Math 116 - 099 Name:
Quiz 10 §§4.8, 8.3

This quiz has a total of 30 points. Please show all work to receive full credit.

1. [9 points] A particle moves along the path given by the parametric equations

$$x(t) = a\cos t \quad y(t) = \sin 2t \quad \text{for } 0 \le t \le 2\pi.$$

where a is a positive constant.

(a) [1 points] At which times $0 \le t \le 2\pi$ does the particle pass through the origin?

(b) [4 points] For what values of a are the two tangent lines to the curve at the origin perpendicular?
Hint: Two lines are perpendicular if the product of their slopes is equal to -1.

(c) [2 points] At what times $0 \le t \le 2\pi$ does the curve have a horizontal tangent line?

(d) [2 points] Find an expression that computes the length of the curve.

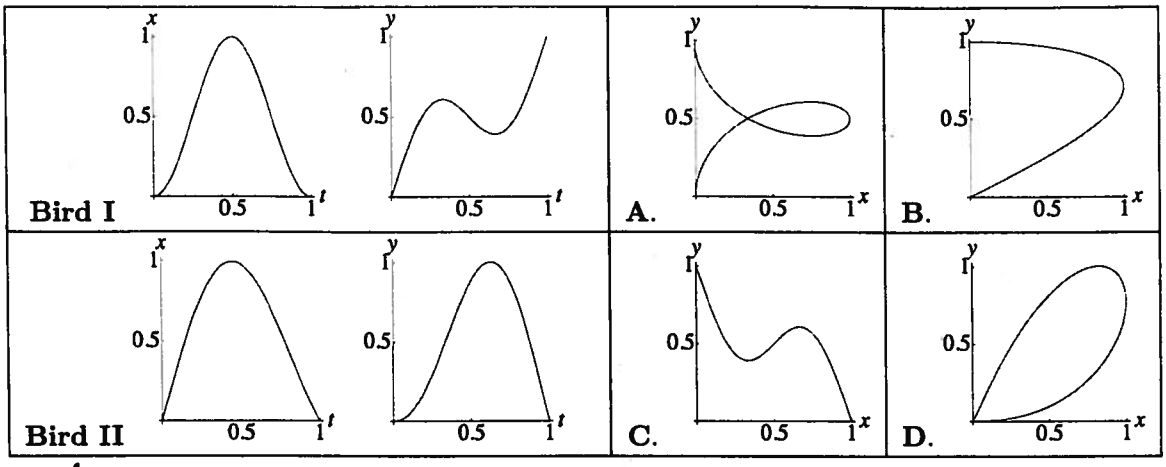
2. [8 points] Casey is doodling one Sunday afternoon and draws an infinity symbol, or lemniscate. The picture doodled is described by the polar curve $r^2 = 16\cos(2\theta)$. (The radius is measured in inches.)

   (a) [2 points] Casey decides to color the inside of the lemniscate red. Write, but do not evaluate, an expression involving one or more integrals that gives the total area, in square inches, that has to be filled in with red.

   (b) [3 points] Casey decides to outline the right half of the lemniscate in blue (the portion to the right of the y-axis). Write, but do not evalute, an expression involving one or more integrals that gives the total length, in inches, of the outline he must draw in blue.

   (c) [3 points] Next Casey doodles another picture of the same lemniscate, but this time also draws over it the circle $r = 2\sqrt{2}$. Casey decides to color the area inside the lemniscate but outside the circle purple. Write, but do not evaluate, an expression involving one or more integrals that gives the total area, in square inches, that gets filled in with purple.

3. [6 points] The x and y positions of two birds in flight, Bird I and Bird II, are graphed below as functions of time t (see figures labeled Bird I and Bird II on the left). To the right, there are four parametric curves, A,B,C,D, showing flight paths of several birds in the x-y plane.



a. [1 points] Is the horizontal velocity of bird I zero at any time $0 < t < 1$? If so, give an approximate t value.

b. [2 points] Based on the plots shown for bird II, consider a parametric curve for the flight path for bird II in the x-y plane. Would the slope of the tangent line to the flight path curve at time $t = 0.9$ be positive, negative, or zero? Justify.

c. [2 points] One of the parametric curves A,B,C,D corresponds to bird I and another corresponds to bird II. Indicate which ones by circling the correct answers:

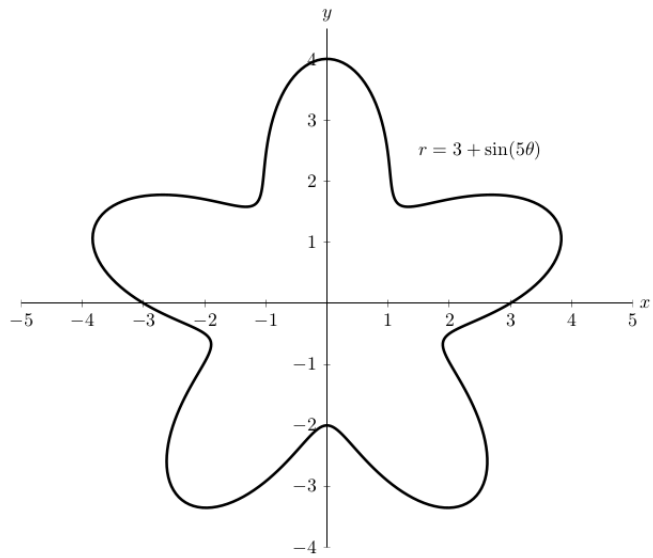
Bird I corresponds to:	A	B	C	D
Bird II corresponds to:	A	B	C	D

d. [1 points] A third bird flies according to the following parametric equations

$$x(t) = 1 - t^3 \quad y(t) = t^2 - t.$$

Find the speed of the bird at $t = 1$. Show all your work.

3. [7 points] Consider the star defined by $r = 3 + \sin(5\theta)$, shown below. It is nice and round.



(a) [2 points] Find the radius of the largest circle centered at the origin which fits entirely within our star.

(b) [3 points] Write, but do not evaluate, an integral that represents the area of one of the five pieces of our star that lie outside the circle $r = 3.5$.

(c) [2 points] If we parametrize our star by

$$(x(\theta), y(\theta)) = (f(\theta)\cos\theta,\ f(\theta)\sin\theta) \qquad \text{where } f(\theta) = 3 + \sin(5\theta),$$

for how many angles θ, in the range $0 \le \theta < 2\pi$, does $\dfrac{x'(\theta)}{y'(\theta)} = 0$? You do not need to calculate the values of θ, just determine how many there are.