The Quest for Primordial Non-Gaussianity

Overview and some recent developments

Dragan Huterer
Physics Department
University of Michigan

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Motivation: testing Inflation

V

\[ \varphi \]

inflaton is slowly rolling

inflation
ends

reheating
Why study non-Gaussianity (NG)?

1. NG presents a window to the very early universe. For example, NG can distinguish between physically distinct models of inflation.

2. Conveniently, NG can be constrained/measured using CMB anisotropy maps and LSS. In particular, there is a rich set of observable quantities that are sensitive to primordial NG.
Non-Gaussianity papers in the past 15 years

Produced by Emiliano Sefusatti

# of articles with “Non-Gaussian” in the title on the ADS data base

Large-Scale Structure
CMB
Inflation / Theory
"non-primordial NG"
Initial conditions in the universe

\[
\frac{\delta T}{T}(\theta, \phi) = \sum_{\ell, m} a_{\ell m} Y_{\ell m}(\theta, \phi) \quad \ell \sim \frac{180^\circ}{\theta}
\]

**Generic inflationary predictions:**

- Flat geometry
- Nearly scale-invariant spectrum of density perturbations
- Background of gravity waves
- (Very nearly) gaussian initial conditions

**Statistical Isotropy:**

\[
\langle a_{\ell m} a_{\ell' m'} \rangle \equiv C_{\ell \ell' m m'} = C_{\ell} \delta_{\ell' \ell} \delta_{m m'}
\]

**Gaussianity:**

\[
\langle a_{\ell m} a_{\ell' m'} a_{\ell'' m''} \rangle = 0
\]
Standard Inflation, with...

1. a single scalar field
2. the canonical kinetic term
3. always slow rolls
4. in Bunch-Davies vacuum
5. in Einstein gravity

produces unobservable NG

Therefore, measurement of nonzero NG would point to a violation of one of the assumptions above

Recall: power spectrum

Define Fourier transform of density fluctuation:

\[ \delta(\vec{r}) = \int \frac{d^3 k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{r}} \delta(\vec{k}) \]

Then the power spectrum \( P(k) \) is defined via

\[ \langle \delta_{\vec{k}_1} \delta_{\vec{k}_2^*} \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2) \, P(k) \]

Sometimes it’s nice to work in harmonic space

\[ a_{\ell m} = 4\pi(-i)^{\ell} \int \frac{d^3 k}{(2\pi)^3} T_{\ell}(k) \delta(\vec{k}) Y_{\ell m}(\hat{k}) \]

Then the angular power spectrum is defined as:

\[ \langle a_{\ell m} a_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} \, C_{\ell} \]
The bispectrum: similar, but for 3-pt function

Fourier space:

\[ \langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \delta_{\vec{k}_3} \rangle = (2\pi)^3 \, \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \, B(\vec{k}_1, \vec{k}_2, \vec{k}_3) \]

Harmonic space:

\[ \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle \equiv B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} \]

and the angle-averaged bispectrum is

\[ B_{\ell_1 \ell_2 \ell_3} \equiv \sqrt{\frac{(2\ell_1 + 1)(2\ell_2 + 1)(2\ell + 1)}{4\pi}} \left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{array} \right) \sum_{m_1 m_2 m_3} \left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{array} \right) B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} \]
3-pt correlation function of CMB anisotropy
⇒ direct window into inflation

e.g. Luo & Schramm 1993

"local"
(eg. multi-field)

"equilateral"
(eg. higher-derivative action; interactions)

NG from 3-point correlation function

Local NG (squeezed triangles) - tests # inflationary fields

\[ \Phi = \Phi_G + f_{NL} (\Phi_G^2 - \langle \Phi_G^2 \rangle) \]

“Equilateral”, “orthogonal” NG- tests inflationary interactions
tests interactions; parameter \( f_{NL}^{eq}, f_{NL}^{orth} \)

Threshold for new physics: \( f_{NL}^{any \ kind} \gtrapprox O(1) \)

Alvarez et al, arXiv:1412.4671
Using publicly available NG maps by Elsner & Wandelt

$f_{NL} = -5000$

$f_{NL} = +5000$

$f_{NL} = 0$ (Gaussian)

$f_{NL} = -500$

$f_{NL} = +500$
Current upper bound on NG is \(~1000\) times smaller than this:
Brief history of NG measurements: 1990’s

Early 1990s; COBE: Gaussian CMB sky (Kogut et al 1996) \[ |f_{NL}| \lessapprox 3000 \] (Komatsu 2002)

1998; COBE: claim of NG at l=16 equilateral bispectrum (Ferreira, Magueijo & Gorski 1998)

but explained by a known systematic effect! (Banday, Zaroubi & Gorski 1999)

