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The application of asymptotic solutions to characterising the process zone in almost complete frictionless contacts

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Abstract

A method is developed for characterising the nature of the plastic zone which develops along the boundary of any notionally complete frictionless contact but where, in practice, there is some small rounding. The approach consists of an outer asymptote, the solution for a semi-infinite square ended rigid punch, whose validity sets the upper limit to the load, and a nested inner asymptote, the solution for a semi-infinite rounded punch, which sets the lower limit to the load. The technique is applied, as an example, to a circular punch, and explicit values of the load given to ensure that the singular field characterises the local stress field to within a given degree of accuracy.

Keywords: Asymptotic analysis; Crack nucleation; Contact problem; Crack initiation

1. Introduction

Fatigue problems arise under conditions of normal loading when a notionally complete contact is pressed onto a relatively brittle material. Experiments to investigate the phenomenon have been carried out on bearing steels, using cylindrical square-ended indenters which give rise to local cracking from the contact boundary (Alfredsson and Olsson, 2000). The intention in the present paper is to provide a rigorous theoretical framework for quantification of the nucleation conditions for such cracks, and to permit

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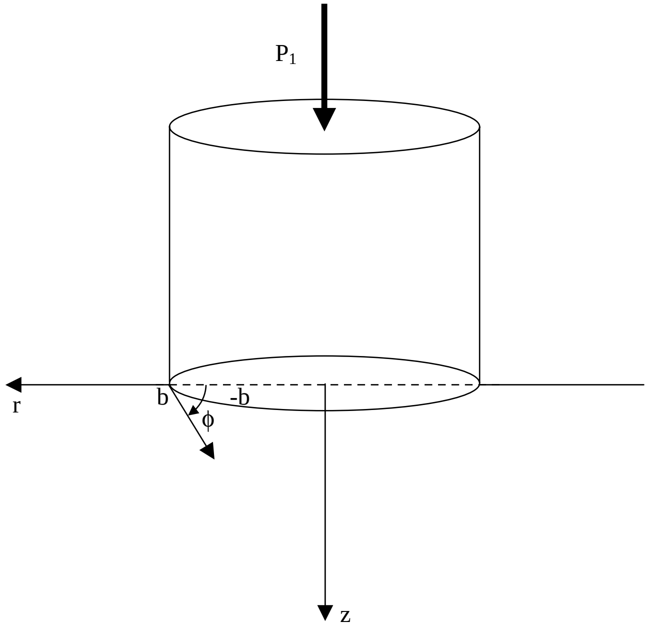

Fig. 1. Schematic of a rigid square-edged cylindrical punch pressed against an incompressible half plane.

the results of one test to be carried over to another rigorously, using a single parameter characterisation. The method may be applied to a punch of *any* cross section shape, provided that the end of punch is flat and parallel with the indented surface.

To illustrate the method, consider the problem of a rigid circular cylindrical punch of radius b, pressing normally into an elastic half-space, Fig. 1. The effects of interfacial friction will not be addressed, so that the solution will apply rigorously only when either (a) the interface is lubricated, or (b) the half-space is incompressible. Eliminating friction here means both that the conditions within the half-space will be dictated by the contact pressure distribution alone, and that a local square-root singularity in the contact pressure distribution will be present adjacent to the contact edge. It is assumed that crack nucleation will occur in a tiny process zone attached to the contact corner, and the characteristic stress/strain distributions within that zone which control nucleation are themselves controlled by the characteristics of an elastic hinterland. If, further, the hinterland is itself within a region in which the stress state is adequately described by the lead, singular term in an expansion of the contact pressure measured from the corner, it follows that the characteristics of the process zone are dictated solely be the intensity of that singular elastic field, and which may be thought of as a generalized stress intensity factor, K^*. This requirement is analogous to conditions of ‘small scale yielding’ in fracture mechanics (Rice, 1968).

If these ‘small process zone’ conditions are satisfied, the crack nucleation condition for a flat indenter of any shape could be determined by solving the corresponding elastic indentation problem and examining the predicted variation of K^* around the edge of the contact area. This linear elastic contact problem might, for example, be solved by traditional finite element methods, using square-root singular elements adjacent to the contact edge. Also, the critical value of K^* could be determined by a single experiment.

