as the most commonly used tool to manage exchange rate exposure, firms in Brazil seem to prefer currency swaps, effectively converting foreign debt into domestic debt by simultaneous transactions in the spot and the forward markets. Broad preference for currency swaps is also consistent with costly hedging, since a swap reduces transaction
costs by allowing companies to arrange in only one contract what may take several transactions (e.g. forward contracts) to replicate.  

Table II shows the number of firms holding financial debt in domestic and foreign currency and those using currency derivatives to hedge their foreign exposure. These data also provide evidence of minor changes in lending and hedging behavior. The number of firms borrowing in both currencies and the number of firms borrowing only in domestic currency both slightly increased during the floating regime. The number of firms that hedge using currency derivatives increases after the collapse of the exchange rate regime but, interestingly, about 80 percent of the firms with dollar debt in the sample still remain unhedged during the floating regime.

<table>
<thead>
<tr>
<th>Table II: Brazil: Exchange rate risk exposure overtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floating ER</td>
</tr>
<tr>
<td>Total Number of firms</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1. Number of firms with no financial debt</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2. Number of firms with only domestic debt</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>3. Number of firms with dollar and domestic debt</td>
</tr>
<tr>
<td>Firms hedged using dollar derivatives</td>
</tr>
<tr>
<td>Firms not hedged</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>4. Number of firms with only dollar debt</td>
</tr>
<tr>
<td>Firms hedged using dollar derivatives</td>
</tr>
<tr>
<td>Firms not hedged</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>5. Number of firms with unknown debt position</td>
</tr>
</tbody>
</table>

Source: Brazil Securities Exchange Commission (CVM). Financial annual reports and explanatory notes to financial statements of companies publicly traded at the Sao Paulo Stock Exchange Market

6 Bonomo et al (2003) pointed out that Brazilian firms prefer currency swaps because they can obtain advantageous swap contracts from local banks. Banks, in turn, are able to offer these contracts because they are not exposed to exchange rate risk since they hold government bonds indexed to the dollar in their portfolios. According to these authors, in the end the hedge appears to be provided by government through bank intermediation.
Overall, the evidence presented in this section seems to support the view that there are no significant changes in the lending and hedging patterns of the Brazilian corporate sector during the period 1996-2001, that is, before and after the currency crisis. It should be mentioned that having a genuine measure of firm-level hedging is difficult, and the previous observations about hedging refer only to the use of currency derivatives. Firms may use different hedging strategies other than derivatives, such as holdings of dollar-denominated assets and revenues from exports, which are not emphasized in the current analysis. The main point that can be emphasized however is that, contrary to prior suggestions, after the large depreciation that followed the collapse of the fixed exchange rate firms in Brazil were still able to borrow significantly in foreign currency, more companies hedged this debt by getting involved in currency derivative markets and there was still a large fraction of firms holding dollar debt which was not financially hedged through currency derivatives. As will be shown in the next section, predictions of the model of optimal debt allocation and hedging are consistent with these empirical facts.

3. A Model of Optimal Currency Composition of Debt and Hedging

Consider a small open economy described by a two-date model \((t = 0, 1)\). The economy is populated by a continuum of risk-neutral firms, domestic banks and foreign banks. Firms are run by wealth-constrained entrepreneurs who need to raise funds to cover their investment outlays. Firms’ investment projects can be financed by borrowing in either domestic currency from local banks or in foreign currency from foreign banks. Given the uncertainty about the exchange rate, firms can choose to hedge their foreign exchange risk by signing forward contracts offered by local banks. At \(t=0\) firms sign debt contracts and make investment, borrowing and hedging decisions. At \(t=1\), exchange rate and investment returns are realized and claims are settled. Agents are protected by limited liability so that no party can end up with negative payoffs.
3.1 Investment Projects

Each firm has access to a project requiring an investment of fixed size $I>0$ at date 0, which yields at date 1 a verifiable return in domestic currency $R$ in case of success and nothing in case of failure. Firms differ only in their initial capital $A$ (which is publicly observable). The distribution of firms is described by the cumulative distribution function $F(A)$. It will be assumed that $A<I$, so that firms need external funding to undertake their investments.

There are three types of investment projects as described in Figure 1: a good project with a high probability of success $P_G$, and a bad and a worse projects each with the same probability of success $P_B$ ($P_G>P_B$). The bad project gives a low private benefit $b$ and the worse project gives a high benefit $B$ to the entrepreneur, with $B>b>0$. Firms face a moral hazard problem in choosing a project. In the absence of proper incentives or outside monitoring, entrepreneurs can divert resources by deliberately reducing the probability of success of a project (from $P_G$ to $P_B$) to enjoy the private benefit.

**Figure 1: Three types of Investment Projects**

<table>
<thead>
<tr>
<th>Project</th>
<th>Good</th>
<th>Bad</th>
<th>Worse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of success</td>
<td>$P_G$</td>
<td>$P_B$</td>
<td>$P_B$</td>
</tr>
<tr>
<td>Private Benefit</td>
<td>0</td>
<td>$b$</td>
<td>$B$</td>
</tr>
</tbody>
</table>

Local banks can monitor firms to reduce the moral hazard problem and eliminate the worse project (B-project). Monitoring is costly so that local banks face a fixed cost $C>0$. Foreign banks are uninformed investors (i.e. they are unable to monitor firms) and have access to alternative projects with a gross rate of return $r^*$ in international markets. Furthermore, it is assumed that given the rate of return on investor capital in domestic currency denoted by $r^f$, only the good project has a positive expected net present value (NPV), even if the private benefit of the firm is included:

\[
P_G R - r^f I > 0 > P_B R - r^f I + B \tag{3.1}
\]
3.2 Financing Decisions and Exchange Rate Risk

In what follows, firms are represented by the index \( f \); domestic banks are represented by the index \( m \) and foreign banks are represented by the index \( u \). At date 0 a representative firm invests all its funds \( A \) and sign debt contracts to borrow in domestic currency an amount \( I_m \) from local banks and in foreign currency an amount \( I_u \) from foreign investors so that:

\[
A + I_m + s_L I_u = I
\]  
(3.2)

where \( s_L >0 \) is the exchange rate at date 0, quoted as \( s_L \) units of domestic currency per unit of foreign currency.

Notice that investment return and exchange rate are both uncertain. Investment projects are subject to two types of bad events: failure (economic default) or bankruptcy due to currency-led default (financial default). Exchange rate fluctuations, therefore, can turn solvent firms into bankrupt firms even when they undertake successful projects. It will also be assumed that in case of any type of default firms’ creditors are left with nothing.

For simplicity, consider only two states of nature about the exchange rate: low (L) and high (H). At date 0 the exchange rate is \( s_L \). If the economy operates under a fixed exchange rate regime then at date 1 \( s_1 = s_L \). Under a floating exchange rate regime, at date 1 the exchange rate can either depreciate to \( s_H > s_L \) with probability \( q \), or remain unchanged with probability \( 1-q \). To avoid losses due to exchange rate fluctuations, companies may decide to hedge their foreign exchange risk by buying currency forward contracts.

Let \( F \) be the one-period forward exchange rate charged to the firm and defined as units of domestic currency per unit of foreign currency. Given the possibility of economic and financial default, it

---

7 It is also assumed that the internal rate of return of investment projects exceeds the market rate \( r \) which means that funds invested in the firm are worth the external rate of return plus an incentive effect.

8 There is no other source of funding (e.g. equity finance) in the model. As is the case of emerging economies in Asia and Latin America, a large part of the funds for investments are provided in the form of bank loans.

9 In case of financial default the firm is declared bankrupt and its residual value goes to pay for bankruptcy costs.
will be assumed that forward markets are efficient so that $F$ is the expected exchange rate at date 0 and is given by:

$$
F = \begin{cases} 
(1-q)s_L + qs_H & \text{if financially solvent in both states } L, H \\
s_L & \text{if financially solvent only in state } L \\
s_H & \text{if financially solvent only in state } H 
\end{cases} \quad (3.3)
$$

Note that $s_L \leq F \leq s_H$. Expression (3.3) states that the forward rate is adjusted according to the risk of financial default in a way such that over-hedging (e.g. infinite hedging in case of zero transaction costs) is ruled out as will be clear in the next paragraphs. Another important feature of the hedging contract is that payments in either direction are contingent on the project succeeding. In other words, when the project fails and state $s_L$ occurs the firm has nothing so that it will not pay out on the hedging contract but it will not receive any payment either if state $s_H$ occurs. This particular feature guarantees that a hedged firm will always be able to obtain a forward contract at the rate $F = (1-q)s_L + q s_H$, which occurs because the hedging contract is offered by domestic banks so that payments in both directions are netted out on average.\(^{11}\)

Let $h$ be the amount of forward contracts, in foreign currency, purchased by a firm and $\phi$ the transaction costs per unit of forward contract. For simplicity, let $\phi$ be also expressed in domestic currency per unit of foreign currency. At date 1 proceeds from hedging operations in domestic currency are given by:

$$
\Pi^H = h (s_i - F) - |\phi| h 
$$

In this economy, a debt contract must specify the amount of each participant’s investment and the payments to each of them under all circumstances so that: 1) if the project fails, it pays zero to all parties 2) At $t=1$ project returns $R$, in case of success, plus proceeds from hedging operations in case of no financial default (if any) are divided among parties, from which the firm receives $R_f(s_i) \geq 0$, domestic

---

\(^{10}\) As shown later, the forward rate is related to interest rates according to a covered interest parity condition.  
\(^{11}\) For simplicity, it is assumed that firms buy forward contracts from domestic banks other than those from which they borrow so that transaction costs of hedging do not go as revenues to domestic lenders.
banks receive \( R_m \geq 0 \), foreign banks receive \( s_i R_u \geq 0 \) and forward sellers receive \( \varphi | h | \geq 0 \). Note that debt payments to lenders are independent of the exchange rate \( s_i \) provided the firm is solvent. However, returns to the entrepreneur (the equity-holder) may depend on \( s_i \).