(and anyway isn’t unexpected given all bispectrum configurations you can measure; Komatsu 2002)
Brief history of NG measurements: 2000’s

Pre-WMAP CMB: all is gaussian (e.g. MAXIMA; Wu et al 2001)

WMAP pre-2008: all is gaussian
(Komatsu et al. 2003; Creminelli, Senatore, Zaldarriaga & Tegmark 2007)

-36 < f_{NL} < 100  (95% CL)

Dec 2007, claim of NG in WMAP
(Yadav & Wandelt arXiv:0712.1148)

27 < f_{NL} < 147  (95% CL)

Max WMAP multipole used
we always use the marginalizing over the synchrotron, free-free, and dust parameters by combining the V- and W-band maps, and foreground (and foreground-reduced (clean) i.e., we use all the data. We use both the raw maps (be-

The values quoted for "V+W" and "Marg." are our best estimate of $f_{NL}^{local}$, $f_{NL}^{equil}$, $f_{NL}^{orthog}$, and $b_{src}$.

### Table 11: Constraints from WMAP (7-yr)

<table>
<thead>
<tr>
<th>Band</th>
<th>Foreground$^b$</th>
<th>$f_{NL}^{local}$</th>
<th>$f_{NL}^{equil}$</th>
<th>$f_{NL}^{orthog}$</th>
<th>$b_{src}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>V+W</td>
<td>Raw</td>
<td>59 ± 21</td>
<td>33 ± 140</td>
<td>−199 ± 104</td>
<td>N/A</td>
</tr>
<tr>
<td>V+W</td>
<td>Clean</td>
<td>42 ± 21</td>
<td>29 ± 140</td>
<td>−198 ± 104</td>
<td>N/A</td>
</tr>
<tr>
<td>V+W</td>
<td>Marg.$^c$</td>
<td>32 ± 21</td>
<td>26 ± 140</td>
<td>−202 ± 104</td>
<td>−0.08 ± 0.12</td>
</tr>
<tr>
<td>V</td>
<td>Marg.</td>
<td>43 ± 24</td>
<td>64 ± 150</td>
<td>−98 ± 115</td>
<td>0.32 ± 0.23</td>
</tr>
<tr>
<td>W</td>
<td>Marg.</td>
<td>39 ± 24</td>
<td>36 ± 154</td>
<td>−257 ± 117</td>
<td>−0.13 ± 0.19</td>
</tr>
</tbody>
</table>

Komatsu et al. 2011
## Constraints from Planck

<table>
<thead>
<tr>
<th>Shape and method</th>
<th>Independent</th>
<th>ISW-lensing subtracted</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SMICA (T)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local</td>
<td>10.2 ± 5.7</td>
<td>2.5 ± 5.7</td>
</tr>
<tr>
<td>Equilateral</td>
<td>-13 ± 70</td>
<td>-16 ± 70</td>
</tr>
<tr>
<td>Orthogonal</td>
<td>-56 ± 33</td>
<td>-34 ± 33</td>
</tr>
<tr>
<td><strong>SMICA (T+E)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local</td>
<td>6.5 ± 5.0</td>
<td>0.8 ± 5.0</td>
</tr>
<tr>
<td>Equilateral</td>
<td>3 ± 43</td>
<td>-4 ± 43</td>
</tr>
<tr>
<td>Orthogonal</td>
<td>-36 ± 21</td>
<td>-26 ± 21</td>
</tr>
</tbody>
</table>
Constraints from Planck: modal expansion

\[ B(k_1, k_2, k_3) = \sum_{p, r, s} \alpha_{prs} q_p(k_1)q_r(k_2)q_s(k_3) \]
Galaxy cluster counts’ sensitivity to NG

Lots of effort in the community to calibrate the non-Gaussian mass function - $dn/d\ln M(M, z)$ - of DM halos

NG initial PDF $\Rightarrow$ sensitivity to counts “on the tail”
A DM halo gets more massive with $f_{NL}>0$ (and v.v.)

Mapping between $M_G$ and $M=M_{NG}$:

$$dN = \int \frac{dP(M|M_G)}{dM} \frac{dN}{dM_G} dM_G$$

Dalal, Doré, Huterer & Shirokov 2008
NG/Gaussian mass function ratios: for fixed M, more sensitivity at higher redshift

Smith & LoVerde 2011; many others going back to 1990s

Unfortunately, cluster counts are weakly sensitive to NG

e.g. $\sigma(f_{NL})=450$ measured from SPT (Williamson et al 2010)

Nevertheless:

• cluster abundance is sensitive to ALL non-Gaussianity
Effects of primordial NG on the bias of virialized objects
Simulations with non-Gaussianity ($f_{NL}$)

- Same initial conditions, different $f_{NL}$
- Slice through a box in a simulation $N_{\text{part}}=512^3$, $L=800$ Mpc/h

Under-dense region evolution decrease with $f_{NL}$
Over-dense region evolution increase with $f_{NL}$

Dalal, Doré, Huterer & Shirokov 2008
...and now with baryons!

