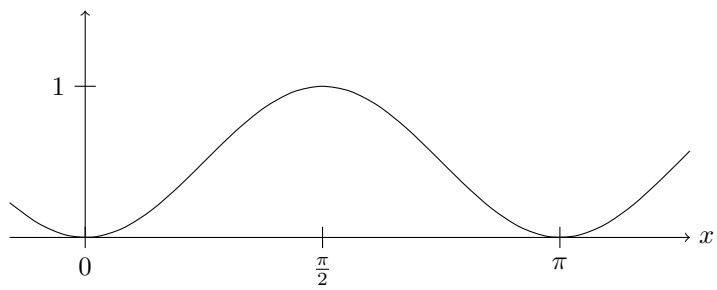
Math 116 - 099 Name:
Quiz 1 §§5.1 - 6.2

This quiz has a total of 24 points. Please show all work to receive full credit.

1. [4 points] Consider the function $\sin^2(x)$ whose graph is shown below:



   (a) [2] Write an expression for the left-hand Riemann sum with $n = 4$ subdivisions which approximates the definite integral

$$\int_0^{\pi} \sin^2(x)dx.$$

   Write out all terms; you do not need to simply the expression.

   (b) [1] On the graph above, sketch the area measured by the Riemann sum in part (a).

   (c) [1] The sum in part (a) is _____ the right-hand Riemann sum.

   larger than smaller than equal to

2. (a) [3] Find a and b which make the following identity hold:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{10}{n} e^{1+10k/n} = \int_a^b e^x dx.$$

   (b) [1] Evaluate $\int_a^b e^x dx$ in terms of a and b.

3. [5 points] Let $f(x)$ be a function such that $f'(x) > 0$ and $f''(x) < 0$ for all x, and let $F(x)$ be an antiderivative of $f(x)$.
Explain whether each expression below gives the exact value of $f(3.1)$, an underestimate of $f(3.1)$, or an overestimate of $f(3.1)$, or if there is not enough information to decide.

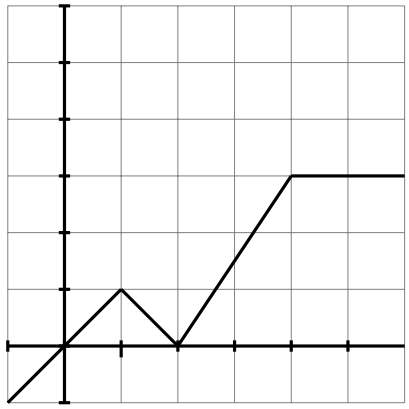
(a) $f(3) + \int_3^{3.1} f'(t)dt$

exact under over not enough info.

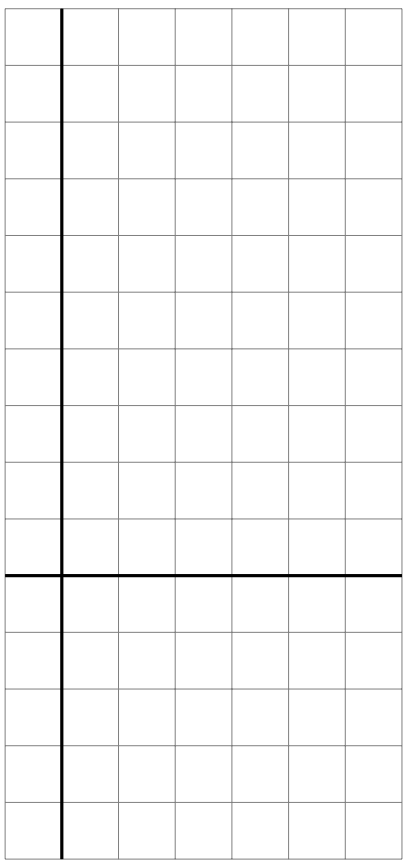
(b) $\dfrac{F(3.11) - F(3.1)}{0.01}$

exact under over not enough info.

4. [5 points] The graph of $y = g(x)$ is given below.



Suppose $G(x)$ is an antiderivative of $g(x)$ and that $G(0) = -3$. Write down the values for $G(-1)$ and $G(5)$ and carefully draw the graph of $G(x)$ below.



$G(-1) =$ _____

$G(5) =$ _____

5. [6 points] For each statement below, circle True or False and give a brief justification.
Let $\text{Avg}(f)$ denote the average value of the function f on the interval $[1, 4]$.

   (a) For any functions f and g, $\text{Avg}(f + g) = \text{Avg}(f) + \text{Avg}(g)$.

   True False

   (b) For any functions f and g, $\text{Avg}(fg) = \text{Avg}(f) \cdot \text{Avg}(g)$.

   True False

   (c) If $f(x)$ is a continuous on the interval $[a, b]$, then $\int_a^b (5 + 3f(x))dx = 5 + 3\int_a^b f(x)dx$.

   True False

6. [Extra Credit!]

   (a) [2] Explain what is wrong with the following argument that $\lim_{n\to\infty}(n\sin(1/n)) = 0$.

$$\lim_{n\to\infty}(n\sin(1/n)) = n\lim_{n\to\infty}(\sin(1/n)) = n(\sin(0)) = n \cdot 0 = 0.$$

   (b) [1] What is the actual value of the limit $\lim_{n\to\infty}(n\sin(1/n))$?
   (Hint: You can use your calculator.)