Math 116 - 031 – Quiz 1 **Name**:

Chapters 5 & 6

1. [6 points] Each of the following questions is worth 2 points.

(a) If $f(x)$ is an even function and $\int_{-2}^{2}\big(3f(x)-5\big)dx = 4$, find $\int_0^2 f(x)dx$.

Solution. Expanding the integral gives $3\displaystyle\int_{-2}^{2} f(x)dx - \int_{-2}^{2} 5dx = 4.$

The first integral equals $6\int_0^2 f(x)dx$ since $f(x)$ is even, and the second integral is $5\cdot 4 = 20$, so:

$$6\int_0^2 f(x)dx - 20 = 4, \text{ so } \int_0^2 f(x)dx = 4.$$

(b) The average value of $f(x) = \frac{10}{x^2}$ on the interval $[1, a]$ is equal to 2. Find the value of a.

Solution. First of all, the antiderivative is $\frac{-10}{x}$. The average value of $f(x)$ on the interval $[1, a]$ is defined by

$$\frac{1}{a-1}\int_1^a \frac{10}{x^2}dx = \frac{1}{a-1}\Big(\frac{-10}{a} + \frac{10}{1}\Big).$$

So, set this equal to 2 and multiply through by $a(a-1)$ to clear the denominators:

$$\frac{1}{a-1}\Big(\frac{-10}{a} + \frac{10}{1}\Big) = 2 \iff -10 + 10a = 2a(a-1).$$

This is a quadratic equation in a with solutions $a = 1, 5$. We reject $a = 1$ (otherwise the $\frac{1}{a-1}$ wouldn't make sense at the beginning), so the answer is $a = 5$.

(c) If the class is collectively answering calculus problems at a rate of $Q(t)$ questions per minute (where t is measured in minutes since you began the quiz), describe in a sentence what $\int_5^{15} Q(t)dt$ means.

Solution. $\int_5^{15} Q(t)dt$ is the total number of calculus questions answered by the class between the 5th and 15th minutes of the quiz.

(d) Let $A(x) = \int_5^{\ln(x)} \frac{\cos(t)}{t}dt$. Compute $A'(x)$.

Solution. Let $y = \ln(x)$. Then, by the chain rule,

$$A'(x) = \frac{d}{dx}\int_5^y \frac{\cos(t)}{t}dt = \Big(\frac{d}{dy}\int_5^y \frac{\cos(t)}{t}dt\Big)\cdot\frac{dy}{dx} = \frac{\cos(y)}{y}\cdot\frac{1}{x} = \frac{\cos(\ln(x))}{x\ln(x)}.$$

(e) Use a left hand sum with $n = 4$ rectangles to approximate $\int_1^3 x\ln(x)dx$. Don't convert to decimal numbers or simplify (you can write in terms of ln).

Solution. With 4 subdivisions, the rectangles will have width $\Delta x = 0.5$, so

$$\begin{aligned}\text{LEFT}(4) &= 0.5(f(1) + f(1.5) + f(2) + f(2.5)) \\ &= 0.5(1\ln(1) + 1.5\ln(1.5) + 2\ln(2) + 2.5\ln(2.5)).\end{aligned}$$

2. [5 points] Poussey and Taystee are working in the Litchfield Library, putting books in their correct locations on the shelves. By working together, they can re-shelve 10 books per minute. Meanwhile, books are being returned to the library at a variable rate of $f(t)$ books per minute (and then need to be re-shelved). Assume there are 21 books waiting to be shelved when they begin their work.

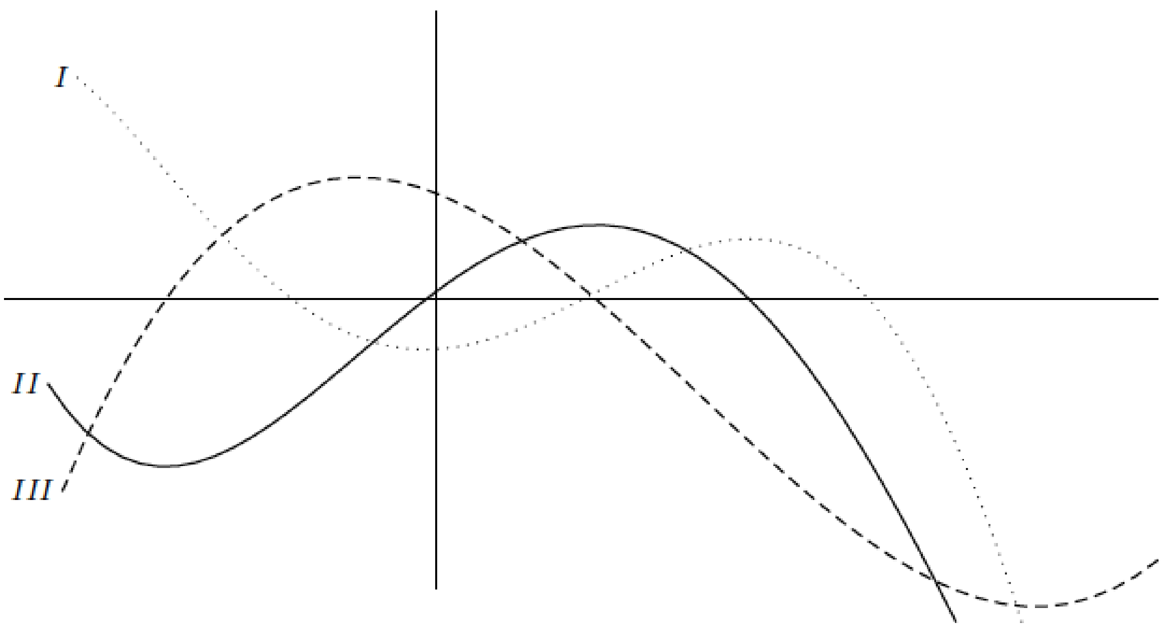
Let $B(t)$ be the number of books waiting to be shelved, t minutes after they start working. Write a formula for $B(t)$ using a definite integral.

Solution. There are initially 21 books waiting to be shelved; and $B(t)$ decreases by 10 per minute and increases at a variable rate according to $f(t)$. So:

$$B(t) = 21 - 10t + \int_0^t f(s)ds \qquad \text{OR} \qquad B(t) = 21 + \int_0^t (f(s) - 10)ds.$$

(Note: I used a dummy variable s in the integrand. $B(t)$ is only a function of t, that is, the upper endpoint of the integral.)

3. [5 points] On the graph below, we have the graphs of f, f' and f''. Identify which is which [3 points] and include a short explanation of your reasoning [2 points].



$I = f$

$II = f'$

$III = f''$

There are many possible ways to figure out the solution. For example, towards the right, III (dashed line) has a local minimum, but neither of the other graphs is zero, so the derivative of III does not appear. Thus $III = f''$.

Next, to figure out which is f', one way is to notice that at $x = 0$, III is positive and the only line with positive slope is II (the solid line), so $II = f'$. That leaves $I = f$ by process of elimination.