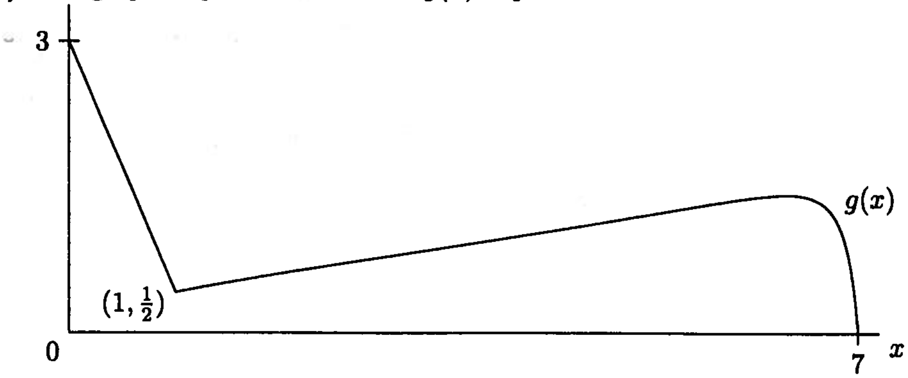
10. [8 points] The graph of part of a function $g(x)$ is pictured below.



a. [4 points] A thumbtack has the shape of the solid obtained by rotating the region bounded by $y = g(x)$, the x-axis and y-axis, about the y-axis. Find an expression involving integrals that gives the volume of the thumbtack. Do not evaluate any integrals.

b. [4 points] A door knob has the shape of the solid obtained by rotating the region bounded by $y = g(x)$, the x-axis and y-axis, about the x-axis. Find an expression involving integrals that gives the volume of the door knob. Do not evaluate any integrals.

7. [10 points] Maize and Blue Jewelry Company is trying to decide on a design for their signature aMaize-ing bracelet. There are two possible designs: type W and type J. The company has done research and the two bracelet designs are equally pleasing to customers. The design for both rings starts with the function $C(x) = \cos\left(\frac{\pi}{2}x\right)$ where all units are in millimeters. Let R be the region enclosed by the graph of $C(x)$ and the graph of $-C(x)$ for $-1 \leq x \leq 1$.

   a. [5 points] The type W bracelet is in the shape of the solid formed by rotating R around the line $x = 50$. Write an integral that gives the volume of the type W bracelet. Include **units.**

   b. [5 points] The type J bracelet is in the shape of the solid formed by rotating R around the line $y = -50$. Write an integral that gives the volume of the type J bracelet. Include **units.**

6. [12 points]

a. [3 points] Let f be a positive, continuous function. Which of the following are antiderivatives of f whose graphs go through the point $(1,0)$? Circle all that apply.

$$\int_0^1 f(t)\,dt \qquad \int_0^x f(t)\,dt + \int_1^0 f(t)\,dt \qquad \int_0^x f(t)\,dt$$

$$\int_2^{2x} f(t/2)\,dt \qquad \frac{1}{2}\int_2^{2x} f(t/2)\,dt$$

b. [3 points] Let R be the region between the x-axis and the graph of some positive, continuous function from $x=a$ to $x=b$. If V is the volume of the solid whose base is R and whose cross-sections parallel to the y-axis are semicircles, what is the volume of the solid whose base is R and whose cross-sections parallel to the y-axis are equilateral triangles?

$$\frac{\sqrt{3}}{4}V \qquad \frac{2\sqrt{3}}{\pi}V \qquad \frac{4\sqrt{3}}{\pi}V \qquad 2\pi V \qquad \frac{2\pi}{\sqrt{3}}V$$

c. [3 points] Which of the following expressions gives the arclength of the graph of $y=\sin(x^2)$ from $x=0$ to $x=\sqrt{\pi}$?

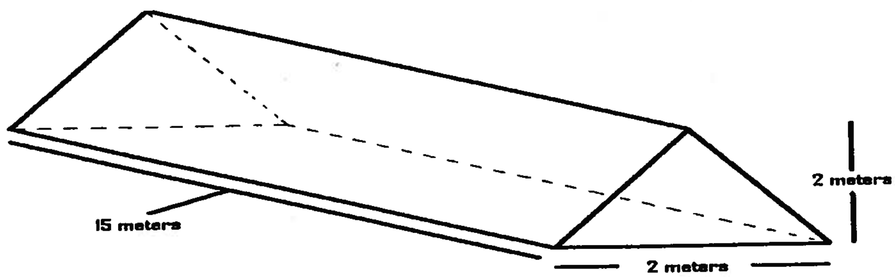
$$\int_0^{\sqrt{\pi}} \sqrt{1+2x^2\sin^2(x^2)}\,dx \qquad \int_0^{\sqrt{\pi}} \sqrt{1+\sin^2(x^2)}\,dx \qquad \int_0^{\pi} \sqrt{1+4x^2\cos(x^2)}\,dx$$

$$\int_0^{\sqrt{\pi}} \sqrt{1+\cos^2(x^2)}\,dx \qquad \int_0^{\sqrt{\pi}} \sqrt{1+4x^2\cos^2(x^2)}\,dx$$

d. [3 points] If the average value of a continuous function is A on $[0,3]$ and B on $[3,5]$, what is its average value on $[0,5]$?

