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Multiscale analysis of moving clusters of microcontacts

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ABSTRACT

Greenwood's approximation for the thermal resistance of a cluster of microcontacts is used recursively to estimate the thermal resistance due to a fractal array of circular contact areas motivated by Archard's contact model. The results are then extended to the case of sliding contacts, using a technique due to Burton. It is found that the total resistance converges on a limit when arbitrarily large numbers of fractal scales are included, but the fine scale features in the contact area have a disproportionate effect at high Peclet number and hence reduce the proportion of frictional heating passing into the moving body.

1. Introduction

When two bodies slide against each other, heat is generated at the interface due to frictional dissipation. The roughness of the surfaces causes this heating to be localized near regions of actual contact and leads to the development of high local temperatures known as 'flash temperatures'. Tribologists have long been interested in the prediction of flash temperatures, not least because they can have a profound effect on the chemical and physical processes taking place at the interface, and hence on the coefficient of friction and the wear rate [1–3].

The associated heat conduction problem depends very much on the nature of the interaction between the rough surfaces. If the material of one body is substantially harder than the other, it is expected that asperities on the hard surface will 'plough' through the surface layers of the softer, which latter will therefore see moving heat sources distributed over the moving actual contact areas. By contrast, the contact areas will be stationary with respect to the harder body. This asymmetry in the process leads to a corresponding asymmetry in the heat flow and particularly in the partition of the frictional heat between the sliding bodies. Jaeger [4] considered the problem of a heat source uniformly distributed over a circle of radius a that moves at speed V over the surface of a conducting half space. The average temperature of the heated circle is a strong function of the Peclet number, defined as

$$\mathrm{Pe} = \frac{Va}{2k}, \qquad (1)$$

where k is the thermal diffusivity of the material. The temperature falls with increasing Peclet number, since the heat source is continually being presented with new cold material as it moves over the surface. In the asymmetric process described above, this causes the softer body to have a substantially lower effective thermal resistance and hance biasses the partition of heat flow towards the softer body.

The problem is complicated by uncertainties about the size of the actual contact areas resulting from the roughness of the surfaces. For many years, this was addressed by approximating the surfaces as a distribution of isolated asperities whose effect was then estimated using the classical contact theory of Greenwood and Williamson [5]. However, the refinement of modern surface measurement techniques has led to the realization that surfaces are multiscale processes and it is not clear at what scale an asperity is appropriately defined. This issue is of particular importance for the frictional heating problem, since the radius of the contact area appears in the Peclet number, so the partition of heat as well as the flash temperature would be expected to be a strong function of the scale at which the asperities were defined. Furthermore, such actual contacts as occur will tend to be clustered near the peaks of the larger wavelength features in the surface, rather than being sparsely distributed, so interaction effects can be expected to be important.

In modern treatments of the contact of rough surfaces, these multiscale effects are generally handled by using a fractal description [6,7]. For elastic contact, both numerical and analytical treatments show that the total area of actual contact decreases without limit as additional scales of refinement are added to the fractal surface [7,8] and if engineering estimates of flash temperature are desired, we seem to be left with an essentially arbitrary choice

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Nomenclature

a	radius of contact area (m)
a_n	radius of actual contact area at scale n (m)
A	total actual contact area (m^2)
D	fractal dimension
f_n	dimensionless contact resistance at scale n
g	dimensionless contact resistance for a single contact area
h	amplitude of Weierstrass series (m)
k	thermal diffusivity (m^2/s)
K	thermal conductivity (W/m K)
N	number of contact areas in a cluster
P	normal contact force (N)
Pe	Peclet number
Q	total heat flow through the contact area (W)
R_n	radius of spherical asperity at scale n (m)
$\overline{T}$	mean contact area temperature (K)
V	sliding speed (m/s)

Greek letter

α	radius of clustered contact area (m)
γ	ratio of contact radius between adjacent scales
μ	coefficient of friction

Subscript

n	index of scale

as to where to truncate the process. However, in the analogous problem of the electrical contact resistance between stationary surfaces, it can be shown that the resistance does indeed have a finite limit under arbitrary degrees of refinement of scale, provided only that the corresponding surface roughness has a highest and lowest point [9]. This leads us to believe that a recursive treatment of the present fractal heat conduction problem should exhibit some similar limiting behaviour.