1.1. Slightly rounded indenters

In practice, there is always likely to be some slight rounding of the edge of the indenter due (for example) to the manufacturing process or to local yielding. From an engineering perspective, we would not expect this to influence the crack nucleation process if the rounding radius is sufficiently small, but this raises the question “How small does the radius need to be?”

Clearly, the rounding radius must be small compared with the linear dimensions of the nominally flat indenter, since otherwise there will not be a K^*-dominated region at the corner. However, the presence of rounding will influence the stress/strain state in its immediate neighbourhood and this would be expected to affect the nucleation criterion significantly unless the radius is small compared with the anticipated process zone.

This heuristic argument can be placed on a rigorous footing using the method of matching asymptotic expansions. The outer field comprises the elastic solution for the sharp-ended punch and defines the local square-root singular stress field characterized by K^*, whilst the inner field represents the modification of this local field by the finite rounding radius. On the scale of the inner field, the other linear dimensions of the

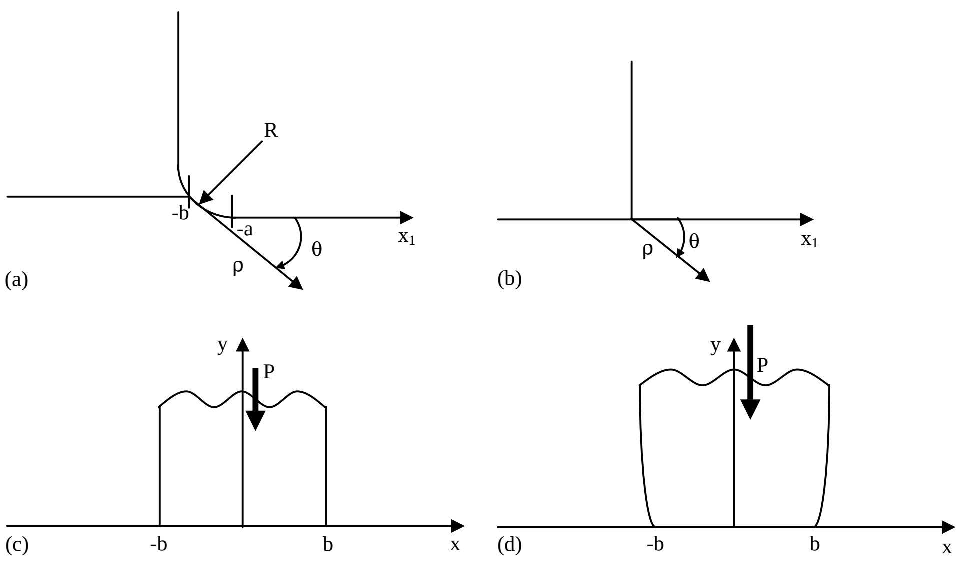

Fig. 2. Schematic of (a) a semi-infinite flat and rounded punch pressed against an elastically similar half plane, (b) a semi-infinite square-ended rigid punch pressed against an incompressible half plane, (c) a finite square ended rigid punch pressed against an incompressible half plane and (d) a finite flat and rounded punch pressed against an elastically similar half plane.

indenter appear infinite and hence the inner field is completely characterized by the solution of the semi-infinite rounded indentation problem of Fig. 2(a). In this problem, when the distance x from the corner is large ($x \gg R$, where R is the rounding radius), the contact pressure will vary with $x^{-1/2}$. The matching process is achieved by choosing parameters such that this inverse square-root field has the same multiplier as that on the singular asymptotic field at the corner in the full solution for the sharp indenter.

Once the solution of the problem of Fig. 2(a) is known, this matched asymptotic argument permits us to write down the solution for a finite flat indenter with rounded corners, provided (i) the solution of the corresponding sharp indenter problem is known and (ii) the rounding radius is sufficiently small for the local field to be completely surrounded by a K^*-dominated field. This is particularly useful for problems in which the geometry demands a numerical solution, since a numerical solution of the rounded indenter problem would be computer-intensive, due to the mesh refinement required at the edge of the contact region.

Also, a detailed examination of the asymptotic fields at the rounded edge will permit us to establish criteria determining when the solution of the sharp indentation problem alone is sufficient to characterize failure.

2. The asymptotic fields

Two asymptotic solutions are required for the general formulation which, once found, may be scaled and used to give universal results for the characteristics of the stress field local to the contact corner.

