Similarly, the boundary condition is free from applied stress and the media are $\xi = 0.2$, $\eta = 0.8$.

By truncating the infinite algebraic Eq. (3.16) to $n = s = 3$ for $K_\alpha a = 0.1$ and $n = s = 4$ for $K_\alpha a = 1.0, 2.0$, we find the coefficients A_n. Figures 2 and 3 show the results of stress concentration factors of calculation.

Now, we conclude this paper with the following discussions:

(a) From the numerical results indicated above, we can see that the effect of anisotropy on dynamic stress concentration is quite significant in engineering sense.

(b) The convergence of Eq. (3.16) depends on wave number $K_\alpha a$ and on cavity shapes. For low $K_\alpha a$, a few terms of the series are sufficient; while for high $K_\alpha a$, the convergence is rather slow. So, in this case, the number of terms needed becomes large in order to get reasonably good results.

(c) For the square cavity case, the mapping function (4.2) maps the unit circle only to "nearly square cavity" with corners as shown in the figure attached. Such shape of course misses the character of sharp corners. This is the weak point of the method of mapping as noted universally in static case. Increasing the number of terms of the mapping functions is a way to make the corners of the figure rather sharp.

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Steady-State Transonic Motion of a Line Load Over an Elastic Half-Space: The Corrected Cole/Huth Solution

H. G. Georgiadis[3] and J. R. Barber[4]

Introduction

Recently, the authors have investigated various *transient* and *steady-state* elastodynamic indentation problems, with a view to elucidating the paradoxes associated with such problems when the edge of the contact area has a speed in the super-Rayleigh/subseismic range (Georgiadis and Barber, 1993). In the course of this investigation, an *integral equation* formulation was developed for the steady-state problem of an indenter moving over a half-plane at constant speed, using the classical solution of Cole and Huth (1958) as a Green's function. However, the resulting equation exhibited different asymptotic behavior at the ends of the contact zone from other published solutions to elastodynamic *crack* and *contact* problems (see, e.g., Brock, 1977; Freund, 1979; Burridge et al., 1979; Georgiadis, 1986; Robinson and Thompson, 1974). Further investigation showed that this inconsistency was attributable to an error in the Cole/Huth solution in the transonic range. The purpose of the present Note is to rederive the solution for this speed range.

The Cole/Huth problem involves a concentrated load moving with a constant speed, V, over the surface of an elastic half-space under plane-stress or plane-strain conditions. This classical problem was formulated within steady-state elastodynamics and solved by a complex-variable method. A generalization involving an *inclined* load, i.e., a formulation including both normal and tangential tractions, was considered by Eringen and Suhubi (1975), but their final results exhibit the same error.

Obviously, the Cole/Huth problem possesses considerable engineering importance. For instance, it is of great interest in soil dynamics, where ground motions and stresses can be produced by blast waves (surface pressure waves due to explosions), or by supersonic aircraft. Other applications are encountered within the context of contact mechanics (see, e.g., Johnson, 1985); for instance, the problem of high-velocity rocket sleds sliding over steel guide rails (Gerstle and Pearsall, 1974). Consequently, this problem has attracted much interest being cited and fully presented in such classical texts as Sneddon (1951), Fung (1965), and Eringen/Suhubi (1975).

This Brief Note sets out to present the correct solution to the steady-state moving load problem for the *transonic* range, i.e., when the velocity of the load is between the shear and the longitudinal-wave velocities. It is this particular velocity range, where the results for displacements and stresses by Cole/Huth (1958) and Eringen/Suhubi (1975) are in error.

Analysis

We shall present very briefly the solution to the Cole/Huth problem for an inclined load. Our approach leads directly to the expressions for the real and imaginary parts of the complex potential function.

Assume that an elastic body in the form of a half-plane is set into motion by an inclined concentrated load moving over the surface with a constant velocity V (see Fig. 1). The longitudinal and shear-wave velocities are defined as $c_1 = [(\lambda + 2\mu)/\rho]^{1/2}$ and $c_2 = (\mu/\rho)^{1/2}$, in terms of the Lamé constants λ, μ and the mass density ρ. The quantities $M_j \equiv V/c_j$ ($j = 1, 2$) are the Mach numbers which define the speed range (subsonic, transonic, supersonic) of the motion.

The steady-state elastodynamic field can be described by introducing a moving coordinate system (x, y) as $x = x' -$

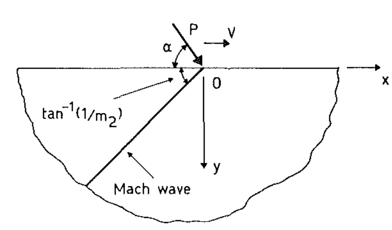

Fig. 1 Steadily moving load over the surface of an elastic half-plane. The Mach wave (shock wave in shear stress) is also shown for the transonic range.