2. Some preliminary considerations

Tian and Kennedy [10] developed a convenient closed-form approximation for the steady-state mean temperature $\overline{T}$ at a uniformly heated circular contact area moving at speed V over the entire range of Peclet number. Their results can be expressed in the notation

$$\overline{T} = \frac{Qg(a)}{K}, \tag{2}$$

where K is the thermal conductivity, Q is the total heat flux through the contact circle and the function

$$g(a) = \frac{0.2191}{a\sqrt{(0.6575 + \mathrm{Pe})}} \tag{3}$$

defines a material-independent measure of the resistance to heat flow from the contact area into the body. It is important to note that the contact radius a appears implicitly in Pe as well as in the explicit multiplier. The expression tends to the exact limiting value

$$g(a) = \frac{8}{3\pi^2 a} \tag{4}$$

when V and hence Pe tend to zero.

2.1. Clustering

Suppose now that we have a set of N contact areas, each of radius a clustered in a circular region of radius α. Suppose further that the total heat flux through these contact areas is Q and that the flux is uniform – i.e. each individual contact area transmits a heat flux Q/N. Arguments of Holm [11] and Greenwood [12] for the electrical conduction problem suggest that the mean temperature for the stationary case ($V = 0$) can be obtained by summing the temperatures due to (i) a heat input Q/N uniformly distributed over a single circular area and (ii) a heat input Q distributed uniformly over the cluster area. This approximation works well if the total area of actual contact $N\pi a^2$ is significantly less than the cluster area $\pi\alpha^2$ and hence

$$\frac{\alpha}{a} \ll \sqrt{N}, \tag{5}$$

but it overestimates the resistance for more densely packed clusters. A better approximation can be obtained by including (iii) a 'constriction alleviation' term [13], such that (i) + (iii) represents the thermal resistance associated with the spreading of the flux Q from a circle of radius a into a concentric tube of radius $\alpha/\sqrt{N}$, corresponding to a cross-sectional area that is $1/N$th of the area of the cluster. This additional term can be approximated as the negative temperature due to the extraction of Q/N over a circle of radius $\alpha/\sqrt{N}$ [14].

Summing these three contributions and using the notation (2) and (3), we obtain

$$\overline{T} = \frac{Qg(a)}{NK} + \frac{Qg(\alpha)}{K} - \frac{Qg(\alpha/\sqrt{N})}{NK}. \tag{6}$$

For the stationary case, this reduces to

$$\overline{T} = \frac{8Q}{3\pi^2 K}\left(\frac{1}{Na} + \frac{1}{\alpha} - \frac{1}{\alpha\sqrt{N}}\right). \tag{7}$$

Notice that in the limit where the entire cluster is in actual contact, we have $N\pi a^2 = \pi\alpha^2$, so the first and third terms in Eqs. (6) and (7) cancel, and the thermal resistance is correctly given by the second (cluster) term alone. Thus, these expressions are exact at the two extremes of sparse contact and dense contact, and can be expected to be give reasonable though approximate predictions in the intermediate range.

2.2. Transient temperatures

Eqs. (6) and (7) define the mean temperature in the actual contact areas after sliding has progressed long enough for a steady thermal state to be established. The time taken to achieve such a state is of the order L^2/k, where L is the largest length scale defining the contact geometry, which might be the radius of the nominal contact area for a pin-on-disk machine, or in the present case, the radius of the cluster α. With typical material properties, this might give time scales in the range 1–100 s, and hence the steady state analysis is appropriate only in cases where contact areas can be predicted to persist, for example, due to a significant difference in hardness between the two materials. In situations where contacts are defined by a very short duration interaction between pairs of asperities on the opposing surfaces, a transient analysis is necessary [15,16].

