

GRAVITATIONAL LENSING AS A PROBE OF QUINTESSENCE

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ABSTRACT

A large number of cosmological studies now suggest that roughly two-thirds of the critical energy density of the universe exists in a component with negative pressure. If the equation of state of such an energy component varies with time, it should in principle be possible to identify such a variation using cosmological probes over a wide range in redshift. Proper detection of any time variation, however, requires cosmological probes beyond the currently studied range in redshift of ~ 0.1 – 1 . We extend our analysis to gravitational lensing statistics at high redshift and suggest that a reliable sample of lensed sources, out to a redshift of ~ 5 , can be used to constrain the variation of the equation of state, provided that both the redshift distribution of lensed sources and the selection function involved with the lensed source discovery process are known. An exciting opportunity to catalog an adequate sample of lensed sources (quasars) for probing quintessence is now available with the ongoing Sloan Digital Sky Survey. Writing $w(z) \approx w_0 + z(dw/dz)_0$, we study the expected accuracy to which the equation of state today (w_0) and its rate of change $[(dw/dz)_0]$ can simultaneously be constrained. Such a determination can rule out some missing-energy candidates, such as classes of quintessence models or a cosmological constant.

Subject headings: cosmology: observations — cosmology: theory — gravitational lensing — quasars: general

1. INTRODUCTION

There is now strong evidence that the matter density of the universe, both baryonic and dark, is smaller than the critical value predicted by inflation. Some of the same cosmological probes that suggest that the ratio of matter density to critical density is less than one prefer the presence of an additional component with energy density such that the total energy density of the universe is in fact the critical value (see, e.g., Lineweaver 1998; Perlmutter et al. 1999; Riess et al. 1998). Type Ia supernovae and other observations have recently provided evidence that the equation of state, w , of this component is $-1 \lesssim w \equiv P/\rho \lesssim -\frac{1}{3}$ (Garnavich et al. 1998; Waga & Miceli 1998; Cooray 1999).

The oldest known candidate for this additional energy is the cosmological constant, characterized by $w = -1$. However, other possibilities have now been considered, including a slowly varying, spatially inhomogeneous scalar field component better known as quintessence (see, e.g., Huey et al. 1998; Zlatev, Wang, & Steinhardt 1998; Wang & Steinhardt 1998). Certain quintessence models, called *tracker models* (Ferreira & Joyce 1998; Zlatev et al. 1998; see Steinhardt, Wang, & Zlatev 1998 for a comprehensive review), have the feature that the energy density of the scalar field, Ω_ϕ , is a fixed fraction of the energy density of the dominant component. Therefore, such models may explain the coincidence problem—why Ω_ϕ is of the same order of magnitude as Ω_m today. In these tracker models, w is a function of redshift and typically varies from $w \sim \frac{1}{3}$ during the radiation-dominated era to $w \sim -0.2$ during the matter-dominated era, finally reaching a value of $w \sim -0.8$ during late epochs (today). Time variation of the equation of state is even more prominent in scalar field models involving pseudo-Nambu-Goldstone bosons (PNGB models) (Frieman et al. 1995). In such models, the field is frozen to its initial value because of the large expansion rate, but it becomes dynamical at some later stage at redshift z_x . Likely values for z_x are roughly

between 3 and 0 (Coble, Dodelson, & Frieman 1997), which means that interesting dynamics—and hence the variation in the equation of state—happen at redshift of a few. Huterer & Turner (1998) point out that distance measurements to Type Ia supernovae offer a possibility to reconstruct the quintessence potential, while Starobinsky (1998) and Wang & Steinhardt (1998) suggested the possibility of using cluster abundances as a function of redshift as well.

In § 2, we study constraints on $w(z)$ based on current Type Ia supernovae distances, gravitational lensing statistics, and globular cluster ages. In § 3, we consider the possibility of imposing reliable constraints on $w(z)$ based on cosmological probes at high redshift, in particular gravitational lensing statistics. We follow the conventions that the Hubble constant, H_0 , is $100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$, that the present matter energy density in units of the closure density is Ω_m , and that the normalized present-day energy density in the unknown component is Ω_ϕ .