$$2A+3B \qquad \frac{2A+3B}{5} \qquad \frac{A+B}{2} \qquad \frac{3A+2B}{5} \qquad \frac{A+B}{5}$$

2. [7 points] Deep beneath Dennison Hall lies a large septic tank. It has the shape of a triangular prism with the dimensions depicted below.



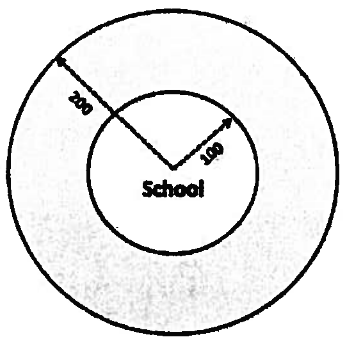
Suppose that the tank described above is full of sewage and that this sewage has a density of $1000(1 + e^{-2x})\ \frac{kg}{m^3}$, where x is the distance in meters above the base of the tank.

a. [5 points] Find a definite integral that computes the mass of the sewage in the tank.

b. [2 points] Compute the value of the integral using your calculator. Do not forget to include the units.

4. [8 points] In a small town, property values close to the school are determined primarily by how far the land is from the school. The function $\delta(r) = \dfrac{1}{ar^2+1}$ gives the value of the land (in thousands of dollars per m^2), where r is the distance (in meters) from the school and a is a positive contant.

a. [5 points] Find a formula containing a definite integral that computes the value of the land that lies in the annulus of inner radius 100 m and outer radius 200 m (figure shown below).



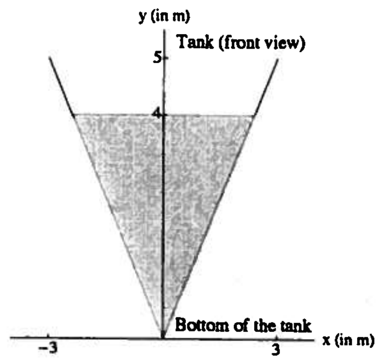
b. [3 points] Calculate the exact value of the land that lies in the annulus of inner radius 100 m and outer radius 200 m. Your answer should contain a. Show all your work.

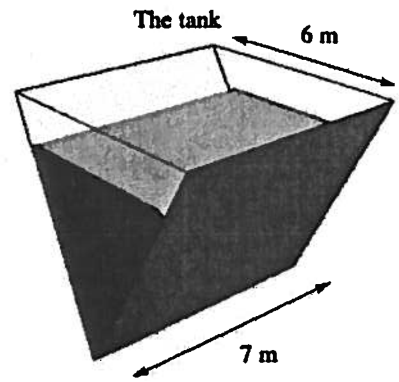
7. [8 points] Alyssa Edwards wants to play a prank on Coco Montrese by spilling a bucket of orange cheese powder on her. To do this Alyssa lifts the bucket at a constant speed from the ground to a height of 10 meters. Unfortunately the bucket has a small hole and the cheese begins leaking out at a constant rate as soon as the bucket leaves the ground. The bucket initially weighs 10kg and when it reaches a height of 10 meters it only weighs 5kg. Recall the gravitational constant is $g = 9.8\text{m/s}^2$.

a. [3 points] Write an expression giving the mass of the bucket $m(h)$ when the bucket is h meters above the ground.

b. [5 points] How much work is required to lift the bucket from the ground to a height of 10 meters? Include units.

6. [11 points] The Math Department has recently acquired a triangular storage tank 6 m wide, 5 m tall and 7 m long, which it will use to store coffee for its graduate students. The tank currently contains a special coffee blend, with a mass density 1033 kg per m^3, up to a depth of 4 m.





a. [8 points] Write an expression that approximates the work done in lifting a horizontal slice of the liquid in the tank that is y meters above the bottom of the tank, with thickness Δy, to the top of the tank. Use $g = 9.8$ m per s^2 for the acceleration due to gravity.

b. [3 points] While grading this exam, the grad students will need coffee. Find a definite integral that computes the work required to pump all the coffee to the top of the tank. Give the units of this integral. You do not need to evaluate it.