3. Fractal geometries and the Archard process

Archard [17] defined a multiscale surface in an attempt to explain features of the relationship between contact resistance and contact pressure. Starting from the classical Hertzian contact of a sphere on a plane, he considered the effect of adding a set of smaller spherical asperities on a single larger sphere, and then a further set of even smaller asperities on these. Although the Hertzian analysis shows a non-linear relation between total contact area and applied load, Archard was able to show that the corresponding relation approached closer and closer to linearity as more fine scales were added. In retrospect, his contact model can be seen as a precursor of modern fractal descriptions of surface roughness [6,7,18].

In the spirit of Archard's model, we construct a fractal contact area by starting (at scale zero) with a single circular area of radius a_0. At scale 1, we then postulate that there exists a uniform distribution of N smaller areas of radius a_1, clustered into the original circle and at scale 2, each of these smaller areas is replaced by a cluster of N smaller circles of each radius a_2. The resulting set of contact areas is illustrated in Fig. 1.

The process can then be continued indefinitely. To maintain self-similarity, we enforce the condition

$$\frac{a_0}{a_1} = \frac{a_1}{a_2} = \cdots = \frac{a_n}{a_{n+1}} \equiv \gamma \quad \text{and hence} \quad a_k = \frac{a_0}{\gamma^k}. \tag{8}$$

Notice incidentally that the total contact area at scale n is

$$A_n = N^n \pi a_n^2 \tag{9}$$

and hence

$$\frac{A_n}{A_{n-1}} = \frac{N\pi a_n^2}{\pi a_{n-1}^2} = \frac{N}{\gamma^2}, \tag{10}$$

using Eq. (8). This quantity clearly cannot exceed unity.

Ciavarella and Demelio [19] have explored the nature of the contact areas developed for a given applied nominal pressure as additional scales are added in Archard's original model of the elastic contact problem. At low values of n, they find that the contact area is not a pure fractal (i.e. it is not self-affine), but at sufficiently large n it approximates a fractal and indeed the total contact area A_n obeys the recursive relation

$$\frac{A_n}{A_{n-1}} = \left(\frac{1}{\gamma}\right)^{D-1}, \tag{11}$$

where the fractal dimension D is defined such that

$$\frac{R_n}{R_{n-1}} = \gamma^D \tag{12}$$

and R_n is the radius of the spherical asperities at scale n. With this definition, D represents the fractal dimension of a two-dimensional section through the three-dimensional surface and hence must lie in the range $1 < D < 2$. Equating (10) and (11), we obtain

$$\gamma = N^{1/(3-D)} \tag{13}$$

implying that γ must lie in the range

$$\sqrt{N} < \gamma < N. \tag{14}$$

If the present fractal contact area is to represent a realistic physical approximation to the contact area for Archard's model.

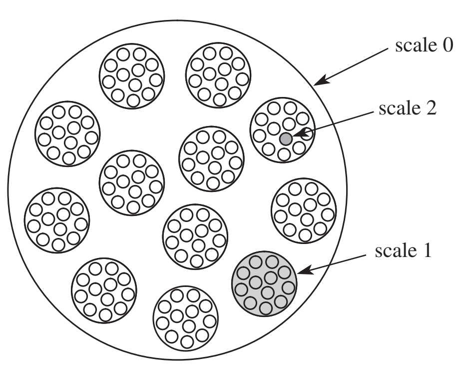

Fig. 1. Contact area defined by the 2-scale Archard process with $N = 12$, $\gamma = 5.24$.

