DISTRIBUTION OF HEAT BETWEEN SLIDING SURFACES

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When sliding takes place between metals of comparable hardness, the mechanism is probably restricted to single asperity interactions of a short duration. In this paper, a solution is derived for the heat conduction in a single interaction of this type and it is shown that, unless there is initially a temperature difference between the surfaces, there can be no meaningful distinction between 'moving' and 'stationary' surfaces. The solution is integrated for an arbitrary configuration and the result is considered with reference to the work of Jaeger (1)† and Blok (2).

INTRODUCTION

IT IS CUSTOMARY, in work on this subject, to draw a sharp distinction between the 'stationary' and the 'moving' surface. Therefore, it is relevant to examine this distinction before proceeding to a more detailed consideration of the problem.

If there is a single, permanent area of contact between the surfaces, the distinction is made by considering the velocity of the surfaces relative to this area of contact. Jaeger (1) and Blok (2) have published solutions for this problem which have been widely applied in subsequent work.

If the load is carried by a number of transient areas of contact, resulting from individual asperity interactions, the moving surface is more difficult to define. It is this case which will be considered in this paper.

'MOVING' AND 'STATIONARY' SURFACES

Consider the mechanism of a single interaction between two asperities. A junction is formed which undergoes considerable deformation and eventually fractures. Green (3) has simulated this process on a large scale using two-dimensional plasticine asperities.

Now, if the initial condition of the asperities is essentially similar and the mechanical properties are comparable, there is no information inherent in the unit event to distinguish between the surfaces. The area of contact will move relative to both surfaces though the total movement will probably be restricted to the order of magnitude of the junction diameter. Even if an occasional asperity ploughs for some distance through the other surface, the event is as likely to be repeated with the surfaces reversed.

Why then should there be any asymmetry in the division of heat between the surfaces other than that resulting from differences in thermal properties?

The answer to this question is to be found in a more practical consideration of friction systems. The usual difference is one of nominal area of contact; thus the motion of the surfaces is defined relative to the nominal area of contact instead of the individual junction.

However, information about the nominal area of contact is not carried in each individual asperity of the surfaces. In order that any supportable distinction be drawn between them, there must be a difference in some parameter of the asperities before the interaction takes place.

The parameter which carries this information is the temperature field in the vicinity of the interacting asperities. If there is no temperature difference between the surfaces in such a system, the distinction between moving and stationary surfaces is meaningless.

Having made this statement, I must repeat the condition that the mechanical properties of the surfaces be comparable. If this were not satisfied there would be a tendency for the harder surface to plough through the softer, thus making the area of contact stationary relative to the harder surface.

It follows that the prime consideration in the distribution of heat between surfaces is the thermal resistance presented by the nominal area of contact and other restrictions to heat flow from the surfaces.

The following analysis is confined to systems in which successive interactions are sufficiently widely spaced for the surrounding temperature fields to be uniform. In order to simplify the mathematics, the field is further restricted to values of $Va/4k > 5$, where V is the relative velocity, a is the junction radius, and k is the thermal diffusivity of either surface.

The transient heat conduction in a single interaction is first considered. It is then shown how the solution may be integrated over a known distribution of junctions to give

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† *References are given in the Appendix.*

the heat transfer rate as a function of velocity and temperature difference between the surfaces. This unknown temperature difference may be eliminated by using the heat conduction equations applicable to the large-scale geometry. An example is worked out using an exponential distribution of junctions and a circular nominal area of contact in two semi-infinite solids.

SINGLE INTERACTION

It is assumed that, before the interaction, the temperature difference between the surfaces in the vicinity of the asperities is T_0. The junction is assumed to have a constant radius a and the duration of the interaction is given by $t = \beta a/V$, where a and V are as defined above and β is a function of the interaction mechanism of the order 2.

The solution of this transient heat conduction problem is the sum of two independent solutions:

(1) The same heat generation but initial temperatures zero;

(2) The initial temperatures known but no heat generation.

Problem (1)

Let the heat passing into surface 1 be q_1 and into surface 2 be q_2. Similarly, ρ_1, ρ_2 are densities, k_1, k_2 thermal diffusivities, and c_1, c_2 specific heats of the two surfaces.

