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THE REBOUND OF ELASTIC BODIES IN OBLIQUE IMPACT

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Introduction

The classical problem of rebound is usually treated by assuming that the colliding bodies are rigid and that the coefficient of friction, μ , is constant. Two contact states can be distinguished: sliding, in which the ratio of friction force to normal force is equal to μ , and rolling, in which the friction force is zero. If sliding gives way to rolling during impact, the friction force will fall discontinuously to zero.

This sudden change in tangential force is unlikely to occur in practice. The current work shows that no discontinuities in tangential force are predicted if the elasticity of the contact area is included in the analysis. This paper reports the close agreement observed between theoretical and experimental results.

Theory

We can improve upon the simple analysis by considering the elastic deformation of the bodies. It is well known that Hertz's theory of contact provides an adequate description of events in normal impact (1,2).

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A significant advance is made, however, when the contribution of tangential elasticity is included. Mindlin (3) has developed a theory of tangential compliance for two identical elastic spheres which are pressed together under a constant normal load. He showed that, for small relative tangential loads, an annulus of micro-slip is generated at the boundary of contact. As tangential load increases, the inner radius of this annulus progressively reduces until, when the critical value of friction force is reached, the surfaces break away in gross-slip. In a later paper, Mindlin and Deresiewicz (4) extended the original theory to cover cases involving more complex loading.

The authors (5,6) have used a similar approach in making a detailed study of elastic impact which discloses interesting deviations from the predictions of the simple theory. To ensure that the contact area is circular and that normal motion is not influenced by tangential effects, attention was restricted to the case of the impact, on an elastic half-space, of a body whose centre of mass is also the centre of curvature of a spherical contact surface. Simple examples of such a body are a sphere or a symmetrical slice from a sphere.

The predictions of this theory agree with those of the simple rigid body theory when gross-slip persists throughout the impact, but the occurrence of micro-slip blurs the boundary between the states of sliding and rolling and changes in friction force become continous. The interface behaves somewhat as a pair of mutually perpendicular non-linear springs which react independently against the body, except that the stiffness of the tangential 'spring' is influenced by the normal compliance. Tangential vibration, distinct from the half-cycle of normal vibration, is excited by the initial conditions and its frequency, characteristically, depends upon the mass distribution of the body.

In the interests of generality, a non-dimensional formulation has been used. The system is characterised by only one non-dimensional parameter

$$\chi = \frac{(1-\nu)(1+\frac{R^2}{k^2})}{(2-\nu)}$$

where R is the radius of the contact surface, k is the radius of gyration and ν is Poisson's ratio for the material. The only independent variable at input is the non-dimensional local angle of incidence, ψ_1, of the subsequent impacting region where

$$\psi = \frac{2(1-\nu)}{\mu(2-\nu)} \cdot \frac{Vx}{Vz_1},$$

Vx is the local tangential velocity of the body relative to the half-space and Vz_1 is the corresponding normal velocity of approach.

Figure 1 shows the relation between the local angles of reflection and incidence, ψ_2 and ψ_1, respectively for a body which is a thin diametral slice cut from a sphere. In this case χ = 1.2558 (ν = 0.28). In contrast to the prediction of simple theory, ψ_2, is negative for low values of ψ_1. Moreover, the range of ψ_1, over which gross-slip can be expected throughout the cycle, exceeds that from simple theory, since tangential elastic recovery of the surfaces can maintain

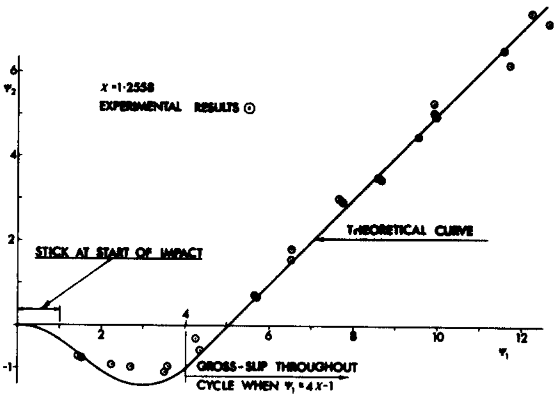

Figure 1. The non-dimensional local angle of reflection, ψ_2, as a function of the corresponding angle of incidence, ψ_1, for a homogeneous solid disc of Poisson's ratio 0.28.

relative motion even when the contact patch of the body has been brought to rest.