3.1. Temperature calculation for the n-scale Archard process

In a physically realistic contact process involving rough surfaces, the individual contacts at a given scale will not be identical, but will depend on the location of the contact in the cluster. Furthermore, random variability in the contact geometry can be expected to increase thermal resistance relative to that with a regular array [20]. However, to permit an analytical approximation to the process, we here make the simplifying assumption that the total frictional heat flux Q is shared equally by the N^n contact areas at scale n. The mean temperature of the contact area for a single cluster ($n = 1$) can then be written down using Eq. (6). We have

$$\alpha = a_0; \quad a = \frac{a_0}{\gamma} \tag{15}$$

and hence

$$\overline{T} = \frac{Q}{K}\left(\frac{g(a_0/\gamma)}{N} + g(a_0) - \frac{g(a_0/\sqrt{N})}{N}\right). \tag{16}$$

Results for larger values of n can be developed by a recursive process. We know that the temperature is linear in Q and inversely proportional to K, so a general expression for the mean temperature at scale n might be written in the generic form

$$\overline{T} = \frac{Q}{K} f_n(N, \gamma, a_0), \tag{17}$$

where $f_n(N, \gamma, a_0)$ is an as yet unknown function. Notice incidentally that with this notation, we already know the functions for $n = 0, 1$, which are

$$f_0(N, \gamma, a_0) = g(a_0), \tag{18}$$

$$f_1(N, \gamma, a_0) = \frac{g(a_0/\gamma)}{N} + g(a_0) - \frac{g\left(a_0/\sqrt{N}\right)}{N}, \tag{19}$$

from Eqs. (2) and (16) respectively.

Suppose now that we have an $(n + 1)$-scale process and we focus attention on a single multiscale cluster of radius a_1. If this cluster existed in isolation, it would comprise an n-scale process through which the total heat flux is Q/N. Its mean temperature would then be given by Eq. (17) under the transformation $Q \to Q/N$, $a_0 \to a_0/\gamma$, or

$$\overline{T} = \frac{Q}{NK} f_n(N, \gamma, a_0/\gamma). \tag{20}$$

Now the interaction of the N such clusters of radius a_1 can be estimated using the same procedure as we used for the one-scale process above. All we need to do is to add the cluster resistance and subtract the alleviation term, each of which is identical to the corresponding terms in Eq. (16). We therefore obtain

$$\overline{T} = \frac{Q}{K}\left(\frac{f_n(N, \gamma, a_0/\gamma)}{N} + g(a_0) - \frac{g\left(a_0/\sqrt{N}\right)}{N}\right). \tag{21}$$

But since this is the mean temperature for the $n + 1$-scale process, we then have

$$f_{n+1}(N,\gamma,a_0)=\frac{f_n(N,\gamma,a_0/\gamma)}{N}+g(a_0)-\frac{g\left(a_0/\sqrt{N}\right)}{N}, \tag{22}$$

which defines a recurrence relation for the functions f_n. The reader can verify that this permits the expression (19) for $n = 1$ to be obtained from Eq. (18) and the process can clearly be extended to any desired scale n. In fact, by writing explicit expressions for the first few of these functions, it can be verified that

$$f_n(N,\gamma,a_0)=\sum_{j=0}^{n}\frac{g(a_0/\gamma^j)}{N^j}-\sum_{j=0}^{n-1}\frac{g\left(a_0/\gamma^j\sqrt{N}\right)}{N^{j+1}}. \tag{23}$$

3.2. The stationary solution

In the special case $V = 0$, the function g is given by the simpler expression (4) and the series in Eq. (23) can be summed in closed-form giving

$$f_n(N,\gamma,a_0)=\frac{8}{3\pi^2 a_0}\left\{\left[1-\left(\frac{\gamma}{N}\right)^{n+1}\right]-\frac{1}{\sqrt{N}}\left[1-\left(\frac{\gamma}{N}\right)^{n}\right]\right\}\Big/\left[1-\frac{\gamma}{N}\right]. \tag{24}$$

Fig. 2 shows the *normalized thermal resistance*

$$\frac{f_n}{f_0}=\frac{3\pi^2 K a_0 \overline{T}}{8Q}$$

for $N = 12$ and four values of γ determined from Eq. (13) with $D = 1.2$, 1.4, 1.6, 1.8 respectively. Notice that at modest values of the fractal dimension D, the result converges rapidly with refinement of scale. Even for $D = 1.8$ which would generally be reckoned as a very high value for a practical surface profile, the thermal resistance has essentially converged by $n = 6$.