Financial default at date 1, is defined as a situation in which firms that undertake a successful investment are unable to meet their financial obligations and become bankrupt. Financial default is then given by the following condition:

\[
R - R_f(s_i) - R_m(s_i) - s_i R_u(s_i) + h (s_i - F) - \varphi | h | < 0 , \text{ where } s_i = s_L, s_H
\]

(3.5)

As mentioned above, the forward rate in expression (3.3) rules out over-hedging. For example, consider the case of a firm wanting to buy a significantly high amount of forward contracts to increase its expected payment when \( s_H \) occurs but default when \( s_L \) occurs. As a result, expression (3.5) becomes positive in state \( H \) but negative in state \( L \), that is, the firm is solvent only in state \( H \). By setting \( F = s_H \) forward sellers eliminate incentives for firms to over-hedge because in such a case the firm will not be paid in the depreciation state.

The preceding analysis implies that the firm hedges its foreign exchange exposure in order to reduce the possibility of financial default. Furthermore, a firm that is hedged ex-ante cannot be insolvent or solvent in only one state. Since financial default may occur in either state \( L \) or \( H \), let \( 1_L \) and \( 1_H \) be indicator variables such that:

\[
1_H = \begin{cases} 
1 & \text{if solvent in state } H \\
0 & \text{if default in state } H 
\end{cases}
\]

(3.6)

\[
1_L = \begin{cases} 
1 & \text{if solvent in state } L \\
0 & \text{if default in state } L 
\end{cases}
\]

Accordingly, expected cash flows to the entrepreneur with zero proceeds from hedging when the firm invests in the good project are:

\[
R_f(s_H) = \begin{cases} 
P_G[R - R_m - s_H R_u] & \text{if solvent in state } H \\
0 & \text{if default in state } H 
\end{cases}
\]

(3.7)

\[
R_f(s_L) = \begin{cases} 
P_G[R - R_m - s_L R_u] & \text{if solvent in state } L \\
0 & \text{if default in state } L 
\end{cases}
\]
3.3 Firm’s Expected Profits

At date 0, a firm with initial capital $A$ and investment $I$ chooses the currency composition of its debt ($I_m$ and $I_u$), creditors’ payments ($R_m$ and $R_u$), its hedging amount ($h$) and its expected payments $R_f(s_L)$ and $R_f(s_H)$ to maximize expected total profits:

$$E[\Pi_{TOT}] = P_g \{(1-q)1_L[R - R_m - s_L R_u] + q1_H[R - R_m - s_H R_u] - \phi| h |\} + r_f(I - I_m - s_L I_u) \quad (3.8)$$

subject to resource constraints, incentive and participation constraints and non-negativity constraints (e.g. limited liability).

In setting up the problem in this way, it is assumed that the firm undertakes only good projects with probability of success $P_g$. This is going to be always the case because only good projects have a NPV>0 which means that if bad projects are undertaken no borrowing is obtained from creditors.

Depending on the exchange rate realization, when the firm is financially solvent, project returns $R$ and proceeds from hedging operations are distributed among all the parties according to the following resource constraints:

$$R_f(s_L) + R_m + s_L R_u = R + h(s_L - F) - \phi| h | \quad (3.9)$$

$$R_f(s_H) + R_m + s_H R_u = R + h(s_H - F) - \phi| h | \quad (3.10)$$

When the firm defaults in either state $H$ or $L$ it pays nothing to parties and the residual value of the firm goes to pay for bankruptcy costs.

Let $1_m$ be an indicator variable such that $1_m = 1$ if the firm borrows from the local bank (with monitoring) and $1_m = 0$ if the firm borrow directly from foreign banks and not from local banks (without monitoring). Given the two states of nature of the exchange rate, the firm invests in a good project whenever it obtains an expected payment greater or equal than the expected payment of a bad project including private benefits, in other words, when:

$$P_g[(1 - q)R_f(s_L) + qR_f(s_H)] \geq P_g[(1 - q)R_f(s_L) + qR_f(s_H)] + 1_m b + (1 - 1_m) b \quad (3.11)$$
This is the firm’s incentive constraint and can also be written as:

\[
(1 - q)R_I(s_L) + qR_I(s_H) \geq 1_m \frac{b}{\Delta p} + (1 - 1_m) \frac{B}{\Delta p}
\]

(3.12)

where \(\Delta p = P_G - P_B > 0\).

### 3.4 Domestic Bank Lending

A local bank monitors the firm and finances the project whenever it receives an expected payment sufficient to cover the fixed monitoring cost \(C\), then the bank’s incentive constraint is given by:

\[
[1_L(1 - q) + 1_H q]R_m \geq 1_m \frac{C}{\Delta p}
\]

(3.13)

On the other hand, the bank is willing to finance the project if it receives at least a net expected payment (net of monitoring costs) equal to the opportunity cost of its funds. The participation constraint for the bank is then given by:

\[
P_G[1_L(1 - q) + 1_H q]R_m - 1_m C \geq r^f I_m
\]

(3.14)

Let \(r\) denote the domestic lending rate that a local bank charges on \(I_m\) funds lent to the firm, defined as:

\[
r = \frac{P_G[1_L(1 - q) + 1_H q]R_m}{I_m}
\]

(3.15)

The minimum domestic rate of return \(r\) acceptable for a bank that decides to finance an investment project in domestic currency is determined by the condition,

\[
\frac{P_G C}{\Delta p} - C = r^f \frac{P_G C}{r\Delta p}
\]

(3.16)

This expression is obtained by combining (3.13) through (3.15), all holding with equality, and assuming that the bank lends to the project so that \(1_m = 1\). This condition, in turn, implies that

\[
r = r^f \frac{P_G}{P_B}
\]

(3.17)
This expression for the domestic lending rate states that the cost of domestic funds incorporates a risk premium relative to the risk-free rate. At the minimum rate of return acceptable for a bank this risk premium is equal to the ratio of success probabilities of the good and bad projects.

Local banks can borrow and lend internationally and are able to replicate a forward contract. Perfect competition and non-arbitrage conditions ensure that banks are indifferent between lending in domestic currency at \( r^f \), converting these funds into foreign currency at date 0 at the spot rate \( s_L \) and lending abroad in foreign currency where they earn \( r^* \), and converting these funds back to domestic currency at the forward rate \( F \). These transactions imply that:

\[
r^f = r^* \frac{F}{s_L} \quad (3.18)
\]

This is a covered interest rate parity condition and can be also expressed in terms of the domestic lending rate \( r \) and the international rate \( r^* \) as:

\[
r = \frac{r^* P_G}{P_B} \frac{F}{s_L} \quad (3.19)
\]

In this expression \( P_G > P_B \) and \( F > s_L \) so that the bank lending rate \( r \) is always greater than the international interest rate \( r^* \) and, therefore, foreign debt is always preferred to domestic debt. As a result, when they have to, firms want to borrow the least they can from a local bank and obtain the rest of their funds from foreign lenders.

The firm will be able to borrow in foreign currency if foreign banks are promised an expected payment greater than or equal to what they could obtain by investing their funds in international capital markets at the rate \( r^* \). The foreign bank participation constraint in foreign currency is then:

\[
P_G [1_L (1 - q) + 1_H q] R_u \geq r^* I_u \quad (3.20)
\]
3.5 Firm’s Profit Maximization

The firm’s problem is to find variables $h$, $I_m$, $I_u$, $R_m$, $R_u$, $R_f(s_l)$, $R_f(s_H)$ given exogenous parameters such as the firm’s initial assets $A$, its fixed investment $I$, the foreign rate of return $r^*$, the probability of currency depreciation $q$ and the cost of hedging $\phi$, in order to:

Maximize

$\max E \left[ \Pi_{\text{TOT}} \right] = P_G \{ (1-q)l_L[R - R_m - s_L R_u] + q l_H[R - R_m - s_H R_u] - \phi | h | \} + r^f (I - A - I_m - s_L I_u)$

subject to

$A + I_m + s_L I_u \geq I$ (1)

If firm is solvent:

$R_f(s_l) + R_m + s_L R_u = R + h (s_L - F) - \phi | h |$ (2)

$R_f(s_H) + R_m + s_H R_u = R + h (s_H - F) - \phi | h |$ (3)

$(1-q) R_f(s_l) + q R_f(s_H) \geq 1_m \frac{b}{\Delta p} + (1-1_m) \frac{B}{\Delta p}$ (4)

$[(1-q)l_L + l_H q] R_m \geq 1_m \frac{C}{\Delta p}$ (5)

$P_G [(1-q)l_L + l_H q] R_m - l_m C \geq r^f I_m$ (6)

$P_G [(1-q)l_L + l_H q] R_u \geq r^f I_u$ (7)

$R_f(s_H) \geq 0$ (8)

$R_f(s_l) \geq 0$ (9)

Conditions (1) through (3) are resource constraints. Conditions (4) and (5) are the incentive constraints of the firm and the domestic bank. Conditions (6) and (7) are the participation constraints for domestic and foreign banks. Conditions (8) and (9) are non-negativity constraints for the firm’s payments. Only one of these two non-negativity constraints will be binding when the firm defaults in one state of the exchange rate but is solvent in the other. Notice that there are 9 constraints to solve for 7 choice variables, which means that in equilibrium at least 7 constraints must be binding.