The temperature at the centre of a circle, radius a, on the surface of a semi-infinite solid and heated at the rate q_1 per unit area is:

$$T = \frac{2q_1}{\rho_1 c_1}\left(\frac{t}{\pi k_1}\right)^{1/2}\left(1-e^{-a^2/4k_1 t}\right)+\frac{q_1 a}{\rho_1 c_1 k_1}\operatorname{erfc}\frac{a}{\sqrt{4k_1 t}}$$

If $Va/4k > 5$ and $t = \beta a/V$ (see above), $\beta a^2/4kt$ is large and we can neglect the terms $e^{-a^2/4kt}$ and erfc $a/\sqrt{4kt}$ which decay rapidly with increasing argument. Under these conditions,

$$T = \frac{2q_1}{\rho_1 c_1}\left(\frac{t}{\pi k_1}\right)^{1/2}$$

and for surface 2,

$$T = \frac{2q_2}{\rho_2 c_2}\left(\frac{t}{\pi k_2}\right)^{1/2}$$

Eliminating T and putting $q_T = q_1+q_2$ we get

$$q_2 = \frac{q_T}{1+\dfrac{\rho_1 c_1\sqrt{k_1}}{\rho_2 c_2\sqrt{k_2}}} \quad . \quad . \quad . \quad . \quad (1)$$

Problem (2)

If there is initially a temperature difference T_0 between the surfaces, the problem is essentially the discharge of a capacitance through a resistance. An exact analytic solution is not possible. However, in the steady state, the ratio of the temperature of the contact area to the heat transfer rate is

$$\frac{T}{Q_2} = \frac{1}{4k_2\rho_2 c_2 a}$$

It is reasonable to assume that the transient behaviour is proportionately similar to the corresponding uniformly distributed heat input solution. This approximation gives

$$T = \int_0^t \frac{Q_2(1-e^{-a^2/4k_2(t-t')})\,dt'}{4a^2\rho_2 c_2(\pi k_2(t-t'))^{1/2}}$$

where Q_2 is the heat transfer rate as a function of t' and T is the interface temperature. Once more, the exponential term may be neglected whence

$$T = \int_0^t \frac{Q_2\,dt'}{4a^2\rho_2 c_2(\pi k_2(t-t'))^{1/2}}$$

$$T_0 - T = \int_0^t \frac{Q_2\,dt'}{4a^2\rho_1 c_1(\pi k_1(t-t'))^{1/2}}$$

It follows that T is a constant equal to

$$\frac{T_0}{1+\dfrac{\rho_2 c_2\sqrt{k_2}}{\rho_1 c_1\sqrt{k_1}}}$$

The remaining integral is

$$\int_0^t \frac{Q_2\,dt'}{4a^2(\pi(t-t'))^{1/2}}$$

and must be independent of time. Q_2 must therefore be proportional to $t^{-1/2}$, whence

$$T = \frac{Q_2\sqrt{t}}{4a^2\rho_2 c_2\sqrt{\pi k_2}}\int_0^t \frac{dt'}{(t'(t-t'))^{1/2}}$$

$$= \frac{Q_2}{4a^2\rho_2 c_2}\left(\frac{t\pi}{k_2}\right)^{1/2}$$

and

$$Q_2 = \frac{4a^2 T_0}{\sqrt{t\pi}\left(\dfrac{1}{\rho_1 c_1\sqrt{k_1}}+\dfrac{1}{\rho_2 c_2\sqrt{k_2}}\right)}$$

The average rate of heat transfer through a junction of radius a is therefore:

$$\frac{\int_0^{\beta a/V} Q_2\,dt}{\beta a/V} = 8a^{3/2}T_0\left(\frac{V}{\beta\pi}\right)\left(\frac{1}{\dfrac{1}{\rho_1 c_1\sqrt{k_1}}+\dfrac{1}{\rho_2 c_2\sqrt{k_2}}}\right) \quad (2)$$

The sum of this value and q_2 from equation (1) gives the solution for the original problem.

