Math 116 - 031 – Quiz 2 **Name**:

Chapter 7

1. (a) [4 points] Consider the integral $\int_0^1 e^{-t^2/2}dt$.

(i) Which of the approximations LEFT(3), RIGHT(3), MID(3) or TRAP(3) gives the ***closest overestimate*** to the exact value of this integral? You don't have to justify your answer.

MID(3) (because $e^{-t^2/2}$ is decreasing **and** concave down on the interval [0,1].)

(ii) Write out and compute the Riemann sum for that (one) estimate. You may use your calculator.

$$\text{MID}(3) = \tfrac{1}{3}(e^{-(1/6)^2/2} + e^{-(3/6)^2/2} + e^{-(5/6)^2/2}).$$

(b) [8 points] Circle true or false. No explanation is necessary.

TRUE false	Say $f(x)$ and $g(x)$ are two functions with the same derivative ($f'(x) = g'(x)$), and suppose that $f(1) = g(1)$ too. Then in fact $f(x)$ and $g(x)$ are equal.
true **FALSE**	If f and g are continuous functions on $[a, b]$, the average of $f(x)g(x)$ over that interval is the average of $f(x)$ times the average of $g(x)$.
true **FALSE**	At least one of LEFT(n), RIGHT(n), MID(n), or TRAP(n) is an underestimate of $\int_a^b f(x)dx$, no matter what $f(x)$ or n is.
TRUE false	The function $A(x) = \int_1^x e^{\cos(t)}dt$ is increasing for every x.

Notes:

(a) Since $f(x)$ and $g(x)$ have the same derivative, $f(x) = g(x) + C$ for some constant C. But $C = 0$ since $f(1) = g(1)$.

(b) We did an example of this in class.

(c) If $f(x)$ changes its direction and/or concavity in the middle of the interval $[a, b]$, there are *no guarantees* as to whether any of the sums will be over- and underestimates. (Alternately, if $f(x)$ is a constant function, then all four sums are exactly correct, not underestimates.)

(d) The derivative is $A'(x) = e^{\cos(x)}$, which is positive.

(c) Let $F'(x) = f(x)$, where $F(x)$ has these values:

x	0	0.5	1	1.5	2
$F(x)$	1	4	9	13	17

and $f(1) = 2$.

Compute the following [4 points each]:

$$\int_{0.5}^{1} \tfrac{1}{x^2} f(\tfrac{1}{x})dx = \int_{2}^{1} f(u)(-du) = \int_{1}^{2} f(u)du = F(2) - F(1) = 8.$$

This used the u-substitution $u = \frac{1}{x}$ and $du = \frac{-1}{x^2}dx$.

$$\int_{1}^{1.5} e^{f(x)} f'(x)dx = \int_{f(1)}^{f(1.5)} e^u du = e^{f(1.5)} - e^{f(1)} = e^{f(1.5)} - 2.$$

This used the u-substitution $u = f(x)$ and $du = f'(x)dx$. (The answer can't be simplified further since we don't have any other values of $f(x)$. I left them out by mistake – oops!)

$$\int_{1}^{2} xf'(x - \tfrac{1}{2})dx = \int_{0.5}^{1.5} (u + \tfrac{1}{2})f'(u)du \qquad \text{(by the substitution } u = x - \tfrac{1}{2},\ du = dx\text{)}$$

$$= (u + \tfrac{1}{2})f(u)\Big|_{0.5}^{1.5} - \int_{0.5}^{1.5} f(u)du \qquad \text{(by integration by parts)}$$

$$= 2f(1.5) - f(0.5) - \big(F(1.5) - F(0.5)\big)$$

$$= 2f(1.5) - f(0.5) - 9.$$

(The answer can't be simplified further.)

(courtesy of a friend from UMinnesota):

OH HI THERE, SIR ISAAC NEWTON, COULD YOU PLEASE FIND ME THE DERIVATIVE OF F(X) WITH RESPECT TO X? AND HURRY!

anagrams to...

HERE, A NEAT IDEA: WE WRITE F(X PLUS H) MINUS F(X) OVER H, DECIDE H IS VERY TINY, FACTOR OUT THIS H PART. DONE! COOL!