3.6 Equilibrium

The determination of the equilibrium as well as closed-form solutions for the currency composition of debt and optimal hedging are explained in detail in appendix 1 of the paper. As an
illustration, consider that both borrowing and hedging decisions are endogenously determined as optimal firms’ responses to a tradeoff between the lower cost of borrowing in foreign debt and the higher risk involved due to exchange rate uncertainty. As explained above, foreign currency debt is preferred to domestic debt so that the firm always tries to borrow directly from foreign banks. The smaller the size of its initial assets $A$, the more the firm demands from international banks. Since exchange rate uncertainty makes foreign debt risky, foreign banks demand a minimum net worth and hedging operations through currency forwards to ensure that the firm is solvent enough to repay its debt. If the firm’s initial net worth is not sufficient to meet the foreign bank’s requirement then the firm must be monitored and borrow from domestic banks first in order to be able to borrow from international markets. Local banks help firms undertake investments projects because domestic lending reduces the net worth requirement. Depending on the size of its net worth, the monitored firm may also need to hedge to demonstrate financial solvency.

Given that the internal rate of return is greater than the external rate on firm capital, entrepreneurs prefer to invest all their funds in the project. Therefore, constraint (1) in the firm’s problem will be binding in most cases. The only exception happens when a firm has to borrow only in domestic currency from local banks. Recall that local banks have to cover the fixed monitoring costs so that they lend all firms the same fixed amount in domestic currency. Therefore, when the firm has a net worth insufficient to borrow only in foreign currency but high enough to borrow the fixed amount in domestic currency and the sources of funds exceed the size of investment, then the firm has to invest its excess funds at the market rate $r^f$.

Another key feature of the model is that hedging decisions are perfectly observed by creditors. An immediate implication is that firms have incentives to hedge their exchange rate risk to reduce the probability of financial default. When affordable and useful, hedging increases profits by expanding the possibility of borrowing in foreign currency at a lower interest rate. The equilibrium level of hedging corresponds to any value within an optimal range defined by constraints (2), (3), (8) and (9) as shown in the appendix. Depending on exogenous parameters, in particular $q$ and $\phi$, the minimum level of hedging in this range can be positive or negative (negative hedging implies that the firm sells forward contracts).
As shown in what follows, intermediate values of $q$ and small enough $\phi$ make the minimum level of hedging positive so that a profit maximizing firm will always choose this minimum level. However, there will be situations when the firm optimally chooses not to hedge. To see this, let $\bar{q}$ be the probability of depreciation that makes the minimum level of hedging equal to zero and $\bar{\phi}$ be the transaction cost in forward markets above which, for any given level of positive $q$, forward contracts are too costly to provide insurance.\(^\text{12}\) When the probability of depreciation is too high, say $q > \bar{q}$, describing for example a situation of extreme exchange rate volatility, firms will find optimal to choose a zero level of hedging. On the other hand, when too costly, say $\phi > \bar{\phi}$, then even if available hedging is not affordable so that firms will also choose not to hedge.

A representative firm solves its profit maximization problem under two situations: when it hedges enough to avoid financial default and when it does not hedge at all.\(^\text{13}\) Given that hedging is a necessary but not a sufficient condition for financial solvency, a firm with sufficiently high net worth can borrow only small amounts of foreign currency debt and be financially solvent without hedging. Therefore, when the firm does not hedge at all, there are two additional possibilities: the firm is solvent even without hedging or the firm defaults if it is not hedged. Each of these situations exists for a monitored company borrowing in both currencies and for a company borrowing only in foreign currency. These multiple cases imply that creditors demand different net worth levels depending on the composition of credit and the firm’s hedging strategy.

Appendix 2 of the paper illustrates the solution to all these net worth requirements. For the purpose of analysis, a brief description of them is in order: upper bar net worth requirements refer to cases of firms borrowing directly from foreign markets while lower bar net worth requirements refer to cases of

\(^{12}\) It can be shown that $\bar{q} = 1 - \left( \frac{b}{\Delta p} \frac{s_H}{s_L} \right) / (R - \frac{C}{\Delta p})$ and $\bar{\phi} = s_H - F = (1 - q)(s_H - s_L)$ respectively.

\(^{13}\) Partly hedging is ruled out because in case of currency depreciation, this level of hedging is not sufficient to avoid default so that the firm is forced to default as if hedging was zero in the first place. With sufficiently small but positive costs of hedging the firm will prefer zero hedging to partly hedging.
firms being monitored and borrowing from domestic banks first. Accordingly, $A_H$ is the minimum net worth for an optimally hedged firm that borrows in both domestic and foreign currency. Similarly, $\overline{A}_H$ is the minimum level of assets required for an optimally hedged firm wanting to borrow only in foreign currency from international banks.

When the firm does not hedge and as a result it is not solvent in state $H$ then the minimum net worth requirements are $A_{NH}$ and $\overline{A}_{NH}$ for firms being monitored by local banks and for firms borrowing directly from foreign banks respectively. If the firm is unhedged but solvent in state $H$ then it faces a higher minimum net worth required by creditors: $A_{SNH}$ if monitored by local banks and $\overline{A}_{SNH}$ if it wants to borrow directly from foreign banks. 14 An unhedged but solvent firm faces the highest net worth requirements. On the other hand, hedging, when affordable, allows the firm to face the lowest net worth requirement. In equilibrium, depending on the size of the firm’s initial assets and its hedging strategy, there are different financing possibilities. To decide the optimal currency composition of its debt (e.g. how much to borrow from each source) and whether it should be hedged or not, the firm compares its initial assets $A$ with these cutoff levels.

4. Model results

*Equilibrium with costless hedging ($\varphi = 0$)*

Costless hedging represents a long run equilibrium in which currency forward markets are competitive and well-developed so that transaction costs are negligible. The next two results illustrate the optimal financing policies when the economy operates under a fixed exchange rate regime and under floating exchange rates as two separate steady state equilibria.

14 As explained in appendix 2, a second and more stringent criterion for unhedged and solvent firms is also considered. This is the case of firms that are solvent and expect to receive the maximum expected payment given by constraint (4) even in the depreciation state. The minimum net worth required to these firms is even higher: $A_S$ if they are monitored and $\overline{A}_S$ if they want to borrow only from international markets.
Lemma 1 In an economy with fixed exchange rates, that is when $q=0$, an optimal strategy for a firm is not to hedge its dollar debt.

Proof: See Appendix

This basic result explains why a fixed exchange rate biases the currency composition of debt for some firms towards foreign currency debt and eliminates incentives to operate in forward markets. By fixing the exchange rate, the government provides a form of public hedging or free risk management to the corporate sector by creating a perception of no foreign exchange risk.$^{15}$ Consequently, some firms that otherwise would not be able to borrow at all, or some others that should be monitored to have access to international capital markets, are now able to obtain foreign currency debt without constraints. This situation creates incentives for entrepreneurs to borrow extensively in foreign currency without hedging and maintain currency mismatches in their balance sheets. As predicted by the balance sheet approach of currency crises, in the event of unexpected and large currency depreciation the corporate sector in this economy would face widespread bankruptcy.

Lemma 2 In an economy with floating exchange rates when $0 < q < q$ in equilibrium the optimal strategy for a firm with net worth $A$ such that $\underline{A}_H < A < \underline{A}_{SNH}$ or $\overline{A}_H < A < \overline{A}_{SNH}$ is to hedge its dollar debt through currency forwards enough to avoid bankruptcy.

Proof: See Appendix

According to this result, as long as the probability of currency depreciation makes forward contracts a useful instrument to deal with exchange rate risk, hedging makes it easier for the firm to obtain funding from foreign banks at a lower cost because it reduces the required net worth. Without transaction costs in forward markets hedging is always preferred to not hedging and any firm that is not solvent in state $H$ has incentives to hedge enough to avoid financial default. This is the case when the firm

$^{15}$ With costless hedging and $q=0$ the cutoff thresholds of the net worth ratios are at their lowest value and the firm is indifferent about how much it hedges since $F= s_L$. 
uses a mixture of domestic and foreign debt or when it borrows only in foreign currency. Note also that hedging is beneficial only for firms that are not solvent in state H; firms with net worth above the minimum requirement for solvency in state H do not need to hedge.

