International Journal of Engineering Science 48 (2010) 2107

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Letter to the Editor

Discussion on the paper "On the torsion of a prism with non-equilateral triangular cross-section", by L.M.B.C. Campos and F.S.R.P. Cunha

In this recent paper[1], Campos and Cunha claim to have identified a closed-form polynomial solution to the Saint Venant torsion problem for a non-equilateral triangular bar. They start from the Prandtl stress function

$$\Theta = By\left(1-\frac{y}{c}-\frac{x}{a}\right)\left(1-\frac{y}{c}+\frac{x}{b}\right), \tag{1}$$

which satisfies the traction-free conditions on the three straight boundaries $y = 0$, $y = c(1 - x/a)$, $y = c(1 + x/b)$, but which does not so far satisfy the governing (Poisson) equation

$$\nabla^2\Theta = C, \tag{2}$$

where C is a constant. The function Θ of Eq. (1) is a third degree polynomial in x, y and hence its Laplacian (2) is generally a linear function of the same variables. In order that the latter be a constant, the coefficients of x, y must both be zero and it is easily shown that these two conditions are satisfied if and only if the parameters a, b, c define an equilateral triangle. This is of course a classical result.

Next, the authors perform a general rotation (θ) and translation (x_0,y_0) of the coordinate system to develop a corresponding polynomial solution in the transformed Cartesian coordinates (u, v), where $x = x_0 + u\cos\theta + v\sin\theta$, $y = y_0 - u\sin\theta + v\cos\theta$. The resulting function is a third degree polynomial in u, v, and its Laplacian is again linear, requiring two conditions to be satisfied if it is to be constant. However, the authors argue that the arbitrary rotation introduces an additional parameter θ, which can be chosen in combination with a, b, c to allow two of these last three parameters to be chosen arbitrarily.

Unfortunately, this argument, though superficially plausible, is incorrect. The Laplacian of a function is invariant under coordinate transformation and hence $\nabla^2\Theta$ will be constant in the rotated coordinate system if and only if it is also constant in the original one. Thus, no additional degree of freedom is introduced by the rotation.

Notwithstanding this, the authors provide an example to 'illustrate' their procedure, this being defined by the stress function

$$\Theta(u,v) = \left[c(v-u)/(2\sqrt{2})\right]\left\{1+\left[(v-u)/(c\sqrt{2})\right]^2-\left[(v+u)/(a\sqrt{2})\right]^2\right\},$$

[authors' Eq. (45)]. Firstly, this is *not* what we obtain if we substitute $\theta = \pi/4$, $a = b$ [authors (43a,b)] in (1), which actually yields

$$\Theta(u,v) = B\left[c(v-u)/(2\sqrt{2})\right]\left\{\left[1-(v-u)/(c\sqrt{2})\right]^2-\left[(v+u)/(a\sqrt{2})\right]^2\right\}.$$

More importantly, the Laplacian of this function is independent of u, v if and only if $c = a\sqrt{3}$, which of course returns us to the original equilateral triangle.

No-one would be more pleased than me to see an addition to the catalogue of closed-form solutions to the torsion problem, but regrettably this is not one of them.

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Available online 1 August 2010

[1] *International Journal of Engineering Science*, Vol. 48 (2010) pp 718–725.

doi:10.1016/j.ijengsci.2010.07.002