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LETTER TO THE EDITOR

Comments on "Frictionally excited thermoelastic instabilities"

In a very interesting series of recent papers[1–3], Burton, Nerlikar and Kilaparti have developed mathematical models for a variety of situations involving the influence of thermal expansion on the distribution of contact pressure between two conforming sliding solids.

Amongst other conclusions, they suggest that instability is more likely to occur with a pair of solids of widely differing thermal conductivities than with similar materials. Berry[4] has observed instabilities over a wide range of speeds and loads with a variety of material combinations, including some cases of similar materials (*e.g.*, hardened steel on hardened steel). His experimental system of two thin cylinders in sliding contact at a plane end face is essentially that shown in Fig. 1 of ref. 1. In general, it is found that the disturbance is stationary with respect to one solid (*i.e.*, in the authors' notation, either c_1 or $c_2=0$), but as wear takes place, the disturbance is periodically transferred to the opposite solid. In each case, the solid relative to which the disturbance is moving acquires a fairly uniform temperature, except presumably in the immediate vicinity of the surface. The magnitude of the observed circumferential variation in temperature is such as to suggest that extensive regions of the distorted solid surface are not in contact. In other words, the conditions are those referred to by Burton *et al.*[3] as "partially contacting".

However, if such a situation is to be established between similar materials with initially conforming surfaces, it must develop from the condition of complete contact analysed by the authors in their first two papers[1,2] and hence this latter condition is presumably unstable for steel on steel even though the friction coefficients and speeds are well below the limiting values quoted by the authors. This result might be attributable to the effect of surface films[2] or perhaps to constriction resistance due to imperfect thermal contact on the microscopic scale. Alternatively, the presence of a slight initial alignment error might precipitate a disturbance constrained to the misaligned solid.

In the authors' analysis of the partial contact configuration[3], the relation between heat input and curvature (authors' eqn. (10)) can be derived more easily and in a more general form by making use of the properties of the solution for a point heat source on the surface of a semi-infinite solid. If heat is released at a rate q at a point on the surface of the semi-infinite solid, the normal surface displacement, v, at a radius r due to thermal distortion in the steady state is

$$v = \frac{q\alpha(1+\nu)}{2\pi K} \ln (r_0/r) \tag{1}$$

where r_0 is an arbitrary reference point at which v is defined as zero to avoid rigid body displacement[5,6]. By analogy with the potential due to a point source in two-dimensional potential theory, it follows that v must satisfy the differential equation

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{q\alpha(1+\nu)}{K} \tag{2}$$

where q is now the steady heat input per unit time, per unit area of surface.

This result could be applied to problems of thermoelastic instability in three dimensions and is independent of the authors' assumption of a Fourier series statement of the temperature field at the surface. For plane strain this equation reduces to

$$\frac{\partial^2 v}{\partial x^2} = \frac{q\alpha(1+\nu)}{K} \tag{3}$$

The corresponding plane stress relation can be obtained from the solution for a line heat source[6]. It is identical with eqn. (3) except that the term $(1+\nu)$ is deleted*.

The tangential frictional stress, which gives rise to the generation of heat, and hence to the instability, will itself tend to produce a normal displacement of the surface, causing the normal contact stress distribution to be asymmetric. The authors' solution could easily be adjusted to take this effect into account and it would be interesting to see how great an asymmetry would result.

An approximate solution, based on the authors' parabolic approximation, can be obtained by assuming a pressure distribution of the form

$$p = p_0(1-(x/l)^2)(1-cx/l); \qquad -l \leqslant x \leqslant l \tag{4}$$

The additional unknown c is found by applying the condition that slope $(\partial v/\partial x)$ as well as curvature $(\partial^2 v/\partial x^2)$ must be zero at $x=0$. Eliminating c between these two equations, we find that the contact length l has to satisfy the quadratic equation

$$3L^2 - 56L + 256 + 3\pi^2\mu^2(1-\nu)^2 = 0 \tag{5}$$

where

$$L = \alpha V E \pi \mu l / K \tag{6}$$

(notation as in ref. 3).

Of the two solutions of this equation, one is inadmissable, since it corresponds to values of c greater than 1, which would involve tensile contact stresses near $x=1$. The remaining solution lies in the range $8 \leqslant L \leqslant 128/15$ depending on the value of μ. The authors' parabolic approximation gives $L=8$ (but see note *) so that tangential stresses would seem to have little effect on the size of the contact area. However, a quite significant asymmetry of contact pressure is produced at values of μ of the order of 0.1–0.5. In physical terms, this asymmetry might induce the contact region to move in the "upstream" direction. Similarly, if tangential

* It will be observed that this result differs from the authors' eqn. (10) by a factor of 2, which can be traced to a corresponding factor in their eqn. (2), which is misquoted from eqn. (A9) of ref. 7.

stresses were introduced into the continuous contact solution[1], they may lead to solutions involving negative values of c_1 or c_2.

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