The previous two equilibrium results when hedging is costless determine a particular ordering of net worth requirements and define an equilibrium segmentation of firms into different categories depending on their demand for bank loans and their hedging strategy, as shown in Figure 2. Well-capitalized firms with net worth $A > \overline{A}_H$ finance their investment directly in foreign currency from international banks. Poorly capitalized firms with $A < \underline{A}_H$ cannot invest at all since they have no access to any type of finance. In between, somewhat capitalized firms with $\underline{A}_H < A < \overline{A}_H$ can invest only to the extent that they are monitored and demand domestic bank loans. Firms in the monitoring region $[\underline{A}_H, \overline{A}_H]$ finance their investment with a mixture of domestic and foreign debt. As already mentioned, whether firms hedge their foreign debt or not also depends on the size of their initial asset. Well-capitalized firms need not hedge if $A > \overline{A}_{SNH}$ but must hedge if $A \in [\overline{A}_H, \overline{A}_{SNH}]$. Similarly, somewhat capitalized firms need not hedge if $A \in [\underline{A}_{SNH}, \overline{A}_H]$ but must hedge if $A \in [\underline{A}_H, \underline{A}_{SNH}]$.

A typical firm within the monitoring region uses a mixture of domestic and foreign currency debt to finance its investment. However, as mentioned above, there are firms with net worth $A < \overline{A}_H$ but $A + I_m > I$ that only demand bank loans in domestic currency and invest their excess funds outside the firm at the market rate. The minimum net worth requirement for these firms is $A_B$ as is given by:

$$A_B(r^*, q) = 1 - \frac{P_B}{r^*} \frac{s_L}{F} \frac{C}{\Delta p}$$

16 This happens when the monitoring cost $C$ is sufficiently high so that local banks lend relatively high amounts in domestic currency. In general, this does not have to be always the case. When domestic debt is small enough so that $A_B > \overline{A}_H$ then firms never borrow only in domestic currency because they have the possibility to borrow only in foreign currency at a lower cost.
The distribution of net worth requirements in the equilibrium segmentation implied by Lemma 2 and depicted in Figure 2 is:

\[ A_H < A_{NH} < A_{SNH} < A_B < \bar{A}_H < \bar{A}_{NH} < \bar{A}_{SNH} \]

Interestingly now, within the monitoring region there are firms borrowing in domestic and foreign currency, some of which must be hedged, and also firms borrowing only in domestic currency that need not hedge. Notice also that \( A_{NH} \) and \( \bar{A}_{NH} \) are irrelevant so they are not shown in the figure.

Lemma 1 and Lemma 2 are consistent with prior predictions in the literature, in particular with the government guarantees approach that emphasizes the incentives for firms to borrow extensively in foreign currency debt without hedging when the exchange rate is fixed. However, another interesting implication derived from Lemma 2 is that not all firms must hedge during a floating regime because some of them have enough net worth to be financially solvent even in the depreciation state.
Equilibrium with costly hedging $\varphi > 0$

In the previous analysis when $\varphi = 0$ net worth requirements $A_{NH}$ and $\overline{A}_{NH}$ are irrelevant because any firm is optimally hedged. The next result refers to the possibility that currency mismatches also arise during floating regimes because hedging activities even if available can be unaffordable for all firms when transaction costs in currency forward markets are sufficiently high.

**Proposition 1**: With costly hedging and a positive probability of depreciation the optimal strategy for a firm in the monitoring region, regarding debt composition and hedging, depend on $q$ and $\varphi$ such that:

a. When $q \geq \overline{q}$ the firm borrows the least it can in domestic currency from local banks and borrow the rest in foreign currency from foreign banks and does not hedge its foreign debt.

b. When $0 < q < \overline{q}$ and $0 < \varphi < (1-q)(s_H - s_L)$ the firm finds it optimal to borrow the least it can in domestic currency from local banks and the rest in foreign currency from foreign banks and:

   b1. If $\varphi$ is small enough then the firm with $A_{NH} < A < A_{SNH}$ will always hedge its foreign debt

   b2. If $\varphi$ is sufficiently high then only some firms with $A_{NH} < A < A_{SNH}$ hedge its foreign debt.

c. When $\varphi \geq (1-q)(s_H - s_L)$ the firm finds it optimal to borrow the least it can in domestic currency from local banks and the rest in foreign currency from foreign banks and does not hedge its foreign debt.

Proof: See Appendix

Figure 3 illustrates all different hedging possibilities implied by Proposition 3. Notice that the minimum net worth requirement to obtain credit in both domestic and foreign currency varies depending on parameters $q$ and $\varphi$. In the first case, extremely high probability of depreciation makes optimal hedging zero so that no firm participates in currency forward markets. A similar situation occurs in the third case when some firms have incentives to hedge but forward contracts are too expensive or inexistent. Although these two situations look similar in that firms do not hedge, they are different and
have different interpretations. Intuitively, higher values of both q and \( \phi \) increase the net worth requirement making it more difficult to borrow in both domestic and foreign currency. In the first situation (Case a), q is too high that the firm is forced to borrow only small amounts in foreign currency so that it can be solvent in state H without hedging. In the third case (Case c), however, a firm that is not solvent in state H is still able to borrow larger amounts of foreign debt even when it cannot afford to hedge. This latter firm has a currency mismatch in its balance sheet and would default if state H happens.

**Figure 3: Different Hedging Strategies for firms within the monitoring region**

- **Case a: q very high**
  - No credit
  - No need to hedge
  - \( \Delta_H \)
  - Net worth

- **Case b1: q moderate**
  - \( \phi \) very small
  - No credit
  - Hedged
  - \( \Delta_H \)
  - \( \Delta_{SNH} \)
  - Net worth

- **Case b2: q very small**
  - \( \phi \) moderate
  - No credit
  - Not hedged
  - Hedged
  - \( \Delta_{NH} \)
  - \( A^* \)
  - \( \Delta_{SNH} \)
  - Net worth

- **Case c: \( \phi \) very high**
  - No credit
  - Not hedged
  - No need to hedge
  - \( \Delta_{NH} \)
  - \( \Delta_{SNH} \)
  - Net worth
The second case in proposition 1 corresponds to intermediate values of \( q \) and \( \varphi \) which allow firms to hedge. When the cost of hedging is small enough any firm needing to hedge to be solvent will always hedge (case b1) and the cutoff level \( A_{NH} \) is irrelevant. However, as the cost of hedging increases some firms have incentives to borrow in both currencies but find it optimal not to hedge their foreign debt. Now \( A_{NH} \) becomes the relevant cutoff level defining the lower limit in the equilibrium segmentation. The proof of proposition 1 describes the cutoff level \( A^* \) below which firms do not hedge because hedging is too costly. These firms would default in the event of currency depreciation (case b2). This particular situation describes the case of economies of scale in hedging operations. Firms with small net worth tend to be highly leveraged and demand more foreign currency debt than larger firms and cannot cover this exposure because the level of hedging required is not affordable. In contrast, firms with higher net worth demand less foreign currency debt and require a lower level of hedging to manage their foreign exchange risk even when forward contracts are somewhat expensive.

The preceding analysis shows that the segmentation of firms depends on exogenous parameters, in particular, those describing the macroeconomic environment such as interest rates, exchange rates, probability of currency depreciation and costs of hedging. The next result illustrates how different exchange rate regimes and different stages of development in forward markets affect the allocation of domestic and foreign debt and hedging behavior.

**Proposition 2:** When the economy moves from fixed to floating exchange rates and in the floating regime

\[ 0 < q < \bar{q} \quad \text{and} \quad \varphi < (1 - q)(s_H - s_L), \text{ all else equal:} \]

i. Fewer firms obtain funding for their investment.

ii. Fewer firms finance their investment borrowing directly from foreign banks.

iii. Some firms borrowing directly from foreign banks during the fixed exchange rate increase their demand for domestic loans from local banks during the floating regime.

Proof: See Appendix
To illustrate these results, consider that in equilibrium, domestic currency borrowing \( I_m \) is a fixed amount and each firm in the economy demands the same minimum amount from local banks. The aggregate demand for domestic bank loans \( I_m \) is then:

\[
D_m(r^*, q) = \left[ F(\overline{A}_I(r^*, q)) - F(\underline{A}_I(r^*, q)) \right] I_m(r^*, q)
\]

where the individual demand \( I_m \) is written as a decreasing function of both \( r^* \) and \( q \). On the other hand, the aggregate demand for foreign currency loans is given by:

\[
D_u(r^*, q) = \int_{\underline{A}_I(r^*, q)}^{\overline{A}_I(r^*, q)} \left[ I - I_m(r^*, q) - A \right] dF(A) + \int_{\overline{A}_I(r^*, q)}^{\infty} \left[ I - A \right] dF(A)
\]

Since \( r^* \) and \( q \) are exogenous parameters and assuming perfect competition in the domestic banking system, the supply of domestic funds is perfectly elastic at the domestic lending rate \( r \) and the supply of foreign loans is perfectly elastic at the international interest rate \( r^* \), which means that \( D_m \) and \( D_u \) determine the aggregate amounts of domestic and foreign lending in equilibrium.