SURFACE TEMPERATURE

The surface temperature may be found from these two solutions and is:

$$T = \frac{\dfrac{2q_T}{\pi a^2}\left(\dfrac{t}{\pi}\right)^{1/2}+\rho_1 c_1\sqrt{k_1}T_0}{\rho_1 c_1\sqrt{k_1}+\rho_2 c_2\sqrt{k_2}}$$

The maximum temperature is reached just before the fracture when $t = \beta a/V$ and is:

$$T_{max} = \frac{\dfrac{2q_T}{\pi a^2}\left(\dfrac{\beta a}{\pi V}\right)^{1/2}+\rho_1 c_1\sqrt{k_1}}{\rho_1 c_1\sqrt{k_1}+\rho_2 c_2\sqrt{k_2}} \quad . \quad . \quad (3)$$

SUMMATION OVER ALL CONTACT AREAS

If we know the number and size distribution of junctions we can integrate this solution over the nominal contact area to give the distribution of heat between the surfaces as a function of T_0. Thus, if there are $N\,\mathrm{d}a$ junctions whose radius lies between a and $a+\mathrm{d}a$, the total heat passing into surface 2 is

$$Q=\int_0^\infty \frac{\pi a^2 q_T N\,\mathrm{d}a}{\left(1+\frac{\rho_1 c_1\sqrt{k_1}}{\rho_2 c_2\sqrt{k_2}}\right)}+\int_0^\infty \frac{8Na^{3/2}T_0(V/\beta\pi)^{1/2}\,\mathrm{d}a}{\left(\frac{1}{\frac{1}{\rho_1 c_1\sqrt{k_1}}+\frac{1}{\rho_2 c_2\sqrt{k_2}}}\right)} \quad . \; . \; . \; (4)$$

Note that $\int_0^\infty \pi a^2 q_T N\,\mathrm{d}a$ is the total rate of heat generation, Q_T.

In general, if the geometry of the system is known, T_0 can be defined as a function of the amount of heat dissipated in surface 1, Q_T-Q, since it is limited by heat transfer from the surface. If the dimension of the nominal area of contact is large in the direction of sliding it will be necessary to allow for variation of temperature in both surfaces with position. Since the dependence on T_0 is linear, it will be sufficient to take the average value over the nominal area.

In particular, if the nominal area is a circle, radius A, the average value of T_0 is given by

$$Q_T-Q=\frac{3\pi^2 k_1\rho_1 c_1 T_0 A}{8} \quad . \; . \; (5)$$

(This value assumes that the initial temperature of all asperities in surface 2 is zero. This should be acceptable providing interactions are well separated and velocity is high.)

Q can be found from equations (4) and (5) by elimination of T_0.

AN EXAMPLE USING AN EXPONENTIAL DISTRIBUTION OF JUNCTIONS

Suppose $N = n\,\mathrm{e}^{-a/\sigma}$

the total load carried by the system may be taken as

$$L=\int_0^\infty p\pi a^2 n\,\mathrm{e}^{-a/\sigma}\,\mathrm{d}a$$

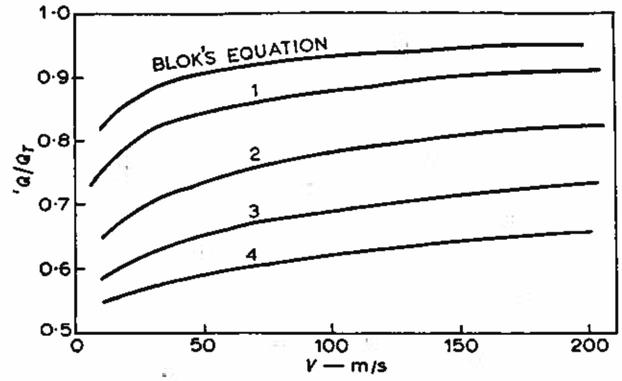
where p is the yield pressure of the softer metal. Whence

$$L = 2n\pi\sigma^3 p \quad \text{or} \quad n=\frac{L}{2\pi\sigma^3 p}$$

Substituting this value into equation (4) and integrating we get

$$Q=\frac{Q_T}{\left(1+\frac{\rho_1 c_1\sqrt{k_1}}{\rho_2 c_2\sqrt{k_2}}\right)}+\frac{\frac{\pi}{3}\left(\frac{V}{\beta\sigma}\right)^{1/2}\frac{LT_0}{p}}{\left(\frac{1}{\rho_1 c_1\sqrt{k_1}}+\frac{1}{\rho_2 c_2\sqrt{k_2}}\right)} \quad (6)$$