2.1. Plane square-edged semi-infinite punch

If the indenter has a sharp corner, the asymptotic field corresponds to the solution of the problem of Fig. 2(b) in which a frictionless semi-infinite square-ended flat punch is pressed into a half plane. There are several possible ways to obtain the solution to this classical problem, but probably the simplest is to start with the solution for a *finite*, square-ended rigid punch of half-width b (Fig. 2(c)), pressed into a half-plane by a normal force P, with no shearing tractions present. The contact pressure distribution is given by (Hills et al., 1993)

$$\frac{bp(x)}{P} = \frac{1}{\pi\sqrt{1-(x/b)^2}} \tag{1}$$

and the corresponding Muskhelishvili potential is

$$\frac{b\Phi(z)}{P} = \frac{\mathrm{i}}{2\pi\sqrt{z^2-1}}, \tag{2}$$

where $\mathrm{i} = \sqrt{-1}$, and z is the normalised complex coordinate $x/b + \mathrm{i}y/b$. By moving the origin to one corner and expanding by the binomial theorem it is trivial to show that the contact pressure varies as $1/\sqrt{\rho}$, where ρ is measured from the contact corner. Instead of using the multiplicative constant appropriate to the finite square-ended rigid punch, however, we choose to write the local pressure in the form

$$p(\rho) = \frac{K^*}{\sqrt{\rho}}, \tag{3}$$

where the generalised stress intensity factor, K^*, may be used to match this asymptotic solution outwardly to *any* finite punch solution exhibiting square root singular behaviour, and inwardly to the solution described in the next section. The corresponding Muskhelishvili potential is given by

$$\Phi(z_1) = \frac{K^*}{2\sqrt{\rho}} \exp(-\mathrm{i}\theta/2), \tag{4}$$

where z_1 is a complex coordinate measured from the contact corner, given in polar form by $z_1 = \rho \exp(\mathrm{i}\theta) \equiv x_1 + \mathrm{i}y_1$, and this may, of course, be used to determine the complete stress field, with the following outcome:

$$\sigma_{xx} = \frac{K^*}{8\sqrt{\rho}} \left[3\cos\left(\frac{\theta}{2}\right) + \cos\left(\frac{5\theta}{2}\right) \right],$$

$$\sigma_{yy} = \frac{K^*}{8\sqrt{\rho}} \left[5\cos\left(\frac{\theta}{2}\right) - \cos\left(\frac{5\theta}{2}\right) \right],$$

$$\sigma_{xy} = \frac{K^*}{8\sqrt{\rho}} \left[\sin\left(\frac{5\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right) \right]. \tag{5}$$

2.2. Plane semi-infinite flat and rounded punch

The solution to the problem depicted in Fig. 2(a) is required. It is the contact pressure distribution near the edge of a semi-infinite punch having a flat end, but with a radiused edge. The most straightforward way of doing this is to use the solution for the *finite* punch with radiused corners as the starting point, Fig. 2(d). A solution for this problem has been available in the Russian literature for some time (Steuerman, 1949), and an accessible solution in English was given by Ciavarella et al. (1998). There, it is shown that the contact pressure is

$$p(x) = -\frac{E^*}{2\pi R}\sqrt{b^2 - x^2} \left[\int_{-b}^{-a} \frac{(a+t)\,\mathrm{d}t}{\sqrt{b^2 - t^2}(t-x)} + \int_{a}^{b} \frac{(t-a)\,\mathrm{d}t}{\sqrt{b^2 - t^2}(t-x)} \right],$$

$$|x| < b, \tag{6}$$

where R is the radius at the ends of the punch, and E^* for the special case of a rigid body on elastic half plane is defined as $E/(1-\nu^2)$, where E is Young's Modulus and ν, Poisson's ratio. First, replace the dummy variable of integration, t, in the second integral by $-t$, so that

$$p(x) = -\frac{E^*}{2\pi R}\sqrt{b^2 - x^2} \int_{-b}^{-a} \frac{(a+t)2t\,\mathrm{d}t}{\sqrt{b^2 - t^2}(t-x)}, \qquad |x| < b. \tag{7}$$