Experimental investigation

Figure 2 shows the essentials of the experimental apparatus. A puck, sliced from the centre of a hard steel ball, is propelled towards a well supported hard steel block, from which it subsequently rebounds. Virtually frictionless

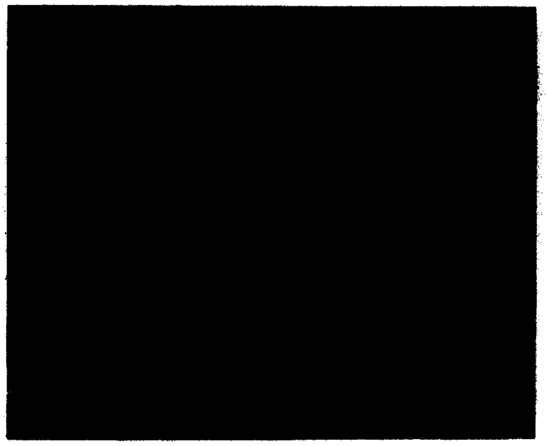

Figure 2. Experimental apparatus with the launching device showing the puck at the instant of impact.

Figure 3. The trajectory of the impact region of the puck before and after impact.

conditions are provided, during incidence and reflection by supporting the puck on an air film maintained by jets in the level base of the apparatus. A pendulum, incorporated into a heavy launching device, provides the initial impulse, whilst consistency of aim is ensured by locating the puck in jaws. The top surface of the puck is marked so that its position at the end of the equal time intervals can be photographed under stroboscopic lighting.

A series of tests was performed using different pucks. Conditions at impact were found to be essentially elastic; typical values of the coefficient of restitution being 0.93. By analysing the photographic recordings, local angles of incidence were calculated. However, a value for the coefficient of friction was required before these results could be compared with the theoretical predictions.

The coefficient of friction was found by conducting a series of tests in the entire gross-slip regime where the simple and more complex theory are indistinguishable. Under these conditions μ is given by the expression

$$\mu = \frac{(Vx_1 - Vx_2)}{2Vz_1\left(1 + \frac{R^2}{k^2}\right)}$$

where Vx_1 and Vx_2 are the initial and final local tangential velocities respectively. A mean value of 0.12 was obtained for μ and this was used for the results plotted in Figure 1.

Discussion

The results show a broad quantitative agreement between experiment and the prediction from elastic theory. Negative values of ψ_2 are abundantly represented, indicating that tangential elasticity is significant in impact when $\psi_1 < 4\chi$. This is visually demonstrated in Figure 3, where the impacting region of the puck is identified by a diamond shaped marker and the impact block by a broad band of reflective tape. The angle of reflection can be assessed by aligning the reflection path of the marker with the jet lattice.

The theoretical analysis is independent of the magnitude of the approach velocity and this is borne out by experimental results where the initial velocity was varied by a factor of 3. The coefficient of friction remains remarkably consistent, being within $\pm$ 5% of the mean value for the whole test series.

Conclusions

The major conclusions are:

i) Tangential elasticity, hitherto disregarded, exerts considerable influence at low angles of incidence. Error will be incurred if the impact response is determined from simple rigid-body theory. As an instance of this, for a disc, the tangential impulse, calculated from simple theory, is only 67% of that predicted by complex theory at $\psi_1 = 2.8$.

ii) The coefficient of friction, in impact between steel bodies, appears to be only slightly affected by wide variations in input conditions.

iii) The response of bodies in oblique impact, though due to highly localised interaction, appears to be closely predicted using the bulk properties of the colliding materials.

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