The augmented Lagrangian (AL) method [1] has drawn more attention recently due to its scalability, simplicity, and fast convergence. One variation of the AL method is to precondition the $\ell_1$ penalty term in the augmented Lagrangian by some positive definite matrix. For example, when choosing a diagonal matrix, we penalize each entry of the split variable differently, so we can have larger steps for those entries that are still far from the solution by decreasing the penalty.

In statistical X-ray CT image reconstruction, the image reconstruction is usually formulated as a PWLS problem, and the ordered-subset (OS) algorithm [5] can be used to accelerate the gradient descent method about $M$ times by grouping the projections into $M$ ordered subsets and updating the image incrementally using the $M$ subset gradients.

In this paper, we propose to combine the AL method with OS, where a diagonal preconditioner is used in the AL method so that the inner minimization problem is another statistically weighted CT problem, and we use it to create your research poster and save valuable time placing titles, subtitles, text, and graphics.

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As can be seen in (5), the inner minimization is another CT reconstruction problem with an updated sinogram and a scaled regularization, and it can be solved using OS algorithms, e.g., [5]. Finally, we have proposed our AL-OS algorithm:

$$\begin{align*}
\mathbf{x}^{(j+1)} &\in \text{argmin}_{\mathbf{x} \in \Omega} \left\{ \frac{1}{2} \| y - A\mathbf{x} \|_W^2 + R(x) \right\} \\
\mathbf{u}^{(j+1)} &\in \text{argmin}_{\mathbf{u} \in \Omega} \left\{ \frac{1}{2} \| \mathbf{u}^{(j)} + \mathbf{d}^{(j)} - \mathbf{A} \mathbf{x}^{(j+1)} \|_G^2 + R(x) \right\} \\
\mathbf{d}^{(j+1)} &= \mathbf{d}^{(j)} - \mathbf{A} \mathbf{x}^{(j+1)} + \mathbf{u}^{(j+1)}.
\end{align*}$$

To find a “good” preconditioner $G$, we analyze a simpler CT problem with a quadratic regularizer and no box constraint. We can show that no matter what regularization parameter is used, $G = W$ leads to a quite fast, with rate $\frac{1}{2}$, convergence of the split variable $u$. Inspired by this, we focus on the diagonal preconditioner $G = \mathbf{n}W$ with $\mathbf{n} > 0$, and the AL iterates become

$$\begin{align*}
\mathbf{x}^{(j+1)} &\in \text{argmin}_{\mathbf{x} \in \Omega} \left\{ \frac{1}{2} \| \mathbf{u}^{(j)} + \mathbf{d}^{(j)} - \mathbf{A} \mathbf{x}^{(j+1)} \|_W^2 + \eta^{-1} R(x) \right\} \\
\mathbf{u}^{(j+1)} &= \frac{1}{\eta} \left( \mathbf{y} + \eta \mathbf{A} \mathbf{x}^{(j+1)} - \mathbf{d}^{(j)} \right) \\
\mathbf{d}^{(j+1)} &= \mathbf{d}^{(j)} - \mathbf{A} \mathbf{x}^{(j+1)} + \mathbf{u}^{(j+1)}.
\end{align*}$$

As can be seen in (5), the inner minimization is another CT reconstruction problem with an updated sinogram and a scaled regularization, and it can be solved using OS algorithms, e.g., [5]. Finally, we have proposed our AL-OS algorithm:

**RESULT**

We evaluate our proposed AL-OS algorithm using a helical CT scan and investigate the effects of the update period and the AL penalty parameter with different values of $P$ and $\eta$. The test sinogram is of size $888 \times 32 \times 7146$ with pitch 0.5312, and the image size is $512 \times 512 \times 109$. Lastly, since the test helical CT scan contains gain fluctuations [7], we include blind imaging, allowing the AL-OS algorithm with OS algorithm to perform gain correction. We use the following objective function [8] in all of our reconstruction algorithms. With this correction, the statistical weighting matrix $W$ and the preconditioning matrix $G$ are “diagonal-plus-rank-one” rather than pure diagonal matrices, which is a simple extension of the proposed diagonal preconditioned AL method. The naming conventions in our experiment are as follows: OS-SQS-M denotes the standard OS algorithm [5] with $M$ subsets, and OS-AL-OS-M-P denotes the proposed AL-OS algorithm with $M$ subsets, the AL penalty parameter $\eta$, and the update period $P$.

**CONCLUSION**

In this paper, we proposed to combine the AL method with OS for solving X-ray CT image reconstruction problems. Inspired by the optimality properties of the AL method for quadratic regularized CT problems, we focused on a diagonal preconditioning matrix $G$ that is proportional to the statistical weighting matrix $W$. Experimental results show that the proposed AL-OS algorithm accelerates the standard OS algorithm remarkably. As can be seen from the convergence rate curves, smaller $\eta$ leads to faster but non-monotone convergence. One possible future work is to consider using continuation, e.g., a decreasing sequence of $\eta$ from 1 to 0, in the proposed AL-OS algorithm for further acceleration.

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**REFERENCES**