To prepare for making the punch semi-infinite, move the origin to the corner $x=-b$, and introduce d, the length scale, by the substitutions

$$\xi = x+b, \quad s=t+b, \quad d=b-a, \tag{8}$$

so that

$$p(\xi) = -\frac{E^*}{2\pi R}\sqrt{\xi\left(1-\frac{\xi}{2b}\right)}\int_0^d \frac{(s-d)(s/b-1)\,\mathrm{d}s}{\sqrt{s(1-s/2b)}[\xi - s + (s^2-\xi^2)/2b]},$$

$$\xi > 0. \tag{9}$$

Now take the limit $b \to \infty$ to achieve the semi-infinite profile:

$$-\frac{2\pi R}{E^*}\,p(\xi) = \sqrt{\xi}\int_0^d \frac{(s-d)\,\mathrm{d}s}{\sqrt{s}(\xi - s)}, \quad \xi > 0. \tag{10}$$

The integral is regular within the interval $d < \xi < \infty$ but Cauchy in the interval $0 \leqslant \xi \leqslant d$, and has the value

$$-\frac{2\pi R}{E^*}\,p(\xi) = 2\sqrt{\xi d} + (\xi - d)\ln\left|\frac{\sqrt{d}-\sqrt{\xi}}{\sqrt{d}+\sqrt{\xi}}\right|, \quad \xi > 0. \tag{11}$$

We note that, when ξ is small

$$-\frac{2\pi R}{E^*}\,p(\xi) \simeq 4\sqrt{\xi d}, \quad 0 < \xi \ll d \tag{12}$$

and when ξ is large

$$-\frac{2\pi R}{E^*}\,p(\xi) \simeq \frac{4}{3}\sqrt{\frac{d^3}{\xi}}, \quad d \ll \xi. \tag{13}$$

Thus, the solution found in Eq. (11) has the required behaviour of being square root singular remote from the radiused end, but square root bounded as the edge of the contact is approached. Before proceeding further, we note that for the solution for large ξ (Eq. (13)) to be consistent with the outer asymptotic, (Eq. (3)), we require

$$-\frac{2\pi R}{E^*} = \frac{4\sqrt{d^3}}{3K^*} \tag{14}$$

and hence we may re-write the solution for the pressure in the form

$$\begin{aligned} p(\xi) &= \frac{3K^*}{4\sqrt{d^3}}\left[2\sqrt{\xi d} + (\xi - d)\ln\left|\frac{\sqrt{d}-\sqrt{\xi}}{\sqrt{d}+\sqrt{\xi}}\right|\right], \quad \xi > 0, \\ &= \frac{3K^*\sqrt{\xi}}{d}, \quad 0 < \xi \ll d, \\ &= \frac{K^*}{\sqrt{\xi}}, \quad d \ll \xi. \end{aligned} \tag{15}$$

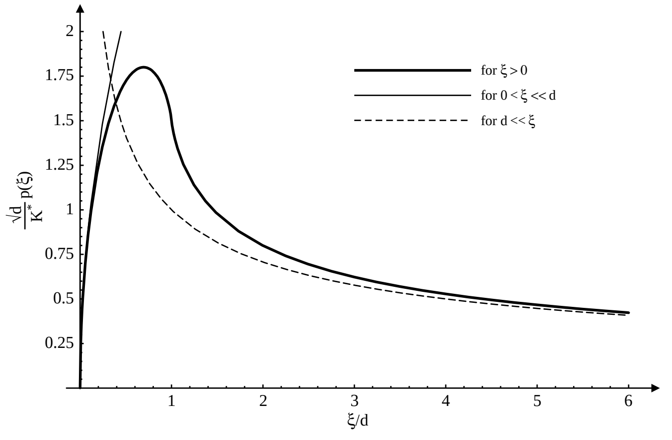

Fig. 3. Plot showing the normalised contact pressure distributions, $p(\xi)\sqrt{d}/K^*$, for a semi-infinite flat and rounded punch (thick solid curve), and its asymptotic behaviour close to (solid thin curve) and remote from (solid dashed curve) the contact edge.