If we specify a circular nominal contact area, equations (5) and (6) yield



Curve (1) $L/Ap = 5\times10^{-3}$ cm; curve (2) 2×10^{-3} cm; curve (3) 10^{-3} cm; curve (4) 5×10^{-4} cm.

Fig. 1. *Variation of Q/Q_T with V*

$$\frac{Q}{Q_T}=\frac{1}{1+\dfrac{\rho_1 c_1\sqrt{k_1}}{\rho_2 c_2\sqrt{k_2}\left[\dfrac{8L}{\pi^3 Ap}\left(\dfrac{V}{\beta\sigma k_1}\right)^{1/2}+1\right]}} \quad (7)$$

Fig. 1 shows the variation of Q/Q_T with V for arbitrary values of the constants ($\beta = 2$, $\rho_1 c_1\sqrt{k_1}/\rho_2 c_2\sqrt{k_2} = 1$, $\sigma = 2\times10^{-3}$ cm, $k_1 = 10^{-1}$ cm^2/s). It is difficult to compare this result with the Jaeger–Blok solution since the latter is strictly not applicable to these conditions. However, since this solution has frequently been used out of its original context it is shown in Fig. 1 in comparison with the present work. The average area of the exponential distribution is taken as the junction size.

If we assume that the load carried by a junction is proportional to its area, equation (3) becomes:

$$T=\frac{\dfrac{2\mu p^{3/4}\beta^{1/2}f^{1/4}V^{1/2}}{\pi^{3/4}}+\rho_1 c_1\sqrt{k_1}T_0}{\rho_1 c_1\sqrt{k_1}+\rho_2 c_2\sqrt{k_2}}$$

where f is the load carried by the junction, μ is the coefficient of friction and p is the yield pressure. For the particular case of $\rho_1 c_1\sqrt{k_1}/\rho_2 c_2\sqrt{k_2} = 1$, $\beta = 2$, this reduces to:

$$T=\frac{0{\cdot}84\mu p^{3/4}V^{1/2}f^{1/4}}{\rho c\sqrt{k}}+\tfrac{1}{2}T_0$$

The corresponding expression from Blok's paper reduces to the form:

$$T=\frac{1{\cdot}02\mu p^{3/4}V^{1/2}f^{1/4}}{\rho c\sqrt{k}}$$

when Va/k is large. Clearly there will be no great difference between these results unless T_0 is large.

CONCLUSIONS

In the equation (7), Q/Q_T is seen to be a function of L/Ap as well as velocity and average junction size. Curves (1), (2), (3) and (4) in Fig. 1 are therefore shown for different values of this parameter. It is seen that as L/Ap becomes larger, the value of Q/Q_T approaches that of Blok's solution for the same velocity. An upper limit is placed on

L/Ap by the assumption of uniform initial temperature fields in the derivation of equation (2). Below this value, the values of Q/Q_T are lower than Blok's solution though showing the same trend with velocity. The explanation for this behaviour is that, as L/Ap becomes smaller, interactions become more separated in time and space. If the junctions distort symmetrically the heat distribution tends to become symmetrical.

The flash temperatures predicted by Blok are not greatly changed by these conditions since the increase in heat flux to the 'stationary' surface, shown in Fig. 1, is offset by the fact that the junction is broken before the steady-state temperature is reached.

A discontinuity in temperature between surfaces in sliding contact has been observed experimentally by Ling and Simkins (4).

Various approximations have been made in deriving the final result presented in Fig. 1 but the general remarks on stationary and moving surfaces and the method of analysis may be applied to other circumstances, subject to the condition imposed regarding interaction mechanisms.

APPENDIX

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