Zhao, Li, Shandera & Jeong, arXiv:1307.5051
Does galaxy/halo bias depend on NG?

$$\text{bias} \equiv \frac{\text{clustering of galaxies}}{\text{clustering of dark matter}} = \left( \frac{\delta \rho}{\rho} \right)_{\text{halos}} \left( \frac{\delta \rho}{\rho} \right)_{\text{DM}}$$

usually nuisance parameter(s)

cosmologists measure

theory predicts

$$\xi_{\text{clusters}}(r) = \left( \frac{r}{25 \text{Mpc}} \right)^{-1.8}$$

$$\xi_{\text{galaxies}}(r) = \left( \frac{r}{5 \text{Mpc}} \right)^{-1.8}$$

Bahcall & Soneira 1983
Bias of dark matter halos

\[ P_h(k, z) = b^2(k, z) P_{DM}(k, z) \]

Simulations and theory both say: large-scale bias is scale-independent (theorem if halo abundance is function of local density and if the short and long modes are uncorrelated)
Scale dependence of NG halo bias

\[ b(k) = b_G + f_{NL} \frac{\text{const}}{k^2} \]

Verified using a variety of theoretical derivations and numerical simulations.

Dalal, Doré, Huterer & Shirokov 2008
\[ \Delta b(k) = f_{NL}(b_G - 1) \delta_c \frac{3 \Omega_M H_0^2}{T(k) D(a) k^2} \]

**Implications:**

- Unique \(1/k^2\) scaling of bias; no free parameters
- Distinct from effect of all other cosmo parameters
- Straightforwardly measured (g-g, g-T,...)
- Extensively tested with numerical simulations; good agreement found
- In general, LSS can probe:
  \[ \Delta b(k) \propto \begin{cases} 
  k^{-2} \text{ (local)} \\
  k^{-1} \text{ (folded)} \\
  k^0 \text{ (equilateral)} \\
  k^{-\alpha} \text{ (generic); } 0 \leq \alpha \leq 3
\end{cases} \]

Dalal et al.; Matarrese & Verde; Slosar et al; Afshordi & Tolley; Desjacques et al; Giannantonio & Porciani; Grossi et al; McDonald; ...
Constraints from current data: SDSS

\[ f_{NL} = 8 \pm 30 \text{ (68\%, QSO)} \]

\[ f_{NL} = 23 \pm 23 \text{ (68\%, all)} \]

Future data forecasts for LSS: \( \sigma(f_{NL}) \approx O(\text{few}) \)
(at least?) as good as, and highly complementary, to Planck CMB
In any case, 2MASS appears to have the least significant covariance effects. Indeed, it appears in Fig. 3 that the observed CCF turns out to be anti-correlated with the NVSS-HEAO correlations. The observed CCF is anti-correlated with the NVSS-HEAO correlations, and the NVSS-HEAO correlations are anti-correlated with the LRG-LRG correlations. While the NVSS-HEAO correlations are anti-correlated, those between the LRG-LRG correlations and the NVSS-HEAO correlations are also anti-correlated. Therefore, lacking a valid reason to include this cut, we thus allow for scale dependence of the bias of any parameter.

We summarize the constraints on $f_{NL}$ found in this work. The constraints on $f_{NL}$ for the LRG-LRG, NVSS-NVSS, NVSS-HEAO, and NVSS-QSO data sets, with and without priors, are shown in Fig. 3. The constraints on $f_{NL}$ for the LRG-LRG, NVSS-NVSS, NVSS-HEAO, and NVSS-QSO data sets, with and without priors, are shown in Fig. 3.

The final constraints on $f_{NL}$ are shown in Fig. 4. The constraints on $f_{NL}$ for the LRG-LRG, NVSS-NVSS, NVSS-HEAO, and NVSS-QSO data sets, with and without priors, are shown in Fig. 3.
### Next Frontier: Large-Scale Structure

<table>
<thead>
<tr>
<th></th>
<th>CMB</th>
<th>LSS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>dimension</strong></td>
<td>2D</td>
<td>3D</td>
</tr>
<tr>
<td><strong># modes</strong></td>
<td>$\propto l_{\text{max}}^2$</td>
<td>$\propto k_{\text{max}}^3$</td>
</tr>
<tr>
<td><strong>systematics &amp; selection func.</strong></td>
<td>relatively clean</td>
<td>relatively messy</td>
</tr>
<tr>
<td><strong>temporal evol.</strong></td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td><strong>can slice in</strong></td>
<td>$\lambda$ only</td>
<td>$\lambda$, $M$, bias...</td>
</tr>
</tbody>
</table>
More general NG models: beyond $f_{NL}$
Current and future constraints on:

\[ f_{\text{NL}}(k) = f_{\text{NL}}^* \left( \frac{k}{k^*} \right)^{n_{f_{\text{NL}}}} \]