The pressure distribution and its asymptotes are displayed in Fig. 3. The maximum contact pressure occurs at $\xi/d \simeq 0.7$ and is

$$p_{\max} \simeq \frac{1.8K^*}{\sqrt{d}}. \tag{16}$$

The Muskhelishvili potential for the *finite* flat and rounded punch problem is defined, in the usual way (Hills et al., 1993), by

$$\Phi(z) = \frac{1}{2\pi i}\int_{-b}^{b}\frac{p(x)\,dx}{x-z}. \tag{17}$$

By using the same transformation of coordinates as before, Eq. (8), we see that this may be re-written as

$$\Phi(z_1) = \frac{1}{2\pi i}\lim_{b\to\infty}\int_{0}^{b}\frac{p(\xi)\,d\xi}{\xi - z_1}. \tag{18}$$

Substituting for the contact pressure defined by the first of Eqs. (15) gives

$$\Phi(z_1) = \frac{3K^*}{8\sqrt{d^3}}\left[2\sqrt{z_1 d} + (z_1 - d)\ln\left|\frac{\sqrt{z_1}-\sqrt{d}}{\sqrt{z_1}+\sqrt{d}}\right|\right], \tag{19}$$

enabling the stress field associated with the inner solution to be evaluated. It may be shown that, for $|z_1|/d \gg 1$ Eq. (19) tends to Eq. (4) so that the entire local stress fields are matched. It would clearly be possible to deduce the individual stress components explicitly, but they would be rather complicated, and it seems preferable, here, to deduce the stress state using complex arithmetic on a computer.

The results in this section are all presented in terms of the dimensionless length x_1/d, since this permits us to present a universal asymptotic solution applicable to a punch with any rounding radius R subject to any loading K^*. However, the contact length d is a dependent variable, which we can determine from Eq. (14) as

$$d = \sqrt[3]{\frac{9\pi^2 R^2 K^{*2}}{4E^{*2}}}. \tag{20}$$

All the quantities on the right-hand side are known, as they are properties of the material combination (E^*), the geometry (R, K^*), or the applied load (K^*). Eq. (20) can then be used to express the maximum contact pressure (Eq. (16)) in terms of the same independent variables as

$$p_{\max} \simeq 1.07 \left(\frac{K^{*2}E^*}{R}\right)^{1/3}. \tag{21}$$

3. Example: the square-ended circular punch

It is again emphasised that the matched asymptotic technique described in Section 1.1 can be applied to a slightly rounded complete contact having *any* planform, in which case the preliminary solution of the corresponding sharp punch problem will probably require a finite element solution. However, in the interests of simplicity, we shall demonstrate its application to the square-ended circular punch of Fig. 1. If the punch is of radius b, Fig. 1, the contact pressure distribution is given by Johnson (1985)

$$\frac{b^2 p(r)}{P_1} = \frac{1}{2\pi\sqrt{1-(r/b)^2}}, \tag{22}$$

where P_1 is the applied total load. The complete internal stress field will be needed for comparison with the outer asymptote in order to determine the conditions of 'small scale yielding'. A method for deriving the internal stress field may be found in Hills et al. (1993), and here only the results will be presented. The individual stress components are given by

$$\begin{aligned}
\frac{b^2\sigma_{zz}}{P_1} &= \frac{t^3}{2\pi(s^2+t^2)^3}(s^2t - 5s^2 - 3s^4 - t^2),\\
\frac{b^2\sigma_{rr}}{P_1} &= \frac{-s^2t}{2\pi}\left(\frac{s^2-3t^2+3s^2t^2-t^4}{(s^2+t^2)^3} - \frac{1}{(1+s^2)(s^2+t^2)}\right),\\
\frac{b^2\sigma_{\theta\theta}}{P_1} &= \frac{-2(1+v)}{2\pi}\,\frac{t}{(s^2+t^2)} - \frac{b^2\sigma_{rr}}{P_1} - \frac{b^2\sigma_{zz}}{P_1},\\
\frac{b^2\sigma_{rz}}{P_1} &= \frac{rst^2(t^2-3s^2)}{2\pi b(s^2+t^2)^3},
\end{aligned} \tag{23}$$

where the two parameters s and t are, in fact, oblate spheroidal coordinates (s,t):

$$z = bst, \qquad r^2 = b^2(1+s^2)(1-t^2), \tag{24}$$

so that

$$s^2 = \frac{1}{2b}[\sqrt{(r^2+z^2-b^2)^2+4z^2}+(r^2+z^2-b^2)],$$

$$t^2 = \frac{1}{2b}[\sqrt{(r^2+z^2-b^2)^2+4z^2}-(r^2+z^2-b^2)]. \tag{25}$$

Introducing a set of polar coordinates (ρ, ϕ) at the edge of contact $z = 0$, $r = b$ (Fig. 1):

$$z = \rho \sin\phi, \quad r = b + \rho\cos\phi, \tag{26}$$

we see that the stress components given by Eq. (23), when $\rho \ll b$ are given by