The converged value can be obtained by allowing $n \to \infty$ in Eq. (24) and it implies that the mean temperature for the entire fractal series is

$$\overline{T}=\frac{8Q}{3\pi^2 K a_0}\left[1-\frac{1}{\sqrt{N}}\right]\Big/\left[1-\frac{\gamma}{N}\right]. \tag{25}$$

The stationary heat conduction problem is mathematically analogous with the electrical contact problem and hence this boundedness result could have been predicted from [9]. More precisely, the tightness of the bounds at scale n is defined by the cumulative amplitude of the residual surface roughness beyond that scale. We know that for the analogous Weierstrass series [7], this cumulative amplitude has the form

$$\frac{h_n}{1-\gamma^{D-2}},$$

where h_n is the amplitude of the nth wave. As the fractal dimension approaches its upper limit $D \to 2$, this expression increases without limit, indicating that progressively more scales need to be included to get a good description of the contact resistance when D is large. This conclusion agrees with the observations from Fig. 2 and from Eqs. (24) and (25).

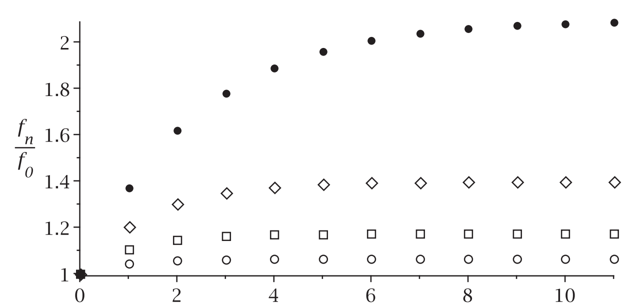

Fig. 2. Normalized thermal resistance of the contact area for a finite Archard process of order n, with $N = 12$ and fractal dimension $D = 1.2$ (○), 1.4 (□), 1.6 (◇) and 1.8 (●).

3.3. The parameters N and γ

The parameters N and γ are artifacts of the Archard process and have no intrinsic relation to the properties of real contacting fractal surfaces. Similar parameters of course appear in other descriptions of fractal geometry such as the Weierstrass series and the Cantor set [21,22]. However, N and γ are related through Eq. (13) to the fractal dimension of the profile D, which is a measurable property and hence only one of these parameters remains essentially arbitrary.

To examine the effect of this remaining parameter, we fixed the fractal dimension at a realistic value of $D = 1.5$, allowed N to take values between 5 and 20, and defined γ through Eq. (13). The results are shown in Fig. 3, which is plotted against the radius ratio a_0/a_n to clarify the relationship between the curves for different combinations of N, γ. In this format, it is clear that convergence with scale refinement occurs within a factor of about 1000 for all values of N, but the converged value, given by Eq. (25) depends upon N, albeit relatively weakly.

3.4. Effect of sliding speed

When the contact areas move over the surface of the half space, an additional parameter is introduced, which is conveniently taken to be the Peclet number at scale zero – i.e.

$$\mathrm{Pe}_0=\frac{Va_0}{2k}. \tag{26}$$

To give the reader some sense of typical magnitudes of Pe_0, we note that a contact circle of radius 1 mm sliding over a steel half space ($k \approx 10 \times 10^{-6}$ m^2/s) at 1 m/s would correspond to a Peclet number $\mathrm{Pe}_0 = 50$.