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Vt, $y = y'$, where (x', y') is a fixed system. Then, for the *transonic* case ($c_2 < V < c_1$), the displacement and stress fields are given in terms of the so-called *potential functions* $W_1(z_1 = x + i\beta_1 y)$ and $W_2(x + m_2 y)$ (Eringen and Suhubi, 1975; Georgiadis, 1986)

$$u_x = 2\mathrm{Re}\,W_1 + 2m_2 W_2, \tag{1a}$$

$$u_y = -2\beta_1 \mathrm{Im}\,W_1 - 2W_2, \tag{1b}$$

$$\sigma_x = 2\mu[(2\beta_1^2 + m_2^2 + 1)\mathrm{Re}\,W_1' + 2m_2 W_2'], \tag{1c}$$

$$\sigma_y = 2\mu[-(1 - m_2^2)\mathrm{Re}\,W_1' - 2m_2 W_2'], \tag{1d}$$

$$\tau_{xy} = 2\mu[-2\beta_1 \mathrm{Im}\,W_1' - (1 - m_2^2)\,W_2'], \tag{1e}$$

where $\beta_1 = (1 - M_1^2)^{1/2}$ and $m_2 = (M_2^2 - 1)^{1/2}$ are real numbers.

The boundary conditions of the problem can be written as

$$\sigma_y(x,0) = -P\,\sin\alpha\;\delta(x), \tag{2a}$$

$$\tau_{xy}(x,0) = -P\,\cos\alpha\;\delta(x), \tag{2b}$$

where $\delta(\bullet)$ is the Dirac delta function, and the angle α defines the inclination of the load, as shown in Fig. 1. Introducing Eqs. (2) into (1) and then eliminating the function $W_2'(x)$ from the resulting system yields a relation between the real and imaginary parts of the function W_1'

$$\mathrm{Re}\,W_1'(x) = \frac{P\bullet f(\alpha,m_2)}{2\mu(1-m_2^2)^2}\,\delta(x) + \frac{4\beta_1 m_2}{(1-m_2^2)^2}\,\mathrm{Im}\,W_1'(x), \tag{3}$$

where $f(\alpha, m_2) \equiv \sin\alpha \bullet (1 - m_2^2) - 2\cos\alpha \bullet m_2$.

The boundary value problem in (3) is a *Riemann-Hilbert problem* (Gakhov, 1966) and can be solved by utilizing the *Hilbert transform* and elements from the theory of singular integral equations (Tricomi, 1985). By applying the operation $\mathrm{Im}\,W_1'(x) = \int_{-\infty}^{\infty}[\mathrm{Re}\,W_1'(\tau)/\pi(x-\tau)]d\tau$ to (3), we get a singular IE which has the solution

$$\mathrm{Re}\,W_1'(x) = \frac{P\bullet f(\alpha,m_2)}{2\mu R^*}\left[(1-m_2^2)^2\delta(x) + \frac{4\beta_1 m_2}{\pi}\frac{1}{x}\right], \tag{4}$$

where $R^* \equiv (1 - m_2^2)^4 + 16\beta_1^2 m_2^2$. Then, $\mathrm{Im}\,W_1'(x)$ follows from (3), whereas $W_2'(x)$ may be obtained from Eqs. (1) and (2). Finally, the functions $\mathrm{Re}\,W_1(x)$, $\mathrm{Im}\,W_1(x)$, and $W_2(x)$ are found by integrating the previous functions and omitting constants of integration, i.e., rigid-body displacements.

The next step involves evaluation of the functions $\mathrm{Re}\,W_1'(z_1)$, $\mathrm{Im}\,W_1'(z_1)$ and $W_2'(x + m_2 y)$ which enter (1) and give the stresses. The first two functions result from $\mathrm{Re}\,W_1'(x)$ through the *Schwarz integral formula* (Churchill et al., 1974)

$$W_1'(z_1) \equiv \mathrm{Re}\,W_1'(z_1) + i\mathrm{Im}\,W_1'(z_1) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\beta_1 y\bullet \mathrm{Re}\,W_1'(\tau) + i(x-\tau)\bullet\mathrm{Re}\,W_1'(\tau)}{(\tau-x)^2 + \beta_1^2 y^2}\,d\tau. \tag{5}$$