$$\frac{b\sigma_{zz}}{P_1} = \frac{1}{4\pi\sqrt{2b\rho}}(\sin\phi\cos(3\phi/2) - 2\sin(\phi/2)),$$

$$\frac{b\sigma_{rr}}{P_1} = \frac{1}{4\pi\sqrt{2b\rho}}(\sin\phi\cos(3\phi/2) + 2\sin(\phi/2)),$$

$$\frac{b\sigma_{\theta\theta}}{P_1} = -\frac{2v\sin(\phi/2)}{2\pi\sqrt{2b\rho}},$$

$$\frac{b\sigma_{rz}}{P_1} = -\frac{1}{4\pi\sqrt{2b\rho}}\sin\phi\sin(3\phi/2). \tag{27}$$

From these equations it can be shown that the generalised stress intensity factor, K^*, consistent with Eq. (19) takes the form

$$K^* = \frac{P_1}{2\pi b\sqrt{2b}}. \tag{28}$$

We can now immediately write down the extent of the contact area and the maximum contact pressure for the case where the punch has a small rounding radius R. From Eqs. (20) and (28), the width of the contact annulus in the rounded portion of the punch is

$$d = \sqrt[3]{\frac{9P_1^2R^2}{32E^{*2}b^3}} \tag{29}$$

and from Eqs. (21) and (27), the maximum contact pressure is

$$p_{\max} = 0.25\left(\frac{P_1^2E^*}{Rb^3}\right)^{1/3}. \tag{30}$$

The exact solution of this problem is given in Ciavarella (1999). In particular, the outer radius of the contact area $b = a + d$ is defined implicitly through the relation

$$P_1 = \frac{E^*}{3R}\left\{(4b^2-a^2)\sqrt{b^2-a^2} - 3b^2a\arccos\left(\frac{a}{b}\right)\right\}. \tag{31}$$

The exact and asymptotic solutions are compared in Fig. 4(a), which shows that the asymptotic method gives very good results even for d/b as large as about 0.1 (and recall that d will generally be significantly less that the rounding radius R). In Fig. 4(b),

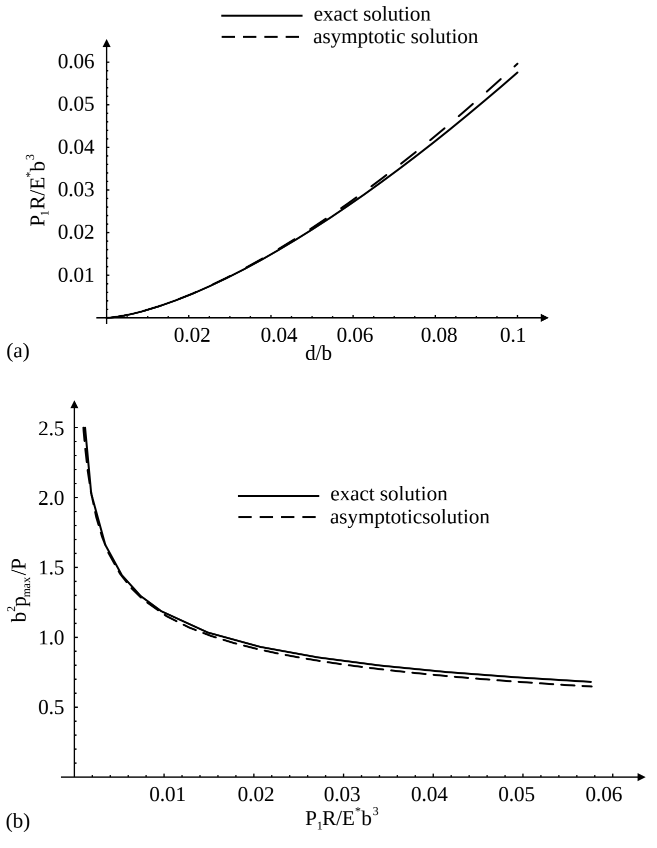

Fig. 4. (a) Plot of the dimensionless applied load, P_1R/E^*b^3, against the curved portion of the contact, d/b and (b) plot of the dimensionless maximum contact pressure, $b^2p_{\max}/P$, against the applied load, P_1R/E^*b^3.

we compare the maximum contact pressure from Eq. (30) with a numerical evaluation of the exact solution. Once again, the asymptotic prediction is extremely good up to $d/b \approx 0.1$, i.e. $(P_1^2R^2/E^{*2}b^6)^{1/3} \simeq 0.15$.