An increase in \( q \) brought by the collapse of the fixed exchange rate regime has an ambiguous effect on \( D_m \) because both cutoff levels \( \overline{A}_I(r^*, q) \) and \( \overline{A}_I(r^*, q) \) increase and there are two opposing effects. A first group of firms with insufficient net worth and those unable to hedge cannot borrow at all and are dropped so that \( D_m \) decreases. Some other firms turn to domestic banks and increase \( D_m \) because they have insufficient net worth or are unable to hedge and cannot borrow only in foreign currency.

The impact of \( q \) on \( D_u \) is ambiguous as well. Firms that were previously borrowing in foreign debt and turned to domestic debt reduce their demand for foreign currency debt so that \( D_u \) drops. However, firms with intermediate net worth and those able to hedge remain in the monitoring region and increase their demand for foreign debt because \( I_m \) decreases for them and, as a result, \( D_u \) increases. In sum, a number of firms borrow less in both currencies and some others borrow more in foreign currency.

The extent to which these changes in the population of firms affect the aggregate currency composition of debt depends on the distribution function \( F(A) \). Possible changes in the population of firms after the collapse of the fixed exchange rate regime are illustrated in the next figure.
Figure 4: Changes in the Equilibrium Segmentation: from fixed to floating exchange rates

Fixed

No Credit | Domestic and Foreign Debt | Only Foreign Debt
---|---|---
Not hedged | Not hedged | Not hedged

\[ \Delta_H \rightarrow \Delta_{SNH} \rightarrow \Delta_B \rightarrow \Delta_H \rightarrow \text{Net worth} \]

Floating

Hedged | Not hedged | Hedged | Not hedged
---|---|---|---

\[ \Delta_H \rightarrow \Delta_{SNH} \rightarrow \Delta_B \rightarrow \Delta_H \rightarrow \text{Net worth} \]

No Credit | Domestic and Foreign Debt | Only Foreign Debt
---|---|---

Figure 4 shows that the switch in the exchange rate regime shifts the monitoring region to the right, making it more difficult for firms to borrow in foreign currency. This figure illustrates changes in hedging behavior than can be significant if the increase in the probability of depreciation is relatively small. However, depending on how large \( q \) and \( \phi \) are, these changes could be small. For example, as \( q \) increases and approximates to \( \bar{q} \), the group of firms that hedge using currency forwards becomes smaller and can be very small if \( \phi \) is also sufficiently high as to prevent some firms from hedging.

4.1. The Brazilian Experience

As a final illustration, consider how the results of the model match up with the currency crisis in Brazil in early 1999. In light of the model results, the lack of major changes in the lending and hedging behavior of the corporate sector in Brazil can be the result of a moderate increase in the probability of
currency depreciation and the existence of somewhat costless hedging. After the floating regime is adopted, some firms borrow less in both domestic and foreign currency and some others borrow more in foreign currency. Moreover, a group of firms borrowing in foreign currency do not hedge either because they cannot afford to buy forward contracts or because their net worth is sufficiently high and they are solvent even in the event of currency depreciation. These changes in the population of firms can offset each other so that the currency composition of debt and hedging activities do not vary significantly across regimes.

Needless to say, there are various other aspects of the macroeconomic environment as well as firm-specific characteristics conditioning the currency composition of lending and the hedging behavior of firms in reality. Moreover, in contrast to what the model assumes, a currency crisis most likely affects the firms’ net worth so that the distribution of firms may not be constant across regimes. Furthermore, other parameters such as the probability of success and the investment payoff R are certainly different across firms and are also affected by the collapse of the exchange rate regime. How companies deal with higher foreign exchange risk definitely depend on changes in these variables, treated as invariant parameters in the model. For example, depending on specific characteristics such as export status, type of ownership or the existence of other sources of funding; firms can adopt hedging strategies other than the use of currency forwards. Nevertheless, the model developed in the paper suggests changes in the behavior of a representative firm and impacts on the population of firms that are broadly consistent with the empirical facts observed in recent currency crises in small open economies, and in particular, in Brazil during the period of 1996 to 2001.

5. Conclusions

This paper introduces hedging decisions in a model of optimal debt allocation at the firm level to understand the sources of currency mismatch in the balance sheet of the corporate sector of countries that recently faced a currency crises. The model explains why some firms with access to foreign currency debt hedge their exchange risk exposure and others do not, as an optimal response to appropriate incentives
given by the macroeconomic environment. Under fixed exchange rates firms borrow extensively in foreign currency and do not hedge because they have no incentives to do so given that government provides a type of free risk management. Under a floating regime, when the probability of currency depreciation is moderate and hedging is affordable firms use currency forwards to hedge their exchange risk exposure and reduce the probability of financial default. Hedging complements net worth required by creditors allowing hedged firms to expand their capacity to access foreign capital markets.

Despite the obvious limitations of a partial equilibrium analysis and some simplifying assumptions, the model is able to provide an analytical framework to determine endogenously the currency composition of credit and the optimal level of hedging at the firm level. Consistent with the empirical evidence in Brazil during 1996 to 2001, the model predicts that with a moderate probability of currency depreciation and somewhat costless hedging, the changes in the population of firms after the economy adopts a floating regime can offset each other so that the currency composition of debt and hedging activities do not vary significantly across regimes.

While the currency composition of debt remained stable in Brazil after the collapse of the exchange rate regime, costless hedging operations seem to provide an effective vehicle to reduce foreign exposure without affecting significantly the aggregate levels of borrowing. A direct policy implication of the model is then the necessary emphasis that policymakers should give to the development of currency derivatives markets to help the corporate sector in smoothing the transition to a free-floating exchange rate regime.
REFERENCES


Appendix

1. Determination of Equilibrium

There are four possible cases in the profit maximization problem of a representative firm. The maximized objective function is different depending on:

i. $1_H=1$ and $1_L=1$ if the firm is solvent in both states
ii. $1_H=1$ and $1_L=0$ if the firm is solvent only in state H
iii. $1_H=0$ and $1_L=1$ if the firm is solvent only in state L
iv. $1_H=0$ and $1_L=0$ if the firm is insolvent in both states

Case (iv) is the uninteresting case and is ruled out because no lender will lend the firm any amount so that it cannot undertake the project. Moreover, given the features of the hedging contracts, firms being solvent only in one state of the exchange rate are firms that chose $h=0$. Therefore, expected profits for the remaining three cases are respectively:

i. $E[\Pi_{TOT}] = P_G [R - R_m - FR_u - \phi | h | ]$
ii. $E[\Pi_{TOT}] = P_G [q (R - R_m - s_H R_u)]$
iii. $E[\Pi_{TOT}] = P_G [(1 - q) (R - R_m - s_L R_u)]$

Case (ii) is also ruled out because it is always dominated by case (i) when the firm is solvent even if it chooses $h=0$. To see this, note that for any $0<q<1$ then $s_H > F$ and profits are greater in case (i) than in case (ii). Therefore, the relevant cases to evaluate are only (i) and (iii), which means that in equilibrium, the firm is always solvent in state L ($1_L=1$) and case (iii) of insolvency in state H happens only when the firm is not hedged at all.

Given that the firm is the residual claimant in debt contracts, it is straightforward to see that constraints (5) through (7) will be binding. This is the case because in order to maximize profits the firm chooses the minimum payments that make its creditors willing to participate financing the project.

A representative firm solves its maximization problem at date 0 using the following algorithm:

i. Suppose that given the lower cost of foreign debt the firm decides to borrow directly from foreign banks ($1_m=0$). Furthermore, suppose the firm judges it is financially solvent without hedging ($1_H=1$, $1_L=1$ and $h=0$). Conditions (5) and (6) jointly determine $I_m=0$ and $R_m=0$. Given $A$ and $I$, the firm finds $I_u$ using (1), which holds with equality, and then $R_u$ is determined by (7). With expected payments $R_m$ and $R_u$ already

32
pinned down, constraint (3) is used to verify if the firm is solvent in state H. If it is, then the firm can in fact borrow directly from foreign markets without hedging.

ii. When the firm is not solvent in state H and decides not to hedge then foreign creditors adjust their expected payment. \( R_u \) depends on \( q \) and is bigger to compensate for the possibility of default. Constraint (8) is binding so that \( R_f(s_{H}) = 0 \) (e.g. the residual value of the firm goes to pay for bankruptcy costs if state H occurs). Constraint (2) gives the firm’s own payment \( R_f(s_L) \). If constraint (4) is met then borrowing in foreign currency without hedging is feasible. Otherwise, the firm could still borrow directly from foreign markets but must use forward contracts to reduce the possibility of default in state H.

iii. If the firm hedges its foreign exchange risk via currency forwards then (2) and (3) combined with (8) and (9) determine an optimal range for hedging \([h, \bar{h}]\) given by:

\[
\frac{s_H R_u + R_m - R}{s_H - F - \varphi} \leq h \leq \frac{R - R_m - s_L R_u}{F - s_L + \varphi}
\]

