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THERMOELASTIC CONTACT OF A ROTATING SPHERE AND A HALF-SPACE

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Summary

An approximate solution is given for the thermoelastic contact problem in which a rotating sphere is loaded against a half-space. Frictional heating causes a redistribution of contact pressure away from the axis of rotation. The contact radius falls with increasing rotational speed, whilst for a given speed there is a limiting value which the radius cannot exceed, no matter how high the load.

Introduction

Solutions have recently been given to a number of thermoelastic contact problems in which the contact stress distribution is modified by the thermal strain due to frictional heat generation. In particular, it is found that, if two perfectly conforming solids slide over each other, there exist stable axisymmetric and two-dimensional configurations in which contact is restricted to a part of the apparent area [1, 2]. Burton and Nerliker [3] have also considered the case of sliding contact between a cylinder and a half-space, for which it is found that the width of the contact area is always smaller than that obtained under isothermal conditions and falls with increasing sliding speed.

In this paper, a related axisymmetric problem is considered in which a sphere is pressed against a plane surface and rotates about its axis of symmetry at constant speed. Previous solutions have been restricted to the limiting case in which the moving solid is a thermal insulator, in order to avoid the mathematical complexities associated with moving sources of heat. The spinning sphere conveniently avoids this problem since a rotating axisymmetric source of heat is thermally indistinguishable from the corresponding stationary source. The contact problem is further simplified by the fact that only circumferential displacement is produced by an axisymmetric torsional traction. Hence, the boundary value problem for determining the normal contact stress is uninfluenced by the frictional stress except *via* its effect on the rate of heat generation.

Statement of the problem

The rate of heat generation per unit area in the contact region is

$$q = q_1 + q_2 = \mu\, \Omega\, r\, p(r) \tag{1}$$

where μ is the coefficient of friction, Ω is the rotational speed and $p(r)$ is the contact pressure at a radius r. The division of heat between the two solids 1 and 2 is determined by considerations of symmetry and continuity of temperature such that

$$\frac{q_1}{K_1} = \frac{q_2}{K_2} \tag{2}$$

where K_1 and K_2 are the conductivities of the two solids.

A second condition to be satisfied within the contact region is

$$\frac{\mathrm{d}}{\mathrm{d}r}\,(u_1 + u_2) \equiv \frac{\mathrm{d}u}{\mathrm{d}r} = -\,\frac{r}{R} \tag{3}$$

where R is the radius of the sphere and u_1, u_2 are the normal surface displacements of the two solids. An indentation of the surface is regarded as a positive displacement.

Outside the contact region,

$$p = 0 \tag{4}$$

and it may be assumed that

$$q_1 = q_2 = 0 \tag{5}$$

i.e. there is no heat loss from the exposed surface.

It is convenient to regard the combined normal surface displacement u $(= u_1 + u_2)$ as the sum of two components: an elastic displacement u_e, which would be produced by the same pressure distribution under isothermal conditions, and a thermoelastic displacement u_t, which would be produced if the surfaces were heated but stress-free.

The thermoelastic displacement is readily found from the equation

$$\nabla_1^2\, u \equiv \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\alpha(1 + \nu)q}{K} \tag{6}$$

which relates the curvature of a stress-free initially plane surface to the local heat input per unit area [4, 5]. Equations (1), (2) and (6) can be combined to give

$$\nabla_1^2\, u_t = \frac{\{\alpha_1(1 + \nu_1) + \alpha_2(1 + \nu_2)\}\,\mu\,\Omega\, r\, p}{K_1 + K_2} \tag{7}$$

It can be shown that the thermoelastic contact area between two convex or plane solids must be simply connected and hence in the present example it must be a circle. The proof is derived from the argument of Section 9 of ref. 6.

Solution

To produce an approximate solution to this problem, following Burton [1 - 3] the pressure distribution $p(r)$ is represented as a finite series of terms and the boundary conditions are used to produce a set of simultaneous equations for the coefficients. The radius a of the contact circle is assumed to be given, and the corresponding total load P is found, notwithstanding the fact that P will usually be the independent variable in a physical situation.

In the limiting case where the rotational speed Ω is zero, the pressure distribution will be of the form

$$\left.\begin{aligned} p(r) &= c(a^2 - r^2)^{1/2} && 0 \leqslant r \leqslant a \\ &= 0 && r > a \end{aligned}\right\} \tag{8}$$

where c is a constant, and it is therefore reasonable to adopt this as the first term of the proposed series.