2. CURRENT CONSTRAINTS ON $w(z)$

Since a generic quintessence model has a time-varying equation of state, one should be able to distinguish it from models where the equation of state is time independent (see, e.g., Turner & White 1997). In this Letter, we write the equation of state of the unknown component as

$$w(z) \approx w_0 + z(dw/dz)_0. \quad (1)$$

This relation should be a good approximation for most quintessence models out to redshift of a few, and, of course, it should be exact for models where $w(z)$ is a constant or changing slowly. Note that negative $(dw/dz)_0$ corresponds to an equation of state that is larger today compared with early epochs. Models where the scalar field is initially frozen typically exhibit such behavior, while, for tracker field models, $(dw/dz)_0 > 0$.

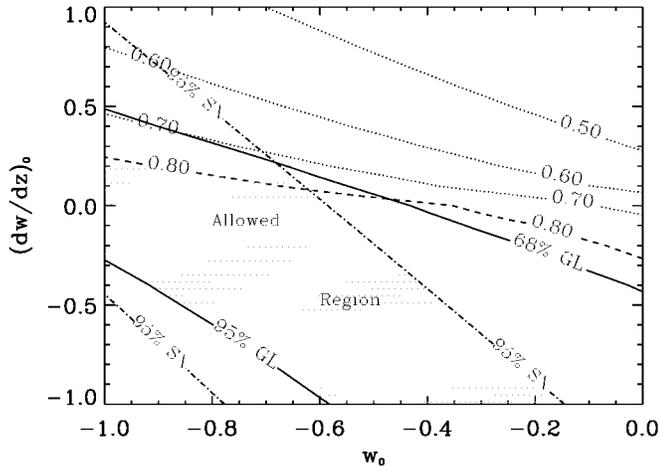


FIG. 1.—Current constraints on $w(z) \approx w_0 + z(dw/dz)_0$. The dot-dashed lines are the current Type Ia supernovae (Riess et al. 1998; Perlmutter et al. 1999) constraints at the 95% confidence level, while the upper limits from gravitational lensing statistics (Cooray 1999) are shown with the solid lines. The age of the universe as a function of H_0 is shown by the dotted lines. A conservative lower limit on this parameter is 0.8 (dashed line) when age of the universe is ~ 14 – 15 Gyr and $H_0 \sim 65$ km s $^{-1}$ Mpc $^{-1}$. The shaded region defines the parameter region that is allowed by current data.

In order to constrain $w(z)$, we extend current published analyses that have so far considered the existence of a redshift-independent equation of state (see, e.g., Cooray 1999; Garavich et al. 1998; Waga & Miceli 1998). Since the only difference between this study and previous ones is that we now allow w to vary with z , formalisms presented in previous papers should also hold, except for the fact that w is now redshift dependent. We refer the reader to previous work for detailed formulae and calculational methods.

In Figure 1, we summarize current constraints on $w(z)$ as given in equation (1). Here we have assumed a flat universe with $\Omega_M = \frac{1}{3}$. Evidence for such low matter density, independent of the nature of the additional energy component, comes primarily from the mass density within galaxy clusters (see, e.g., Evrard 1997). As shown in Figure 1, there is a wide range of possibilities for $w(z)$, and the allowed parameter space is consistent with the degeneracies discussed in the literature (see, e.g., Zlatev et al. 1998). Even though the $w \sim -\frac{1}{3}$ model has been ruled out by combined Type Ia supernovae, gravitational lensing, and globular cluster ages, we now note that a model in which $w_0 \sim -\frac{1}{3}$ but $(dw/dz)_0 \sim -0.9$ is fully consistent with the current observational data.

In order to test whether one can constrain $w(z)$ better than the current data, we increased the Type Ia supernova sample between redshifts of 0.1 and 1; however, the degeneracy between w_0 and $(dw/dz)_0$ did not change appreciably. Increasing the upper redshift of supernova samples decreased the degeneracies; thus, cosmological probes at high redshifts are needed to distinguish properly redshift-dependent $w(z)$ from a constant w model. A probe to a much higher redshift is provided by the cosmic microwave background (CMB) anisotropy data; however, as pointed out in Huterer & Turner (1998) and Huey et al. (1998), CMB anisotropy is not a strong probe of $w(z)$. This is due to the fact that $w(z)$ affects mostly the lower multipoles, which cannot be measured precisely because of cosmic variance. Also, supernovae and galaxy clusters are unlikely to constrain $w(z)$ in the near future given that current observational