4. Effect of the applied load

The asymptotic fields for the sharp and the slightly rounded punch are fundamentally different, in that the former is singular at the contact edge for all loads, whereas the latter is everywhere bounded and indeed the contact pressure tends to zero at the edge of the contact. Thus, for the sharp punch, there will be a region in which the stresses exceed any given yield criterion for any applied load, whereas for the rounded punch, there will be a threshold load at which such a region starts to develop. However, as the load is increased, both 'failure-zones' will increase in size and will tend to become approximately co-extensive when their linear dimensions are large compared with the rounding radius. If the load is increased further, we shall reach a condition where the failure zone is not small compared with the other dimensions of the indenter, in which case there will be no K^*-dominated zone. For a given problem, we can therefore identify three load ranges:

(1) *Light loads*: At sufficiently light loads, the potential failure zone is significantly affected by rounding of the indenter. The matched asymptotic method can be used to determine the corresponding stress fields, but it is essential that the rounded asymptotic of Section 2.2 be used in any failure determination.

(2) *Intermediate loads*: In this range, the potential failure zone is large compared with the rounding radius and hence the asymptotic field for the sharp punch provides an adequate basis for assessing the probability of failure.

(3) *Large loads*: If the load is sufficiently large, the potential failure zone will become comparable with the linear dimensions of the punch and there will be no K^*-dominated field. In this range it is essential to use the full three-dimensional solution of the indentation problem to assess the probability of failure. However, in this process, the rounding of the punch can be neglected.

The procedure is clearly simplest in the intermediate load range, since then all we have to do is determine K^* from the linear elastic sharp punch solution and assess failure from the corresponding 'sharp' asymptotic field. In the following section, we shall determine the upper and lower bounds on the load for the applicability of this simple procedure.

4.1. Lower bound

The lower bound to the permissible applied load will be investigated first, because of its universal nature. The inner asymptotic elastic solution has been scaled to match the outer asymptote so that, remote from the contact corner, the two are equivalent. We are therefore in a position to compare the stress states implied by the outer and inner asymptotes, and to plot their divergence. Clearly, any component of the stress state could be used as a measure of this, but it seems appropriate to use the yield parameter, $\sqrt{J_2}$, as this is the key quantity controlling the process zone.

In Fig. 5, we plot the location of the plastic front implied by (i) the sharp asymptotic solution and (ii) the rounded asymptotic solution for various values of the dimensionless loading parameter

$$\Pi = \left(\frac{2E^* K^{*2}}{3\pi R k^3}\right)^{1/3}, \tag{32}$$

where k is the yield stress in pure shear. The figure is displayed for the case where the half plane is compressible, so that $\nu = 0.5$ and hence a purely hydrostatic state of stress exists at the surface. The figure is scaled with the length d of the contact zone in the rounded solution, since this permits a universal plot to be presented. We recall that d can be expressed in terms of independent geometric and loading parameters through Eq. (20).

Notice that there is some ambiguity in the relative lateral position of the two sets of contours, since we might (for example) identify the corner of the sharp punch with the