Profit maximization implies that for any positive and small \( \varphi \) the firm chooses the minimum level to avoid default so that optimal hedging is \( h = \underline{h} \) with \( R_m = 0 \). As a result, constraint (8) is binding and \( R_f(s_{H}) = 0 \). Note that when \( \varphi = 0 \) there are multiple solutions for the optimal hedge because the firm is indifferent choosing any value within the range \([h, \bar{h}]\). If constraint (4) is met then borrowing in foreign currency and hedging is feasible. The firm determines whether hedging is optimal or not by comparing profits with the not hedging case. If constraint (4) is not met then the firm cannot borrow directly from foreign markets and must turn to domestic banks first.

iv. Suppose the firm borrows from domestic banks \((1_m = 1)\) and believes it is financially solvent without hedging \((1_H = 1, 1_L = 1 \text{ and } h = 0)\). As before, constraints (5) and (6) jointly determine \( I_m \) and \( R_m \). These two variables are now:

\[
R_m = \frac{C}{\Delta p (1 - q + 1_H q)} \quad \text{and} \quad I_m = \frac{P_B s_L C}{r * F \Delta p}
\]

---

17 Two cases of solvency in state H are considered. First, the firm is solvent if it receives the maximum expected payment in state H, that is \( R_f(s_{H}) = b/\Delta p \), even after paying its creditors (constraints (8) and (9) are both non binding). A less stringent second criterion for solvency is when the firm at least guarantees repayment in state H even though it is left with nothing so that \( R_f(s_{H}) = 0 \) (constraint (8) is binding but constraint (9) is non binding).
Notice that when the firm is solvent in state H, \( R_m \) does not depend on \( q \). Given \( I \) and \( A \), and having determined \( I_m \) then: if \( A + I_m > I \) then resource constraint (1) is not binding, the firm borrows in domestic currency only and invests excess funds \( I - A - I_m \) at the market rate \( r_f \). If \( A + I_m \leq I \) then constraint (1) is binding and the firms uses it to determine \( I_u \) so that the currency composition of debt is pinned down. As in steps (ii) and (iii) of this algorithm, the firm can borrow now in both currencies without hedging if it is solvent in state H or can face higher payments if it is not solvent in state H and does not hedge. Whether the firm must hedge or not depends on exogenous parameters (in particular \( q \) and \( \phi \)). The firm solves for its optimal hedging decision by comparing profits in each case. If the firm must hedge the optimal range for hedging is \([h, \overline{h}]\) with \( R_m > 0 \). As before, the minimum level \( h = h \) is chosen when \( \phi > 0 \) or any level within \([h, \overline{h}]\) when \( \phi = 0 \). Finally, if constraint (4) is not met even with a positive level of hedging then the firm is poorly capitalized (too small net worth) and will not be able to borrow at all.

2. Determination of firm’s minimum net worth requirements in equilibrium

To find the minimum net worth requirements under different hedging strategies suppose that instead of using its initial assets as an exogenous variable, the entrepreneur wants to determine the size of \( A \) necessary to maximize expected profits and meet all the constraints. Net worth \( A \) can then be treated as an additional endogenous variable that the entrepreneur finds in equilibrium and compare with her actual initial assets, renamed as \( A_0 \), to determine for example whether the firm is solvent without hedging, or whether the firm needs to borrow from local banks and be hedged.

I. The firm borrows a mixture of domestic and foreign currency debt \((1_m = 1)\) so that constraint (1) is binding. There are three different cases:

a. The firm is solvent in both states only if hedged \((1_H = 1 \text{ and } 1_L = 1)\). There are 9 constraints to solve for 8 unknowns \((h, I_m, I_u, R_m, R_u, R_f(s_L), R_f(s_H), A)\) so that one constraint is non-binding. Constraints (2) and (3) combined with (8) and (9) defines the range \([h, \overline{h}]\) for optimal hedging \( h \) and for any positive and small transaction cost \( \phi \), the firm chooses \( h = h \). This condition is equivalent to have constraint (8) binding and (9) non-binding. Therefore, the firm hedges enough to avoid default and it gets zero cash flow in the devaluation state. Having constraints (1) through (8) holding with equality, in equilibrium:

\[
R_f(s_L) = \frac{b}{\Delta p(1 - q)}, \quad R_f(s_H) = 0, \quad R_m = \frac{C}{\Delta p}, \quad I_m = \frac{P_B s_L C}{r * F \Delta p}
\]

34
\[
R_u = \frac{s_L}{F + \varphi} \left( R - \frac{b + C}{\Delta p} + \frac{b \varphi}{\Delta p (s_{H} - s_{L}) (1-q)} \right), \quad I_u = \frac{P_G}{r^*} \frac{s_L}{F} \left[ R - \frac{(b + c)}{\Delta p} \right]
\]

\[
h = \frac{1}{F + \varphi} \left[ R - \frac{C}{\Delta p} - \frac{b}{\Delta p (1-q) (s_{H} - s_{L})} \right]
\]

The minimum A required by foreign banks is:

\[
\Delta_H (r^*, q, \varphi) = I - \frac{P_B}{r^*} \frac{s_L}{F} \frac{C}{\Delta p} - \frac{P_G}{r^*} \left[ R - \frac{C}{\Delta p} - \frac{b}{\Delta p (1-q) (s_{H} - s_{L})} \right] \frac{s_L}{F + \varphi}
\]

and total expected profits when \( A = \Delta_H \) are:

\[
E [\Pi_{TOT}] = P_G \left( R - \frac{C}{\Delta p} - \frac{b}{\Delta p (1-q) (s_{H} - s_{L})} \right) \frac{s_L}{F + \varphi}
\]

b. The firm is solvent in both states even if it is not hedged: \((1_{H}=1, 1_{L}=1 \text{ and } h=0)\). This situation requires higher net worth A. The first criterion for solvency is that the firm is at least able to pay its debt even if it has to give up its own expected payment during the depreciation state, that is \( R_t (s_{H})=0 \). A firm will be able to invest if in return it expects to get higher payments during the non-depreciation state. Therefore constraints (4) and (9) are not binding and there are 7 binding constraints to solve for 7 unknowns \((I_m, I_u, R_m, R_u, R_f (s_{H}), R_f (s_{L}), A)\). The equilibrium solutions in this case are:

\[
R_f (s_{L}) = (R - \frac{C}{\Delta p}) \left( \frac{s_{H} - s_{L}}{s_{H}} \right), \quad R_f (s_{H}) = 0, \quad h=0, \quad R_m = \frac{C}{\Delta p}, \quad R_u = \frac{1}{s_{H}} (R - \frac{C}{\Delta p})
\]

\[
I_m = \frac{P_B}{r^*} \frac{s_L}{F} \frac{C}{\Delta p}, \quad I_u = \frac{1}{s_{H}} \frac{P_G}{r^*} \left[ R - \frac{C}{\Delta p} \right]
\]

The minimum A required by foreign banks is now:

\[
\Delta_{SNH} (r^*) = I - \frac{P_B}{r^*} \frac{s_L}{F} \frac{C}{\Delta p} - \frac{P_G}{r^*} \frac{s_L}{s_{H}} \left[ R - \frac{C}{\Delta p} \right]
\]

As can be seen, when \( h>0 \) then \( \Delta_H < \Delta_{SNH} \) which means that creditors would demand higher net worth compared to the case when firms must hedge to be solvent. Also the firm gets a higher payment in the good state, that is, \( R_f (s_{H}) > b/(\Delta p(1-q)) \), as expected. Total expected profits when \( A = \Delta_{SNH} \) are:

\[
E [\Pi_{TOT}] = P_G (1-q) (R - \frac{C}{\Delta p}) \left( \frac{s_{H} - s_{L}}{s_{H}} \right)
\]
c. The firm is solvent in both states even if it is not hedged \((1_H=1, 1_L=1\) and \(h=0)\) but in this case the firm still gets the maximum expected payment during the depreciation state, that is, \(R_f(s_H) = \frac{b}{\Delta p}\) and the firm gets higher payments during the non-depreciation state. Unlike the previous case, now the net worth required is greater than \(\Delta_{SNH}\). Constraints (4), (8) and (9) are not binding so that there are 6 binding constraints to solve for 6 unknowns \((I_m, I_u, R_m, R_u, R_f(s_L), A)\). The equilibrium solutions in this case are:

\[
R_f(s_L) = \left( R - \frac{C}{\Delta p} \right) \frac{s_H - s_L}{s_H} + \frac{1}{s_H} \frac{b}{\Delta p}, \quad R_f(s_H) = \frac{b}{\Delta p}, \quad h=0, \quad R_m = \frac{C}{\Delta p}, \quad R_u = \frac{1}{s_H} \frac{b}{\Delta p} - \frac{C}{\Delta p}
\]

The minimum \(A\) required by creditors is now:

\[
\Delta_S(r, q) = 1 - \frac{P_B s_L}{r^* F} \frac{C}{\Delta p} - \frac{P_G s_L}{r^* s_H} \left( R - \frac{b}{\Delta p} - \frac{C}{\Delta p} \right)
\]

As can be seen, \(\Delta_{SNH} < \Delta_S\) which means that this is an even higher net worth compared to the case when the firm is solvent but gets zero expected payment. As expected \(R_f(s_L) > \frac{b}{\Delta p(1-q)}\).