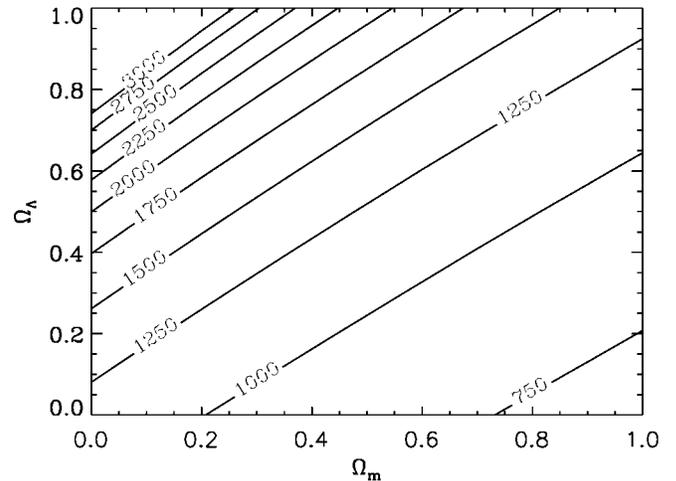


FIG. 2.—Expected number of multiply imaged quasars with image separations between $1''$ and $6''$ in the SDSS data. We have only counted lensed quasars with at least two images greater than a magnitude limit of 21. The expected number is a constant along the $\Omega_M - \Omega_\Lambda$ lines. For $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$, we expect ~ 2000 lensed sources to be detected within the SDSS.

programs are not likely to recover them at high redshifts ($z \gtrsim 2$).

Thus, an alternative probe to high redshifts is needed. In the next section, we consider the possibility of using gravitational lensing statistics, in particular the redshift distribution of strongly lensed sources, as a probe of $w(z)$. Such statistics, in principle, probe the volume of the universe out to a redshift of ~ 5 . In the past, lensing statistics were hampered by the lack of large samples of lensed sources with a well-known selection function and their redshift distributions, which are all needed to constrain $w(z)$. An exciting possibility is now provided by the upcoming high-quality data from the Sloan Digital Sky Survey¹ (SDSS; Gunn & Knapp 1993), which is going to image π steradians of the sky down to a 1σ magnitude limit of ~ 23 . Since no high-redshift ($z \gtrsim 2$) cosmological probes yet exist, gravitational lensing statistics may be the prime candidate to study $w(z)$.

3. GRAVITATIONAL LENSING STATISTICS

In order to calculate the expected number of lensed quasars, in particular, considering the SDSS, we extend previous calculations in Wallington & Narayan (1993; also the notes of S. Dodelson & D. MacMinn 1997, unpublished) and Cheng & Krauss (1998). Our calculation follows that of Cooray, Quashnock, & Miller (1999), in which we calculated the number of lensed galaxies in the Hubble Deep Field. We follow the magnification bias (see, e.g., Kochanek 1991) calculation in Cheng & Krauss (1998). We calculate the expected number of lensed sources as a function of the magnitude limit of the SDSS and select sources with image separations between $1''$ and $6''$. This range is selected based on image resolution and source confusion limits. Our prediction, shown in Figure 2 as a function of Ω_M and Ω_Λ , is based on the current determination of the quasar luminosity function, which is likely to be updated once adequate quasars statistics are available from the SDSS.

As shown, there are about 2000 lensed quasars down to a

¹ See <http://www.sdss.org>, which is maintained at Apache Point Observatory under the auspices of the Astrophysical Research Consortium.