Some general comments about the mean temperature in the case of relative motion can be made with reference to the preceeding section. Each term in the series (23) is smaller than that for $V = 0$, so the thermal resistance is certainly lower than that produced with the same geometry under stationary conditions and hence in the case of two sliding conductors of similar conductivity, the partition of heat will still be biassed in favour of the body that moves relative to the contact area geometry. It also implies that the infinite series has a bounded sum. Indeed, the Peclet number at scale n

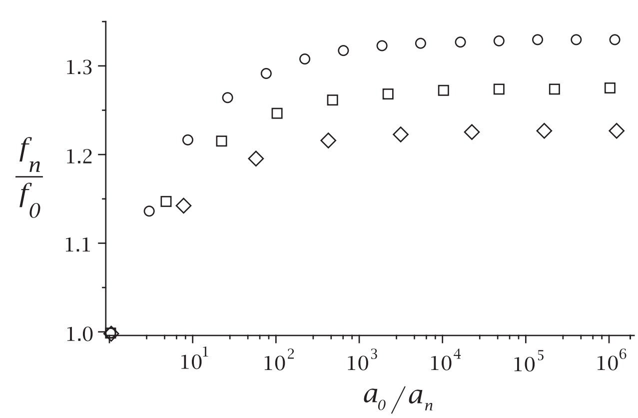

Fig. 3. Effect of the parameter N on the normalized thermal resistance. $N = 5$ (○), 10 (□), and 20 (◇).

$$\mathrm{Pe}_n = \frac{Va_n}{2k} = \frac{Va_0}{2k\gamma^n} = \frac{\mathrm{Pe}_0}{\gamma^n} \tag{27}$$

and hence at sufficiently large values of n, the additional thermal resistance due to each new scale of contact area refinement will be identical to that in the stationary case.

Fig. 4 shows the effect of scale refinement on the thermal resistance of the contact area for $\mathrm{Pe}_0 = 50$. These results are normalized with respect to the *stationary* thermal resistance at scale zero. The results exhibit essentially the same pattern as Fig. 2. In particular, convergence occurs in the first few scales except for the case $D = 1.8$.

Similar curves were obtained over a wide range of Peclet numbers ($\mathrm{Pe}_0 = 0 - 10^5$) and exhibited essentially similar characteristics, except that a few more scales were required for convergence at extremely high values. This can be explained by observing that the zeroth-order resistance is then extremely small, so the contribution at intermediate scales (where the Peclet number Pe_n is smaller) has a proportionately larger effect.

Fig. 5 shows the converged normalized resistance as a function of the zeroth-order Peclet number Pe_0 for $N = 12$ and $D = 1.5$. The dashed line in Fig. 5 represents the zeroth order resistance defined by Eqs. (2) and (3) with $a = a_0$. Notice that the effect of the additional scales of fractal roughness is approximately constant for Peclet numbers up to around 10. It follows that for $\mathrm{Pe} < 10$, a reasonable approximation to the entire fractal series can be obtained from the expression

$$\overline{T} = \frac{8Q}{3\pi^2 K a_0}\left\{\frac{\sqrt{0.6575}}{\sqrt{(0.6575 + \mathrm{Pe}_0)}} + \left(1 - \frac{1}{\sqrt{N}}\right)\Big/\left(1 - \frac{\gamma}{N}\right) - 1\right\}, \tag{28}$$

from Eqs. (3) and (25). This low Peclet number approximation is shown dotted in Fig. 5.