Also:

Halos of mass \( M \) probe NG on scale \( k \sim M^{-1/3} \)

Shandera, Dalal & Huterer 2012
• Huge spectroscopic survey on Mayall telescope (Arizona)
• ~5000 fibres, ~15,000 sqdeg, ~20 million spectra
• LRG in 0 < z < 1, ELG in 0 < z < 1.5, QSO 2.2 < z < 3.5
• Great for dark energy (RSD, BAO)
• Great for NG - 3D P(k, z), bispectrum...
• start 2018, funding DOE + institutions
Systematic Errors: (photometric) calibration errors
For the NG measurements, photo-z but also:

(photometric) calibration errors

- **Detector sensitivity**: sensitivity of the pixels on the camera vary along the focal plane. Sensitivity of a given pixel can change with time.

- **Observing conditions**: spatial and temporal variations.

- **Bright objects**: The light from foreground bright stars and galaxies affects the sky subtraction procedure, which impairs the surveys' completeness near bright objects.

- **Dust extinction**: Dust in the Milky Way absorbs light from the distant galaxies.

- **Star-galaxy separation**: In photometric surveys, faint stars can be erroneously included in the galaxy sample. Conversely, galaxies are sometimes misclassified as stars and culled from the sample. Remember, stars are *not* randomly distributed across the sky.

- **Deblending**: Galaxy images can overlap, and it can be difficult to cleanly separate photometric and spectroscopic measurements for the blended objects.

Huterer, Cunha & Fang 2013
Figure 2. The angular power spectrum in the four redshift slices of quasars. The choice of symbols is the same as for LRGs in fig. 1. We note again that bins in each redshift slice that do not appear contaminated can still be dropped because their cross-power with another redshift slice is significantly contaminated and one cannot tell a priori which redshift slice is responsible for the contamination.

Given value of $A_{NL} k_p=0.1Mpc$ $^{-1}$, modifications to the power spectrum in the presence of primordial non-Gaussianity come in at the largest measured scales (i.e. at small $k$). This is no longer true when we allow for deviations from the local ansatz. In particular, as we increase the value of $\alpha$, non-Gaussian corrections become significant at smaller scales (close to matter-radiation equality) which are better measured, strongly constraining models of inflation that give $\alpha>2$. On the other hand, for $0<\alpha<2$, non-Gaussian corrections are only significant at much larger scales, which are eventually limited by systematics.

Next we examine the constraints on $\alpha$ for a small fixed value of $A_{NL}$. In fig. 3 we see the results for LRGs (not shown).

QSO power spectra from SDSS; open circle points not used since they may be systematics-contaminated!

Similar results for LRGs (not shown)
LSS calibration errors: example maps, power spectra

- dominate on large angular scales
- can be measured, removed using same or other data

Leistedt et al 2013

Huterer, Cunha & Fang 2013; Shafer & Huterer 2015
Dark Energy Survey (2012)

Harvard-CfA survey (1980s)

21cm mapping

Euclid and WFIRST (~202X)

LSST (~2018)
SPHEREx proposal for telescope dedicated to measuring NG (and other science)

- 97 bands (!) with Linearly Variable Filters (LVF)
- $\lambda$ between 0.75 and 4 $\mu$m
- small (20cm) telescope, big field of view
- whole sky out to $z \sim 1$

**goal:** $\sigma(f_{NL}) \leq 1$  

Conclusions:

- Primordial NG directly tests inflation:
  - How many fields
  - What interactions, couplings
- Constraints from WMAP, Planck are superb and consistent with zero NG
- Extremely good prospects for testing with galaxy surveys, at smaller scales than CMB
Advances in Astronomy special issue on “Testing the Gaussianity and Statistical Isotropy of the Universe”
http://www.hindawi.com/journals/aa/2010/si.gsiu/

15 review articles (all also on arXiv)

Testing the Gaussianity and Statistical Isotropy of the Universe
Guest Editors: Dragan Huterer, Eiichiro Komatsu, and Sarah Shandera

Non-Gaussianity from Large-Scale Structure Surveys, Licia Verde
Volume 2010 (2010), Article ID 768675, 15 pages

Non-Gaussianity and Statistical Anisotropy from Vector Field Populated Inflationary Models, Emanuela Dimastrogiovanni, Nicola Bartolo, Sabino Matarrese, and Antonio Riotto
Volume 2010 (2010), Article ID 752670, 21 pages

Cosmic Strings and Their Induced Non-Gaussianities in the Cosmic Microwave Background,