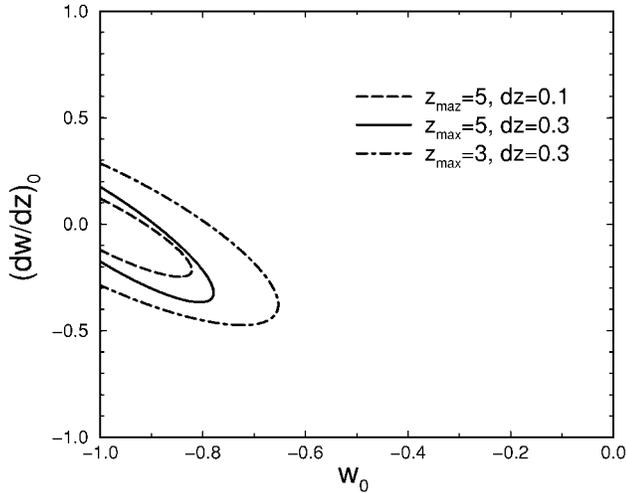


FIG. 3.—Constraints (2σ) on w_0 and $(dw/dz)_0$ using the lensed source redshift distribution that is expected from the SDSS for a fiducial cosmological model of $w_0 = -1$, $(dw/dz)_0 = 0$, and marginalized over Ω_M ($\Omega_M = 0.3 \pm 0.1$ and $\Omega_\Lambda = 1 - \Omega_M$). We show the expected 2σ errors when the redshift distribution of lensed sources is known with an accuracy of 0.1 and 0.3, while the maximum redshift probed by lensing statistics is 3 and 5.

limiting magnitude of 21 that could, in principle, be detected from the SDSS data if $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$. This number drops to about ~ 600 when $\Omega_M = 1.0$; however, such a cosmological model is already ruled out by current data. In this calculation, we have ignored effects that are due to extinction and dust; this is an issue with no consensus among various studies (Malhotra, Rhoades, & Turner 1997; Falco, Kochanek, & Muñoz 1998). It is likely that a combined analysis of statistics from ongoing lensed radio sources and optical searches may increase the knowledge of such systematic effects. The SDSS lensed quasars lie in redshifts out to $z \sim 5$. Detection of such high-redshift lensed sources is likely to be aided by the five-color imaging data, including the z -filter. This possibility has already been demonstrated by the detection of some of the highest redshift quasars known today using the SDSS first-year test images (Fan et al. 1998).

4. LENSING CONSTRAINTS ON $w(z)$: PROSPECTS

In order to estimate the accuracy to which one can constrain $w(z)$ based on gravitational lensing statistics, we take a Fisher matrix approach with parameters w_0 and $(dw/dz)_0$ and assume a flat universe. As stated in the literature (see, e.g., Tegmark, Taylor, & Heavens 1997), the Fisher matrix analysis allows one to estimate the best statistical errors on parameters calculated from a given data set. The Fisher matrix F is given by

$$F_{ij} = - \left\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \right\rangle_x, \quad (2)$$

where L is the likelihood of observing data set \mathbf{x} given the parameters $p_1 \dots p_n$.

We bin the observations (number of lensed quasars) in redshift bins of width Δz out to a maximum redshift z_{\max} . Since the selection function for the quasar discovery process is still unknown, we adopt a Poisson likelihood function at each redshift bin Δz centered at redshift z . The expected number of lensed sources, N_{exp} , in each redshift bin takes into account the

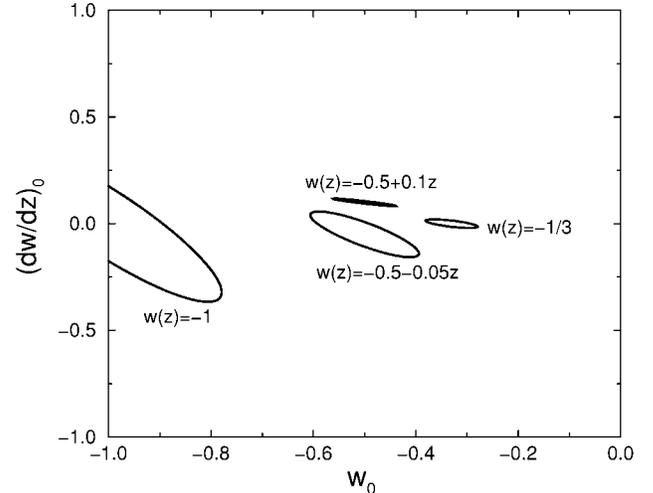


FIG. 4.—Constraints (2σ) on w_0 and $(dw/dz)_0$ using the lensed source redshift distribution that is expected from the SDSS for four fiducial cosmological models when the redshift of lensed sources is known with an accuracy of 0.3 out to a redshift of 5. In all cases, we assumed a flat universe and marginalized over Ω_M ($\Omega_M = 0.3 \pm 0.1$).

