1 Logistics

Next meeting: Friday March 15, 9:30 - 11am

Expectations for next meeting:

- Problem 9: mountain / valley labellings for hexagon crease pattern
- Problem 10: using the formula for spherical triangles relating external angles to (spherical) area, prove the generalization to spherical n-gons.
- [**Writing**] Continue writing up relevant discussion from this week in draft of final report

2 Hexagon configuration space

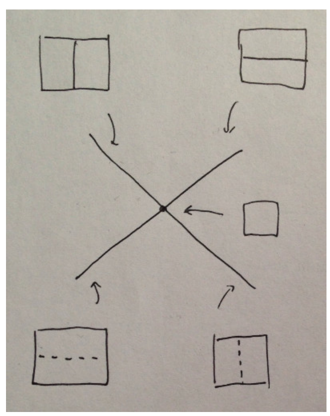
2.1 Mountain / valley diagrams

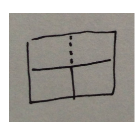
To understand a topological space it often helps to cut it up into smaller, more manageable pieces.

For a fold configuration which is "near flat," we say a crease is a *mountain fold* if it is higher than a secant line between its two adjacent flat regions, and a *valley fold* if it is lower. (We take the nearby flat configuration as reference for what "higher" and "lower" mean.)

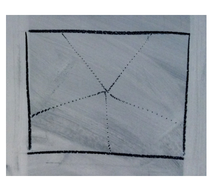
Suppose we label each crease in a configuration with "mountain" or "valley" or neither. Visually we can indicate these respectively by a solid line, a dotted line, or no line.

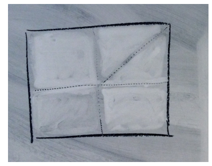
Example 1. For two perpendicular creases (so there are 4 total crease vectors), the following shows possible mountain/valley labellings:



The following mountain/valley labelling is not possible: 

Problem 8. In the following diagrams with 5 creases coming from a central vertex, which mountain/valley labellings are possible? How do these fit together in configuration space near the unfolded state?

(a) creases with equal spacings 

(b) creases perpendicular plus one at 45° angle 

Discussion:

(a) In this crease configuration, the unit null cone near the unfolded state is two disjoint circles. If we split up this space according to mountain/valley labellings, there are 10 one-dimensional regions and 10 zero-dimensional regions.

[INSERT DIAGRAM]

(b) If we split up this space according to mountain/valley labellings, there are 4 one-dimensional regions and 4 zero-dimensional regions.

[INSERT DIAGRAM]

Problem 9. In a diagram with 6 creases coming together at a vertex at equal angles, the unit null cone near the flat space is two disjoint spheres. We consider cutting up these spheres into regions according to mountain/valley labellings.

(a) How many zero-dimensional, one-dimensional, and two-dimensional regions are there on each sphere?

(b) What shapes are the two-dimensional regions? How do they fit together? (i.e. draw out a diagram)