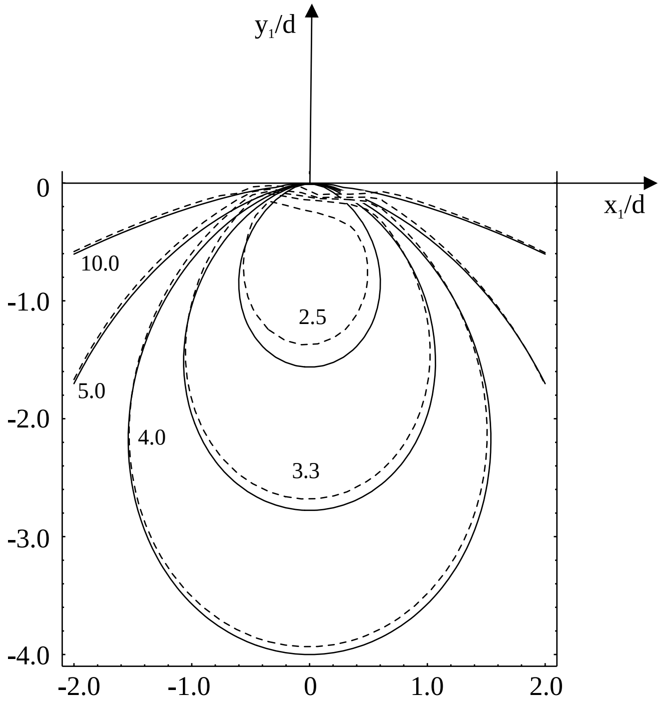

Fig. 5. Plot showing contours of the plastic yield fronts, $(2E^*K^{*2}/3\pi Rk^3)^{1/3}$, implied by the elastic asymptotic solutions of the semi-infinite flat and rounded punch (dashed curves) and the semi-infinite square-ended punch (solid curves).

end of the flat section of the rounded punch, or with some point between a and b in Fig. 2(a). Assuming the material to be homogeneous, failure may be influenced by the size and shape of the failure zone, but not by its lateral position. We have therefore plotted Fig. 5 so as to achieve the best fit at large values of the loading parameter.

A comparison of the contours in Fig. 5 shows that the sharp and rounded solutions are essentially identical for

$$\Pi > 3, \tag{33}$$

except for a small region near the corner of the punch. This criterion therefore establishes a lower bound on the loading parameter Π for the simple 'intermediate load' procedure to be used.

4.2. Small scale yielding (upper bound)

The upper bound to the permitted size of the plasticity (process) zone is set by the position of the plastic front at maximum load in the cycle. It is important that the plastic front be well within a region in which the elastic hinterland is controlled by the singular field. Thus, this restriction is dependent on the geometry of the problem,

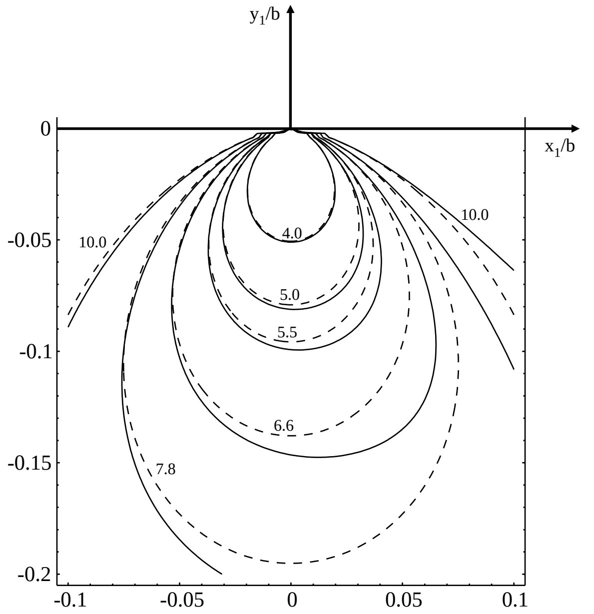

Fig. 6. Plot showing contours of the plastic yield fronts, P_1/b^2k, as implied by the full field solution for a circular square-ended rigid punch (solid curves) and the semi-infinite square-ended rigid punch solution (dashed contours).

and hinges on a comparison of the full field solution of the finite punch problem with the asymptotic singular field solution.

Fig. 6 shows contours of the plastic front for the example of Section 3 at various values of the loading parameter P_1/b^2k. The solid curves relate to the exact solution of the problem, whilst the dashed curves represent the asymptotic (K^*-dominated) solution. The results show that the contours are virtually indistinguishable for $P_1/b^2k < 4$, but progressively larger errors associated with the finite geometry of the system are incurred at larger loads. This establishes the upper bound load for K^*-dominance. The analogous procedure in crack problems is usually referred to as ‘small scale yielding’.

5. Conclusions

A straightforward procedure has been proposed for characterising the process zone present at the boundary of general almost complete contacts. A universal solution for the effects of rounding has been produced, permitting the local field due to rounding to be deduced from the simpler solution for the corresponding square-edged punch problem, provided that no shearing tractions arise along the interface. The method is illustrated by application to the indentation of a half-space by a flat and rounded cylindrical punch.

If the process zone is large compared with the rounding radius, rounding will have a negligible effect on failure, which can therefore be characterised using experiments on nominally sharp punches. A comparison of the sharp and rounded asymptotic stress fields enable us to establish a universal criterion for this simplification to be appropriate.

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