3.5. Implications for the partition of heat

In a practical problem involving two conducting bodies, the proportion of the total heat flux Q passing into the 'moving' body depends on the effective resistance of the two bodies. For example, if the mean temperature in the contact area for a given heat flux Q is $\overline{T}_1$ for body 1 and $\overline{T}_2$ for body 2, the proportion of the total heat flowing into body 1 will be defined by

$$\frac{Q_1}{Q_1 + Q_2} = \frac{\overline{T}_2}{\overline{T}_1 + \overline{T}_2}. \tag{29}$$

Here we implicitly adopt a frame of reference that is stationary with respect to the contact areas, so the moving body is that which sees a moving source of heat. If only a single contact area of radius a_0 is considered, these resistances are given by Eq. (3) for the moving body and Eq. (4) for the stationary body, each with $a = a_0$ and for typical tribological applications, Pe_0 is generally sufficiently high to ensure that most of the heat flows into the moving body. If we now add additional scales of fractal roughness, both effective resistances will increase, but generally the proportional increase in resistance is greater for the moving body, and this will have the effect of reducing the imbalance of heat flux.

Fig. 6 illustrates this effect for sliding of two bodies of equal conductivity, with $N = 12$, $D = 1.5$. The heat flux Q_1 into the stationary body as a proportion of the total heat flux $Q_1 + Q_2$ is plotted as a function of Pe_0. The prediction based only on the zeroth-order contact area is shown as a dashed line. When the Peclet number is small, the resistance of the moving body is close to that of the stationary body, the additional fine scale resistance has an equal effect on both bodies, and both calculations predict an equal partition of heat between the bodies. As the Peclet number increases, the proportion of heat flowing into the stationary body decreases, but this effect is reduced when the fractal roughness is taken into account.

3.6. A numerical example

To illustrate the effect of these results in a flash temperature calculation, we revisit the numerical example treated by Archard and Rowntree [2], based on their experimental observations of phase transformation due to frictional heating. The sliding bodies comprised a pair of crossed cylinder of carbon steel for which we take representative values $K = 47$ W/m K, $k = 13.27 \times 10^{-6}$ m^2/s and the contact area was estimated at $a_0 = 0.280$ mm from the width of the wear track and at $a_0 = 0.255$ mm from Hertzian contact calculations. The applied normal force was 250 N and the friction coefficient μ was measured to be in the range 0.34–0.39. Both cylinders were caused to rotate and translate so as to present

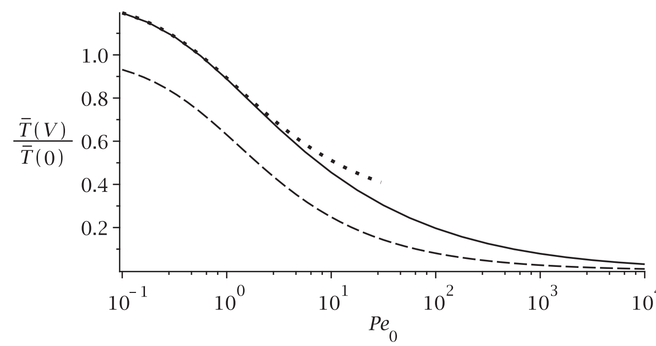

Fig. 5. Normalized mean temperature for the entire fractal series as a function of Pe_0 for the case $N = 12$, $D = 1.5$. The dashed line defines the first term in the series and the dotted line is given by Eq. (28).

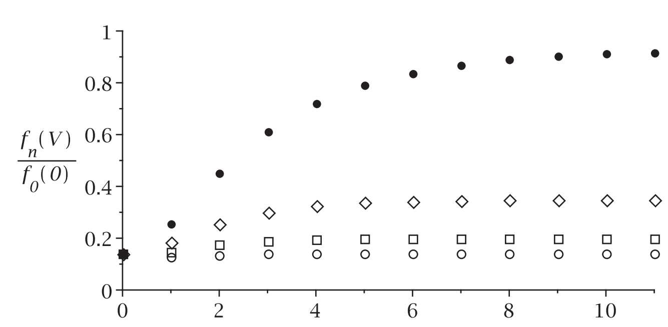

Fig. 4. Normalized thermal resistance of the contact area for a finite Archard process of order n, with $N = 12$, $\mathrm{Pe}_0 = 50$ and fractal dimension $D = 1.2$ (○), 1.4 (□), 1.6 (◇) and 1.8 (●).