In all the previous cases, it is assumed that the monitoring technology is socially valuable. As a result \(\Delta_H < \Delta_H, \Delta_{NH} < \Delta_{NH}\) and \(\Delta_S < \Delta_S\). Total expected profits when \(A = \Delta_S\) are:

\[
E[\Pi_{TOT}] = P_G \left[ (1-q)(R - \frac{C}{\Delta p}) \frac{s_H - s_L}{s_H} + \frac{F}{s_H} \frac{b}{\Delta p} \right]
\]

d. The firm is not solvent if it is not hedged \((1_H=0, 1_L=1\) and \(h=0)\). Since the firm is not solvent in state \(H\) then (2) is ruled out and (8) is binding so that \(R_f(s_H)=0\). Moreover, (9) is non-binding so that \(R_f(s_L) > 0\). Therefore, there are 7 constraints to solve for 7 unknowns \((I_m, I_u, R_m, R_u, R_f(s_L), R_f(s_H), A)\). The equilibrium solution is:

\[
R_f(s_L) = \frac{b}{\Delta p(1-q)}, \quad R_f(s_H) = 0, \quad h=0, \quad R_m = \frac{C}{\Delta p(1-q)}, \quad I_m = \frac{P_B s_L}{r^* F} \frac{C}{\Delta p}, \quad I_u = (1-q) \frac{P_G}{r^*} \left[ R - \frac{(b+c)}{\Delta p} \right]
\]

\[18\text{ Monitoring is valuable when } C[P_G - \frac{s_L}{F} P_u] < P_G [B - b]\]
The minimum net worth requirement is:

\[ A_{NH}(r^*, q) = 1 - \frac{P_B}{r^*} \frac{s_L}{F} \frac{C}{\Delta p} - (1 - q) \frac{P_G}{r} \left[ R - \frac{C}{\Delta p(1 - q)} - \frac{b}{\Delta p(1 - q)} \right] \]

and total expected profits when \( A = A_{NH} \) are:

\[ E[\Pi_{TOT}] = P_G \frac{b}{\Delta p} \]

II. Firm borrows only in foreign currency from international banks \((1_m = 0)\). There are also three different cases as well:

a. The firm is solvent in both states if hedged \((1_H = 1 \text{ and } 1_L = 1)\). There are 7 constraints to solve for 6 unknowns \((h, I_u, R_u, R_f(s_L), R_f(s_H), A)\) so that one constraint is non-binding. As before, constraints (2) and (3) combined with \((8') \text{ and } (9')\) define the range \([h, \bar{h}]\) for optimal hedging \(h\). For any positive and small transaction cost \(\phi\), the firm chooses the minimum level to avoid default so that optimal hedging is \(h = \bar{h}\). This result implies that \((8)\) is binding and \((9)\) is non-binding. The firm hedges enough to avoid default and it gets zero cash flow in the devaluation state. In equilibrium:

\[ R_f(s_L) = \frac{B}{\Delta p(1 - q)}, \quad R_f(s_H) = 0, \quad R_m = 0, \quad I_m = 0, \quad I_u = \frac{P_G}{r^*} \frac{s_L}{F} \left[ R - \frac{B}{\Delta p} \right] \]

\[ R_u = \frac{s_L}{F + \phi} \left( R - \frac{b + C}{\Delta p} + \frac{b \phi}{\Delta p(s_H - s_L)(1 - q)} \right), \quad h = \frac{1}{F + \phi} \left[ R - \frac{B}{\Delta p(1 - q)(s_H - s_L)} \right] \]

The minimum \(A\) required by foreign banks is:

\[ \bar{A}_H(r^*, q, \phi) = 1 - \frac{P_G}{r^*} \left[ R - \frac{B}{\Delta p(1 - q)} \left( \frac{(1 - q)(s_H - s_L) - \phi}{s_H - s_L} \right) \right] \frac{s_L}{F + \phi} \]

and total expected profits when \( A = \bar{A}_H \) are:

\[ E[\Pi_{TOT}] = P_G \frac{B}{\Delta p} - \frac{P_G \phi}{F + \phi} \left[ R - \frac{B}{\Delta p(1 - q)} \frac{s_H}{(s_H - s_L)} \right] \]

b. The firm is solvent in both states even if it is not hedged: \((1_H = 1 \text{ and } 1_L = 1 \text{ and } h = 0)\). Intuitively again, this firm has high enough net worth (i.e. higher than the minimum \(A\) when it has to hedge to be solvent). As
in the case of $I_n=1$ now constraints (4) and (9) are not binding and there are 5 binding constraints to solve for 5 unknowns ($I_u$, $R_u$, $R_f(s_L)$, $R_f(s_H)$, $A$). The equilibrium solutions are:

$$R_f(s_L) = R\left(\frac{s_H - s_L}{s_H}\right), R_f(s_H) = 0, h=0, R_m = 0, R_u = \frac{1}{s_H} R,$$

$$I_m = 0, I_u = \frac{1}{s_H} \frac{P_G}{r} R$$

The minimum $A$ required by foreign banks is now:

$$\bar{A}_{SNH}(r^*) = 1 - \frac{P_G}{r} \frac{s_L}{s_H} R$$

and total expected profits when $A = \bar{A}_{SNH}$ are: $E[\Pi_{TOT}] = P_G \frac{B}{\Delta p}$

Note that when $h>0$ then $\bar{A}_H < \bar{A}_{SNH}$ which means that banks demand higher net worth compared to the case when firms must hedged to be solvent. Also, the firm gets a higher payment in the good state, that is, $R_f(s_L) > B/\Delta p(1-q)$, as expected.

c. The firm is solvent in both states even if it is not hedged as before ($I_n=1, I_L=1$ and $h=0$) but in this case the firm still gets the maximum expected payment during the depreciation state, that is $R_f(s_H) = b/\Delta p$ and the firm gets higher payments during the non-depreciation state. Unlike the previous case, now the net worth required is higher than $\bar{A}_{SNH}$. Constraints (4) and (8) and (9) are not binding and there are 4 binding constraints to solve for 4 unknowns ($I_u$, $R_u$, $R_f(s_L)$, $A$). The equilibrium solutions are:

$$R_f(s_L) = R\left(\frac{s_H - s_L}{s_H}\right) + \frac{1}{s_H} \frac{B}{\Delta p}, R_f(s_H) = \frac{B}{\Delta p}, h=0, R_m = 0, R_u = \frac{1}{s_H} (R - \frac{B}{\Delta p})$$

$$I_m = 0, I_u = \frac{1}{s_H} \frac{P_G}{r} [R - \frac{B}{\Delta p}]$$

The minimum A required by foreign banks is now:

$$\bar{A}_S(r^*) = 1 - \frac{P_G}{r} \frac{s_L}{s_H} [R - \frac{B}{\Delta p}]$$

As can be seen, $\bar{A}_{SNH} < \bar{A}_S$ and as expected $R_f(s_L) > B/\Delta p(1-q)]$. Total expected profits when $A = \bar{A}_S$ are:

$$E[\Pi_{TOT}] = P_G \left[ (1-q)R\left(\frac{s_H - s_L}{s_H}\right) + \frac{F}{s_H} \frac{B}{\Delta p}\right]$$
d. The firm is not solvent if it is not hedged \((1_H=0, 1_L=1\) and \(h=0\)). Constraint (2) is ruled out and constraint (8) is binding so that \(R_f (s_H)=0\). Constraint (9) is non-binding so that \(R_f (s_L) > 0\). Therefore, there are 5 constraints to solve for 5 unknowns \((I_m, R_m, R_f(s_L), R_f(s_H), A)\). The equilibrium solution is:

\[
R_f (s_L) = \frac{B}{\Delta p(1-q)}, \quad R_f (s_H) = 0, \quad h=0, \quad R_m = 0, \quad R_u = \frac{1}{s_L} \left( R - \frac{B}{\Delta p(1-q)} \right)
\]

\[I_m = 0, \quad I_u = (1-q) \frac{P_G}{r^*} \left[ R - \frac{B}{\Delta p(1-q)} \right]
\]

and the minimum net worth requirement is:

\[
\overline{A}_{NH}(r^*, q) = I - (1-q) \frac{P_G}{r^*} \left[ R - \frac{B}{\Delta p(1-q)} \right]
\]

Total expected profits when \(A = \overline{A}_{NH}\) are:

\[E \left[ \Pi_{TOT} \right] = P_G \frac{B}{\Delta p} \]

III. The firm borrows only in domestic currency from local banks: then \(R_u = I_u = 0\). It can be shown that the firm is indifferent between hedging and not hedging if \(\phi=0\). Hence, without loss of generality, it can be concluded that in this case the firm is always solvent and does not hedge because it does not need to. Constraints (8) and (9) are non-binding and constraint (7) is not relevant. Profit maximizing firms pay domestic banks just the least banks demand to participate so that constraints (5) and (6) are binding and the only possible non-binding constraint is (4). Therefore, by making constraint (1) bind to solve for the minimum net worth \(A\), there are 5 binding constraints to solve for 5 unknowns \((I_m, R_m, R_f(s_L), R_f(s_H), A)\). The equilibrium solution is given by:

\[
R_f (s_L) = R - \frac{C}{\Delta p}, \quad R_f (s_H) = R - \frac{C}{\Delta p}, \quad h=0, \quad R_m = \frac{C}{\Delta p}, \quad R_u = 0 \quad I_m = \frac{P_B s_L C}{r^* F \Delta p}, \quad I_u = 0
\]