magnitude limit, the range of allowed image separations, as well as the magnification bias. With these approximations, we can now write the Fisher matrix as

$$F_{ij} = \sum_{\Delta z} \frac{1}{N_{\text{exp}}(z, \Delta z)} \frac{\partial N_{\text{exp}}(z, \Delta z)}{\partial p_i} \frac{\partial N_{\text{exp}}(z, \Delta z)}{\partial p_j}. \quad (3)$$

This form for the Fisher matrix is equivalent to the one derived from a Gaussian distribution, when the uncertainty (σ) of the distribution is taken to be equal to the shot-noise term $[N_{\text{exp}}(z, \Delta z)]^{1/2}$.

In Figure 3, we show the expected (2σ) uncertainties in w_0 and $(dw/dz)_0$. Here we considered a flat universe with three parameters: Ω_M , w_0 , and $(dw/dz)_0$. We marginalized over Ω_M , allowing $\Omega_M = 0.3 \pm 0.1$ (1σ), and considered a fiducial model with $w_0 = -1$ and $(dw/dz)_0 = 0$. The three curves in Figure 3 show variation of the constraint region with the redshift bin width Δz (which is roughly equal to the uncertainty in the redshift determination) and with the maximum redshift of detected quasars z_{\max} . Photometric redshifts now allow redshift determinations with an accuracy on the order of 0.1 68% of the time and on the order of 0.3 100% of the time (see, e.g., Hogg et al. 1998). It is apparent that the size of the constraint region decreases significantly with increasing z_{\max} and decreasing Δz . Note that one can quite safely assume that $z_{\max} \approx 5$ for quasars in a survey such as the SDSS. Additionally, these calculations assume a limiting magnitude of 21, much lower than the expected 1σ limiting magnitude of 23 for the SDSS data.

Figure 4 shows constraints in the $[w_0, (dw/dz)_0]$ -plane for four fiducial models. The models shown are the cosmological constant [$w(z) = -1$], non-Abelian cosmic strings [$w(z) = -\frac{1}{3}$; Spergel & Pen 1997], and two quintessence models exhibiting a variation in w at small z [$w(z) = -0.5 + 0.1z$ and $w(z) = -0.5 - 0.05z$]. In all cases, we assumed a flat universe and marginalized over Ω_M ($\Omega_M = 0.3 \pm 0.1$). We also assumed that redshifts of lensed objects are determined with an accuracy of $\Delta z = 0.3$ and that we have data out to redshift of $z_{\max} = 5$. This figure shows that the strength of the constraints depends

strongly on the fiducial model. In fact, we found that fiducial models for which $w_0 + z(dw/dz)_0 \sim -1$ (for z of order unity) give weaker constraints. This result is not surprising and can be understood by simply using the fact that the number of lensed objects out to a redshift of z is roughly proportional to the volume of the universe $V(z)$. Since dV/dp_i [where p_i is either w_0 or $(dw/dz)_0$] is proportional to $(1+z)^{1+w_0+z(dw/dz)_0}$, we see that the expected number of lensed quasars varies slowly with p_i if $w_0 + z(dw/dz)_0 \sim -1$. In that case, our constraint region will be relatively large.

5. SUMMARY AND CONCLUSIONS

In this Letter, we considered the possibility of constraining quintessence models that have been suggested to explain the missing-energy density of the universe. We suggested gravitational lensing statistics, which can be used as a probe of the equation of state of the missing component, $w(z)$. An exciting possibility to obtain an adequate sample of lensed quasars and

their redshifts comes from the Sloan Digital Sky Survey. Writing $w(z) \approx w_0 + z(dw/dz)_0$, we studied the expected accuracy to which the equation of state today w_0 and its rate of change $(dw/dz)_0$ can simultaneously be constrained. Adopting some conservative assumptions about the quality of the data from SDSS and assuming a flat universe with $\Omega_M = 0.3 \pm 0.1$, we conclude that tight constraints on these two parameters can indeed be obtained. The strength of the constraints depends not only on the quality of the lensing data from the SDSS but also on the fiducial model [true values of w_0 and $(dw/dz)_0$]. In particular, fiducial models for which $w_0 + z(dw/dz)_0 \sim -1$ (for z of order unity) give weaker constraints on w_0 and $(dw/dz)_0$.

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