The total elastic displacement gradient due to this pressure distribution is given by

$$\left.\begin{aligned} \frac{\mathrm{d}u_e}{\mathrm{d}r} &= \frac{c(1-\nu)}{2G}\left\{\frac{a(r^2-a^2)^{1/2}}{r} - r \arcsin\left(\frac{a}{r}\right)\right\} && r > a \\ &= -\frac{\pi\, c(1-\nu)r}{4G} && a \geqslant r \geqslant 0 \end{aligned}\right\} \tag{9}$$

where

$$\frac{1-\nu}{G} \equiv \frac{1-\nu_1}{G_1} + \frac{1-\nu_2}{G_2} \tag{10}$$

and ν_1, ν_2, G_1 and G_2 are respectively Poisson's ratio and the modulus of rigidity for the two solids.

The thermal displacement gradient, obtained by substituting into eqn. (7) and integrating, is

$$\left.\begin{aligned} \frac{\mathrm{d}u_t}{\mathrm{d}r} &= \frac{\pi\, c\, \mu\, \Omega\, \alpha(1+\nu)a^4}{16\, Kr} && r > a \\ &= \frac{c\, \mu\, \Omega\, \alpha(1+\nu)}{8\, Kr}\left\{a^4 \arcsin\left(\frac{r}{a}\right) - r(a^2-2r^2)(a^2-r^2)^{1/2}\right\} && a \geqslant r \geqslant 0 \end{aligned}\right\} \tag{11}$$

where

$$\frac{\alpha(1+\nu)}{K} \equiv \frac{\alpha_1(1+\nu_1) + \alpha_2(1+\nu_2)}{K_1 + K_2} \tag{12}$$

A convenient way of representing a more general pressure distribution is to superimpose a series of terms of the same form as eqn. (8) but in which the radius of the loaded circle varies between 0 and a. For example, dividing the contact area into n equally spaced concentric rings gives the form

$$p(r) = \sum_{i=j}^{n} c_i \left(\frac{a^2 i^2}{n^2} - r^2\right)^{1/2} \tag{13}$$

where j is the smallest integer greater than nr/a. The total elastic and thermal displacements due to such a pressure distribution can be written down from eqns. (9) and (11).

To generate a set of simultaneous equations for the coefficients c_i, the condition (3) is applied at the n stations $r = ja/n$ ($j = 1, ..., n$), obtaining

$$\sum_{i=1}^{n} (a_{ji} + \beta a^2 b_{ji})\, x_i = -j \qquad \text{for } j = 1, 2, \ldots, n \tag{14}$$

where

$$\begin{aligned} a_{ji} &= -j \qquad i \geqslant j \\ &= \frac{2}{\pi}\left\{\frac{i(j^2 - i^2)^{1/2}}{j} - j \arcsin\left(\frac{i}{j}\right)\right\} \qquad j > i \\ b_{ji} &= \frac{1}{2\pi j n^2}\left\{ i^4 \arcsin\left(\frac{j}{i}\right) - j(i^2 - 2j^2)(i^2 - j^2)^{1/2}\right\} \quad i \geqslant j \\ &= \frac{i^4}{4jn^2} \qquad j > i \end{aligned} \tag{15}$$

$$x_i = \frac{\pi\, c_i R(1-\nu)}{4G} \tag{16}$$

$$\beta = \frac{\mu\, \Omega\, \alpha(1+\nu)G}{K(1-\nu)} \tag{17}$$

The total load corresponding to the pressure distribution of eqn. (13) is

$$P = \frac{8Ga^3}{3(1-\nu)R} \sum_{i=1}^{n} \frac{i^3 x_i}{n^3} \tag{18}$$

whilst the torque transmitted is

$$T = \frac{\pi G a^4}{2(1-\nu)R} \sum_{i=1}^{n} \frac{i^4 x_i}{n^4} \tag{19}$$

When $\beta = 0$, eqn. (14) has the trivial solution

$$x_i = \delta_{in} \tag{20}$$

and

$$P = P_{\mathrm{H}} = \frac{8Ga^3}{3(1-\nu)R} \tag{21}$$

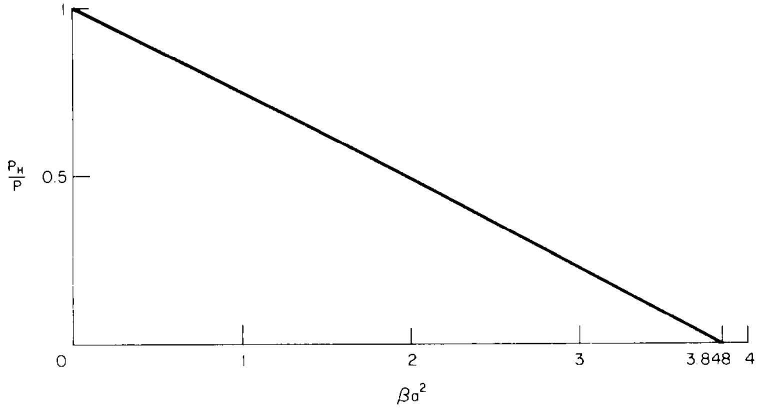

Fig. 1. The relationship between P_H/P and βa^2 determined from eqn. (14) using a series of 18 terms.

$$T = T_H = \frac{\pi G a^4}{2(1-\nu)R} \tag{22}$$

reduce to the corresponding isothermal values.