The minimum net worth requirement is:

\[
A_B (r^*, q) = I - \frac{P_B s_L C}{r^* F \Delta p}
\]

and total expected profits when \(A = A_B\) are:

\[E \left[ \Pi_{TOT} \right] = P_G \left( R - \frac{C}{\Delta p} \right) + r^f \left( I - \frac{P_B s_L C}{r^* F \Delta p} \right)\]
3. Proof of Lemma 1:

Under fixed exchange rates: q=0. If \( \varphi = 0 \) then the net worth requirements for being hedged and being not hedged are equal, that is, \( A_H = A_{NH} < A_{SNH} \) and \( \widebar{A}_H = \widebar{A}_{NH} < \widebar{A}_{SNH} \). Firms with net worth \( A \) such that \( A \geq A_{SNH} \) or \( A \geq \widebar{A}_{SNH} \) need not hedge. Therefore, the relevant region of net worth requirement for which a firm could hedge are \( A_H < A < A_{SNH} \) if the firm borrow from local banks and \( \widebar{A}_H < A < \widebar{A}_{SNH} \) if the firm borrows directly in foreign currency. For any net worth \( A \) within these regions profits are given by:

\[
E[\Pi_{TOT}] = P_G \left[ (R - R_m - s_L R_u) - \varphi \right]
\]

When \( \varphi \geq 0 \), this expression is strictly lower if the firm is hedged given that \( q \leq \widebar{q} \) ensures a positive level of hedging when the firm decides to hedge. Therefore, not hedging is preferred to hedging for firms in the above regions. 

4. Proof of Lemma 2

When hedging is costless ( \( \varphi = 0 \) ) a hedging firm faces a lower net worth requirement relative to a firm that decides not to hedge, that is, \( A_H < A_{NH} \) and \( \widebar{A}_H < \widebar{A}_{NH} \). Consider a firm with net worth \( A_{NH} \leq A < A_{SNH} \) within the monitoring region having to choose between hedging or not. If the firm decides not to hedge then \( R_m = \frac{C}{\Delta p (1-q)} \) and \( R_u = \frac{r^* I_u}{P_G (1-q)} \), both of which are lower than \( R_m = \frac{C}{\Delta p} \) and \( R_u = \frac{r^* I_u}{P_G} \) respectively for any \( q > 0 \) when the firm hedges. Therefore, profits are given by:

\[
E[\Pi_{TOT}] = P_G \left( R - \frac{C}{\Delta p} - \frac{F}{P_G} r^* I_u \right) \text{ if hedged and}
\]

\[
E[\Pi_{TOT}] = P_G \left[ R(1-q) - \frac{C}{\Delta p} - \frac{s_L}{P_G} r^* I_u \right] \text{ if not hedged}
\]

Notice that a pair of similar expressions can be obtained for firms with \( \widebar{A}_{NH} \leq A < \widebar{A}_{SNH} \) borrowing directly from foreign banks (the only difference is that \( R_m = 0 \) so that \( C/\Delta p \) does not appear in
the profit expressions). Costless hedging implies that \( \text{Et} \left[ \Pi_{\text{TOT}}^H \right] > \text{Et} \left[ \Pi_{\text{TOT}}^\text{NH} \right] \) and hedging is preferred over not hedging, when not hedging implies insolvency in state H. In cases in which \( \underline{\Pi}_{\text{NH}} \geq \underline{\Pi}_{\text{SNH}} \) and \( \overline{\Pi}_{\text{NH}} \geq \overline{\Pi}_{\text{SNH}} \), net worth requirements \( \underline{\Pi}_{\text{NH}} \) and \( \overline{\Pi}_{\text{NH}} \) are irrelevant because any firm with \( A \geq \underline{\Pi}_{\text{NH}} \) or \( A \geq \overline{\Pi}_{\text{NH}} \) is solvent enough and need not hedge.

5. Proof of Proposition 1:

This proposition refers to monitored firms with \( A < \overline{\Pi}_H \). There are three different cases so that each will be separately proven.

Case (a): When \( q \geq \overline{q} \) then, \( \underline{\Pi}_{\text{SNH}} \leq \underline{\Pi}_H < \overline{\Pi}_{\text{NH}} \), and \( h \leq 0 \). If \( \varphi = 0 \) the firm could choose a negative level of hedging, that is, the firm could sell rather than buy currency forwards since hedging is a fair game and does not affect profits. If \( \varphi > 0 \) to maximize profits the firm must choose the minimum level of hedging and this cannot be negative so that optimal hedging is zero. Therefore, for any non-negative cost of hedging, zero hedging is optimal. An entrepreneur with net worth \( A \) such \( \underline{\Pi}_{\text{SNH}} \leq A \leq \underline{\Pi}_H \) cannot borrow at all because the firm’s expected payment \( q R_f(s_H) + (1-q) R_f(s_L) < b/\Delta p \). Therefore, the minimum net worth cutoff is \( \underline{\Pi}_H \).

Case (c): When \( \varphi \geq (1-q)(s_H - s_L) \) the firm also prefers zero hedging because a negative level would reduce expected total profits so that zero hedging is also optimal.

Case (b) When \( 0 < q < \overline{q} \) and \( \varphi < (1-q)(s_H - s_L) \) then \( \underline{\Pi}_H < \underline{\Pi}_{\text{SNH}} \) and \( h > 0 \). Two possible cases arise:

(b1) If \( q \) is sufficiently high such that \( \underline{\Pi}_{\text{NH}} \) is larger than \( A_{\text{SNH}} \) then any \( 0 < \varphi < (1-q)(s_H - s_L) \) guarantees that the optimal level of hedging is positive and equal to \( h \) since \( \text{Et} \left[ \Pi_{\text{TOT}}^H \right] > \text{Et} \left[ \Pi_{\text{TOT}}^\text{NH} \right] \) so that hedging is preferred over not hedging for firms with net worth \( A \) such that \( A < \underline{\Pi}_{\text{SNH}} \).

(b2) If \( q \) is sufficiently small such that both cutoff levels \( \underline{\Pi}_{\text{NH}} \) and \( \underline{\Pi}_H \) are both smaller than \( A_{\text{SNH}} \) then the hedging decision depends on whether \( \underline{\Pi}_H < \underline{\Pi}_{\text{NH}} \) or \( \underline{\Pi}_H > \underline{\Pi}_{\text{NH}} \). There exists a cost of hedging defined by \( \varphi^*(q) = \frac{FRq - (F - s_L)(R - b/\Delta p - c/\Delta p)}{(1-q)R - b/\Delta p - c/\Delta p - (s_L b/\Delta p)}/(s_H - F) \) which makes firms indifferent between
hedging and not hedging, that is, $A_H = A_{NH}$ and therefore $E_t[\Pi_{TOT}]^H = E_t[\Pi_{TOT}]^{NH}$. For this particular $q$ it is the case that $\varphi^*(q) < (1-q)(s_H - s_L)$.

i. When $\varphi < \varphi^*(q) < (1-q)(s_H - s_L)$ then $A_H < A_{NH}$ and $E_t[\Pi_{TOT}]^H > E_t[\Pi_{TOT}]^{NH}$ and hedging is preferred over not hedging when the firm has net worth $A$ such that $A_H < A < A_{SNH}$.

ii. When $\varphi^*(q) < \varphi < (1-q)(s_H - s_L)$ then $A_{NH} < A_H$ and $E_t[\Pi_{TOT}]^H < E_t[\Pi_{TOT}]^{NH}$. Moreover, given parameters $\varphi$ and $q$ there exists a net worth $A^*$ with $A_H < A^* < A_{SNH}$ that makes $E_t[\Pi_{TOT}]^H = E_t[\Pi_{TOT}]^{NH}$ such that:

- If $A_{NH} < A < A^*$ then $E_t[\Pi_{TOT}]^H < E_t[\Pi_{TOT}]^{NH}$ so that not hedging is preferred over hedging.
- If $A^* < A < A_{SNH}$ then $E_t[\Pi_{TOT}]^H > E_t[\Pi_{TOT}]^{NH}$ so that hedging is preferred over not hedging.

Since $A^* > A_H$, not any firm with $A > A_H$ will always prefer to hedge. However, in sum, in case (b) for any transaction cost $\varphi$ small enough (e.g. not prohibitive) any firm with $A > A_H$ will always prefer to hedge.

6. Proof of Proposition 2:

The results follow directly from the minimum net worth requirements defining the monitoring region $A_H$ and $\bar{A}_H$ (or $A_{NH}$ and $\bar{A}_{NH}$ if hedging costs are high but not prohibitive as to make some firms prefer to be unhedged). First, higher $q$ increases cutoff net worth defining the lower limit of the monitoring region so that poorly capitalized firms lose their funding implying that aggregate investment drops. Second, higher $q$ also increases the upper limit of the monitoring region so that some firms borrowing only in foreign currency from international banks during the fixed exchange rate regime lie within the monitoring region after $q$ increases. As a result, fewer firms are able to borrow directly from foreign banks and those moving to the monitoring region demand more domestic debt from local banks.