Math 116 - 099

Quiz 8 §§ 9.1 - 9.4 Name:

4. [8 points] Determine whether the following series converge or diverge.
Fully justify your answer. Show all work and include any convergence tests used.

a. [4 points] $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sin\left(\frac{1}{n}\right)}$

Circle one: **Converges** **Diverges**

Justification:

b. [4 points] $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n^2 + 3^n}$

Circle one: **Converges** **Diverges**

Justification:

3. [12 points]

a. [6 points] State whether each of the following series converges or diverges. Indicate which test you use to decide. Show all of your work to receive full credit.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(n)}{\sqrt{n^3}}$

b. [6 points] Decide whether each of the following series converges absolutely, converges conditionally or diverges. Circle your answer. No justification required.

1. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\sqrt{n^2+1}}{n^2+n+8}$

Converges absolutely   Converges conditionally   Diverges

2. $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^{3n}}{5^n}$

Converges absolutely   Converges conditionally   Diverges

13. [10 points] Suppose a_n and b_n are sequences with the following properties.

- $\sum_{n=1}^{\infty} a_n$ converges.
- $n \le b_n \le e^n$.

For each of the following statements, decide whether the statement is always true, sometimes true, or never true. Circle your answer. No justification is necessary. **You only need to answer 5 of the 7 questions.** Only answer the 5 questions you want graded. If it is unclear which 5 questions are being answered, the first 5 questions you answer will be graded.

a. [2 points] The sequence $\frac{1}{b_n}$ diverges.

ALWAYS **SOMETIMES** **NEVER**

b. [2 points] The sequence a_n is bounded.

ALWAYS **SOMETIMES** **NEVER**

c. [2 points] The series $\sum_{n=1}^{\infty} \frac{1}{b_n}$ diverges.

ALWAYS **SOMETIMES** **NEVER**

d. [2 points] The series $\sum_{n=1}^{\infty} e^{-a_n}$ converges.

ALWAYS **SOMETIMES** **NEVER**

e. [2 points] The series $\sum_{n=1}^{\infty} a_n^2$ diverges.

ALWAYS **SOMETIMES** **NEVER**

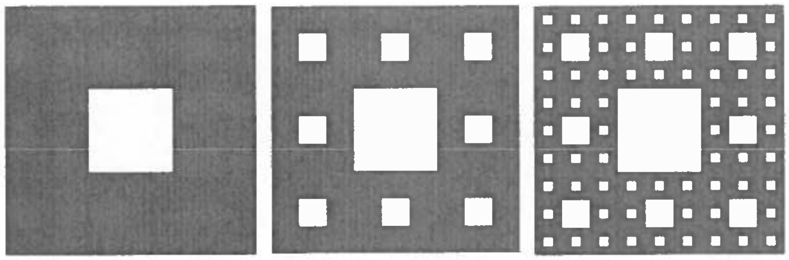
f. [2 points] The series $\sum_{n=1}^{\infty} a_n b_n$ converges.

ALWAYS **SOMETIMES** **NEVER**

g. [2 points] The series $\sum_{n=1}^{\infty} \frac{b_n}{n!}$ converges.

ALWAYS **SOMETIMES** **NEVER**

4. (8 points) The Sierpinski Carpet is an example of a mathematical object called a fractal. It is constructed by removing the center one-ninth of a square of side 1, then removing the centers of the eight smaller remaining squares, and so on. (The figure below shows the first three steps of the construction.)



At the n-th step of the process, 8^{n-1} squares are removed, each with area $\left(\frac{1}{9}\right)^n$. Thus, the area removed at the n-th step is $A_n = \left(\frac{8^{n-1}}{9^n}\right)$. There are infinitely many steps in the process.

(a) (2 pts.) Find the limit of the sequence $A_1, A_2, A_3, \ldots$.

(b) (2 pts.) Write a mathematical expression that represents A, the *total* sum of the areas of the removed squares after infinitely many steps of the process.

(c) (4 pts.) Exactly how much area is removed in all? Show your work.