In combining (4) and (5) integrals which need to be evaluated are found in Tables (e.g., Petit Bois, 1961), and finally we obtain

$$\mathrm{Re}\,W_1'(z_1) = \frac{Bx + \pi^{-1}\beta_1 A y}{x^2 + \beta_1^2 y^2}, \tag{6a}$$

$$\mathrm{Im}\,W_1'(z_1) = \left(\pi^{-1}Ax + \frac{Bx^2}{\beta_1 y}\right)\frac{1}{x^2+\beta_1^2 y^2} - \frac{B}{\beta_1 y}, \tag{6b}$$

where the constants A and B are given as

$$A = P(2\mu R^*)^{-1}\bullet f(\alpha,m_2)\bullet(1-m_2^2)^2, \tag{7a}$$

$$B = P(2\pi\mu R^*)^{-1}\bullet f(\alpha,m_2)\bullet 4\beta_1 m_2. \tag{7b}$$

We also find

$$W_2'(x+m_2 y) = \frac{P}{\mu(1-m_2^2)}[(1/2)\cos\alpha + 4\beta_1^2 m_2 (R^*)^{-1}\bullet f(\alpha,m_2)]\bullet\delta(x+m_2 y) - \frac{P\beta_1(1-m_2^2)\bullet f(\alpha,m_2)}{\pi\mu R^*}\frac{1}{x+m_2 y}. \tag{8}$$

In a similar way, we can find the functions $\mathrm{Re}\,W_1(z_1)$, $\mathrm{Im}\,W_1(z_1)$, and $W_2(x + m_2 y)$, which are required for the determination of subsurface displacements,

$$\mathrm{Re}\,W_1(z_1) = \frac{P\bullet f(\alpha,m_2)}{2\pi\mu R^*}[4\beta_1 m_2\bullet\log(r_1) - (1-m_2^2)^2\bullet\theta_1], \tag{9a}$$

$$\mathrm{Im}\,W_1(z_1) = \frac{P\bullet f(\alpha,m_2)}{2\pi\mu R^*}[4\beta_1 m_2\bullet\theta_1 + (1-m_2^2)^2\bullet\log(r_1)], \tag{9b}$$

$$W_2(x+m_2 y) = \frac{P}{\mu(1-m_2^2)}[(1/2)\cos\alpha + 4\beta_1^2 m_2(R^*)^{-1}\bullet f(\alpha,m_2)]\bullet H(x+m_2 y) - 2\beta_1(1-m_2^2)^{-1}\bullet A\bullet\log(|x+m_2 y|), \tag{10}$$

where $r_1 = (x^2 + \beta_1^2 y^2)^{1/2}$, $\theta_1 = \tan^{-1}(\beta_1 y/x)$, $0 < \theta_1 < \pi$, and $H(\bullet)$ is the Heaviside step function.

Neither Cole and Huth (1958) nor Eringen and Suhubi (1975) give expressions for $\mathrm{Re}\,W_1'$, $\mathrm{Im}\,W_1'$, $\mathrm{Re}\,W_1$, $\mathrm{Im}\,W_1$ (our Eqs. (6), (9)), but their expressions for W_2', W_2 are identical with our Eqs. (8), (10). However, as will be shown in the next section, the final expressions for the stress and displacement fields given by these authors are incorrect.

Results and Conclusions

Having available the functions given by Eqs. (6)–(10), one can readily obtain the stress and displacement field by substituting in Eq. (1). In particular, the expressions for the surface displacements $u_y(x, 0)$, $u_x(x, 0)$ and the normal stress immediately beneath the load $\sigma_y(0, y)$ are found to be

$$u_y(x,0) = \frac{P}{\mu}\left[\frac{\beta_1(1-m_2^4)^4\bullet f(\alpha,m_2)}{\pi R^*}\bullet\log(|x|) + \frac{1}{(1-m_2^2)}\left[\frac{4\beta_1^2 m_2(1+m_2^2)\bullet f(\alpha,m_2)}{R^*} - \cos\alpha\right]\bullet H(-x)\right], \tag{11}$$

$$u_x(x,0) = \frac{2P\beta_1 m_2(1+m_2^2)\bullet f(\alpha,m_2)}{\pi\mu R^*}\bullet\log(|x|) + \frac{P}{\mu(1-m_2^2)}\left[\frac{f(\alpha,m_2)}{R^*}[(1-m_2^2)^3 + 8\beta_1^2 m_2^2] + \cos\alpha\bullet m_2\right]\bullet[1-H(-x)], \tag{12}$$

$$\sigma_y(0,y) = \frac{P(1-m_2^2)\bullet f(\alpha,m_2)}{\pi\beta_1 R^*}[4\beta_1^2 - (1-m_2^2)^2]\frac{1}{y} - \frac{4Pm_2}{(1-m_2^2)}[(1/2)\cos\alpha + 4\beta_1^2 m_2(R^*)^{-1}\bullet f(\alpha,m_2)]\bullet\delta(m_2 y). \tag{13}$$

Equations (11)–(13) differ significantly from the corresponding expressions given by Cole and Huth (1958) and Eringen and Suhubi (1975). These researchers do not give enough detail in their analyses for the cause of the difference to be identified with certainty, but a possible source of error could be an incorrect separation of their complex potentials into real and imaginary parts.