Results

Figure 1 shows the relationship between P_H/P — the reciprocal of the non-dimensional load — and the parameter βa^2. These results were obtained by solving eqn. (14) with a series of 18 terms. As β is increased (*e.g.* by increasing speed), the load required to establish a given size of contact area increases. Alternatively, if the load P is maintained constant, the contact radius falls with increasing speed, approaching the asymptotic expression

$$a = \left(\frac{3.848\ K(1-\nu)}{\mu\ \Omega\ \alpha(1+\nu)G}\right)^{1/2} \tag{23}$$

at very high speeds.

If load is increased at constant speed, this expression defines a constant limit which the contact radius approaches but cannot exceed. (In the corresponding isothermal case, the contact radius increases with load without limit.)

Equation (23) also defines the contact radius which will be obtained if both surfaces are initially plane, *i.e.* if the right-hand side of eqn. (3) tends to zero. Note that the contact area so obtained is independent of load, since the resulting equation is homogeneous. Increasing the load will merely cause a proportional increase of contact pressure throughout the contact area. It is observed from Fig. 1 that the relation between P_H/P and βa^2 can be closely approximated by a straight line between its end points, corresponding to the equation

$$\frac{8Ga^3}{3(1-\nu)PR} + \frac{\beta a^2}{3.848} - 1 = 0 \tag{24}$$

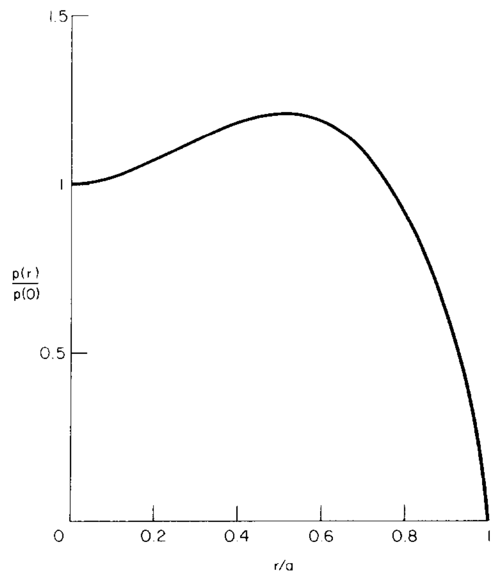

Fig. 2. Contact pressure distribution between two plane solids in rotating contact.

Figure 2 shows the contact pressure distribution obtained as the limiting value $\beta a^2 = 3.848$ is approached. As expected, the greater thermal expansion associated with the greater sliding speed near the periphery of the contact circle causes a shift of contact pressure outwards from the centre in comparison with the isothermal case. One effect of this redistribution of pressure is to increase the transmitted torque for a given load in comparison with the Hertzian torque for the same contact radius. However, this effect is small (<3%) and is completely dominated by the reduction in torque consequent upon the reduction of contact radius. At large speeds and loads, the torque tends asymptotically to

$$T = 1.189\, P \left(\frac{K(1-\nu)}{\mu\, \Omega\, \alpha(1+\nu)G} \right)^{1/2} \tag{25}$$

The concave punch

If eqn. (14) is solved for values of βa^2 in the range $3.848 < \beta a^2 < 8.714$, a solution is obtained in which the pressure distribution is everywhere negative. A physical interpretation can be given to this result by changing the sign of R in eqn. (3), in which case corresponding positive pressures are obtained. Thus the solution applies to the case where the punch is initially concave in the contact region. It would be necessary to preheat one or both solids to establish such a configuration, but frictional heating would then be able to sustain it. As before, the contact radius approaches the value given by eqn. (23) at high speeds and loads.

The contact pressure distribution becomes more pronouncedly non-Hertzian for the concave punch as load and speed are reduced and the contact stress required at the centre of the circle becomes tensile when

$$P^{2/3}\,\Omega < \frac{13.1\,K}{\mu\,\alpha(1+\nu)}\left(\frac{1-\nu}{G\,R^2}\right)^{1/3} \tag{26}$$

It seems probable that stable solutions with annular contact regions might exist at lower loads and speeds, but the present method is not well suited to treating this case. Note that the requirement that the contact area be simply connected (which was invoked to justify the assumption of a circular area) does not apply if the contacting solids are concave.

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