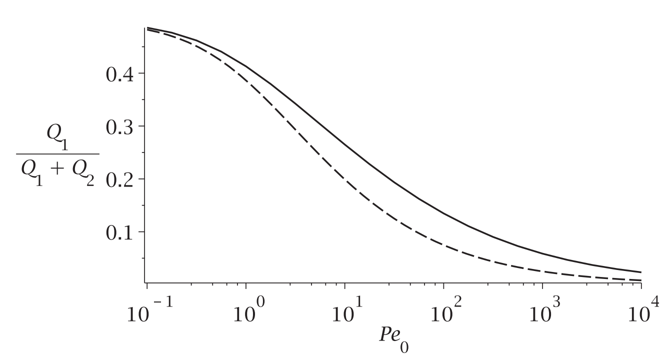

Fig. 6. Heat flux Q_1 into the stationary body as a proportion of the total heat flux $Q_1 + Q_2$. The dashed line shows the result obtained using only the zeroth-order resistances.

continually new sliding surfaces to the frictional process, the corresponding relative velocities being $V_1 = 0.23$ m/s, $V_2 = 4.6$ m/s. Since these two relative velocities are orthogonal, the resultant sliding velocity was $V = \sqrt{V_1^2 + V_2^2} = 4.606$ m/s. Archard and Rowntree used a high Peclet number approximation to Eq. (3) for the faster moving body 2 and the stationary value (4) for the slower and in fact this latter approximation is quite severe, because the Peclet number associated with V_1 is 2.42 (using $a_0 = 0.280$ mm) at which Fig. 5 shows that the thermal resistance is less than half that of a stationary body. The division of heat based on the nominal contact area is found from Eq. (29), using Eqs. (2) and (3) or the dashed line in Fig. 5 for the resistances (with Pe = 48.5 for body 2). We conclude that 20% of the total heat flux flows into the slower moving body 1 and the mean temperature of the nominal contact area is then obtained as 773 °C, using $Q_1 + Q_2 = \mu PV$ and adding the measured 'bulk' temperature of 30 °C reported in [2]. If instead we use the values $a_0 = 0.255$ mm and $\mu = 0.39$, the partition is essentially unchanged, but the mean temperature is increased to 1008 °C. Archard and Rowntree predicted a smaller proportion (15%) to body 1 and a significantly higher mean temperature, which is a consequence of their assumption of the stationary resistance for this body.

If we now use the multiscale analysis with $N = 12$ and $D = 1.5$, and in particular Eqs. (20), (23) with n sufficiently large to ensure convergence, we find that a higher proportion, 26.7% of the total heat flows into body 1 and the mean temperature of the fine scale actual contact areas is predicted to be 1553 °C. This of course exceeds the melting temperature for carbon steels and hence implies that at some intermediate scale (value of n), melting will occur, effectively truncating the fractal process through material flow. More precisely, we might anticipate local grain structures to develop by phase transformation, based on the size of the contact area a_n at which the predicted temperature approaches the melting temperature.

4. Conclusions

The principal result of this paper is the demonstration that the thermal resistance to heat flow from a fractal contact area to the extremities of a half space is bounded in the fractal limit, even though the latter involves strictly an infinite set of areas of measure zero. This result remains true for the case where the contact areas move over the surface of the half space. These results are established here using a particular fractal contact area distribution due to Archard, but it is clear that they are of more general application, since the solution for a stationary surface places bounds on the resistance for the moving contact case, and these bounds are themselves bounded by the theorem established in [9]. It follows that the error implied by the use of (for example) a finite element approximation to a fractal thermal contact problem can be expected to converge with increasing mesh refinement.

Fractal roughness tends to reduce the bias of heat flux into the moving body, particularly at high Peclet number, and this will generally have the effect of raising the mean temperature of the nominal contact area.

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