A check on the correctness of the present analysis and results are provided by our previous findings on the asymptotics of moving *contact* zones (Georgiadis and Barber, 1993). For the case of *normal* load, i.e., when $\alpha = \pi/2$, and for the $u_y(x, 0)$ displacement (which was utilized as a Green's function in Georgiadis and Barber, 1991), the Cole/Huth expression is in error by a factor $(2/M_2^2)$ multiplying the $H(-x)$ term.

With this correction, the asymptotic behavior of the stress and displacement field at the edges of the moving contact zone becomes consistent with that obtained in all other published solutions of elastodynamic crack and contact problems (Brock, 1977; Freund, 1979; Burridge et al., 1979; Georgiadis, 1986; Robinson and Thompson, 1974) involving the edge of a crack or contact zone moving at a speed in the transonic range.

In closing, we mention that the respective *transient* problem was considered by Payton (1967). In principle, one could get the present *steady-state* results by Payton's analysis, as time tends to infinity in the transient problem. However, the latter work does not provide pertinent results for field quantities in the interior of the half-space and, moreover, only the *horizontal* surface displacement caused by a *normal* load was worked out. Notice that we provide results for the more general case of an *inclined* load and stresses and displacements at all field points. It is felt thus, by also taking into account the very complicated expressions in Payton's analysis, that a *direct* steady-state analysis (as the present one) is preferable in some instances over a limiting procedure of exploiting already obtained transient results. This is especially true when one tries to correct some established and well-known analyses, as we did in the present case.

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Capillary-Gravity Waves Generated Against a Vertical Cliff in a Fluid of Finite Depth

A. K. Pramanik[5] and D. Banik[5]

1 Introduction

This is the problem of two-dimensional capillary-gravity waves generated by some free surface oscillatory pressure distribution which moves with a uniform velocity. The fluid is incompressible, inviscid and is of uniform finite depth h and is bounded on one side by a vertical cliff.

This problem without the cliff has been studied by Pramanik and Majumdar (1984). The present problem with infinitely large depth has been discussed by Pramanik and Majumdar (1988). To understand the motivation of our paper we state the main results of the paper of Pramanik and Majumdar (1984). The ultimate steady state consists of six progressive waves, four gravity waves, and two capillary waves. There exists in the (a, b, c) space, where a, b, c are the nondimensional forms of the parameters of the problem, a surface called the critical surface, which divides the space into several regions in each of which the propagation is different.

The aim of the present paper is (i) to fully characterize the critical surfaces for all possible values of the parameters, (ii) to determine the waves for all possible values of the parameters, and (iii) to find the effect of the cliff on the reflection of waves.

As is already stated in Pramanik and Majumdar (1984), the waves were determined on the basis of two sections of the critical surface by the plane c = constant. However, the complete characterization of critical surface is possible. In this paper the critical surfaces are determined for all possible values of a, b, c. It is found that these surfaces divide the whole positive quadrant of the (a, b, c) space into five distinct regions for (a, b, c) in each of which the propagation of waves is different and the waves for all cases are determined. It is known that for (a, b, c) outside these surfaces, the waves are with constant amplitude while the amplitude is unbounded for (a, b, c) on the critical surfaces.

Previously, in linear theory, these waves for (a, b, c) on critical surfaces were not of interest where essentially nonlinear theory is to be developed for the complete understanding of the waves. However, to develop the nonlinear theory (Akylas, 1984) one has to take into account the order of the unboundedness on the critical surfaces. Motivated by this idea, waves are also determined for (a, b, c) on the critical surfaces.

Regarding the effect of the cliff it is found that one gravity wave is reflected for certain values of (a, b, c). In (a, b, c) space there is a surface called the surface of reflection, such that for (a, b, c) on one side of this surface, including those on the surface, reflection occurs. And the amplitude of the reflected wave remains the same as the original waves for all (a, b, c), excepting for those forming a curve on the surface of reflections. For (a, b, c) of this curve, the amplitude is found to be reduced.

2 Formulation and Formal Solution

We take the x-axis along the undisturbed free surface and

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