1 Logistics

Next meeting: Tuesday April 2, 11:30 - 1pm

Expectations for next meeting:

- mountain / valley labellings for hexagon crease pattern

  Problem 9: In a diagram with 6 creases coming together at a vertex at equal angles, the unit null cone near the flat space is two disjoint spheres. We consider cutting up these spheres into regions according to mountain/valley labellings.

  (b) What shapes are the two-dimensional regions? How do they fit together?
  ($\rightarrow$ draw the diagrams we made in this meeting)

- angle constraints on hexagon crease pattern

  Problem 11: Suppose we want to fix three consecutive creases on a hexagon at fold angles θ_1, θ_2, and θ_3. (By "fold angle" we mean flat $\Leftrightarrow$ $\theta_i = 0$.)

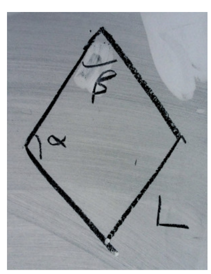
  (a) What combinations of $\theta_1, \theta_2, \theta_3$ are possible for a hexagon? Find exact equations to characterize this. (Hint: it may help to use `https://en.wikipedia.org/wiki/Rodrigues%27_rotation_formula`)
  For example, if $\theta_2 = 0$ and $\theta_1, \theta_3 > 0$ then this is possible (and we can compute $\theta_4, \theta_5, \theta_6$), but if $\theta_2 = 0$, $\theta_1 > 0$, $\theta_3 < 0$ then this is not possible for the hexagon.

  Particular sub-problem:

  Problem 11 (c) Suppose $\theta_1 = \theta_3 = 0$ and θ_2 is arbitrary. Then one possibility is that $\theta_4 = \theta_6 = 0$ and $\theta_5 = \theta_2$. A second possibility is that crease number 5 is "flipped inward", so that $\theta_5 = -\theta_2$, and θ_4 and θ_6 are at some positive angle. What is the angle of $\theta_4 = \theta_6$ as a function of θ_2, in this case?

  Sub-sub-problem:

  Problem 11 (d) Consider a spherical rhombus, i.e. a quadrilateral on a unit sphere with all four side lengths L. Show that opposite pairs of angles are equal, say α and β.



  Show that α and β satisfy the relation

$$\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{1}{\cos L}.$$

  Graph this relation on α, β for $L = \frac{\pi}{3}$ (i.e. the spherical length of a hexagon edge).

  (Check: As $L \to 0$, why does this agree with the relation $\alpha + \beta = \pi$ for flat geometry?)

- [**Writing**] Write up notes for this meeting, and continue writing up relevant discussion from this week in draft of final report
- [**Final project**] Think about whether you would rather create a final report in article form, or a hexagon visualization program as a final project, or both (in addition to the poster)