Math 116 - 099 Name: Solutions

Practice problems §§6.4 - 7.2

1. Compute the following derivatives and integrals.

(a) $\frac{d}{dx}\left(x\int_x^{1/2} \arctan(\sqrt{t})dt\right)$ $= \frac{d}{dx}\left(-x\int_{1/2}^{x} \arctan\sqrt{t}\, dt\right)$

$= -\int_{1/2}^{x} \arctan\sqrt{t}\, dt - x\left(\frac{d}{dx}\int_{1/2}^{x} \arctan\sqrt{t}\, dt\right)$

$= \boxed{-\int_{1/2}^{x} \arctan\sqrt{t}\, dt - x \arctan\sqrt{x}}$

(b) $\frac{d}{dx}\int_1^{\cos(x^3)} e^{-t^2}dt$

$= \left(\frac{d}{dx}\cos(x^3)\right)\cdot e^{-\cos^2(x^3)} = \boxed{-3x^2 \sin(x^3)\, e^{-\cos^2(x^3)}}$

(c) $\int_5^6 (4g(-2t+12)+4)dt$, given that $\int_0^2 g(t)dt = 13$.

$= 4\int_5^6 g(-2t+12)\,dt + \int_5^6 4\,dt \quad = 4\int_{w=-2(5)+12}^{w=-2(6)+12} g(w)\cdot\left(-\frac{1}{2}dw\right) + 4(6-5)$

$w = -2t+12$

$dw = -2\,dt \quad \Rightarrow \quad dt = -\frac{1}{2}dw$

$= -\frac{4}{2}\int_2^0 g(w)dw + 4 = 2(13)+4 = \boxed{30}$

(d) $\int_1^{e^3} \ln(x)dx$ (Hint: use integration by parts, with $u = \ln(x)$ and $v' = 1$.)

$u' = \frac{1}{x} \qquad v = x$

$= x\ln(x)\Big|_1^{e^3} - \int_1^{e^3}\frac{x}{x}dx = (3e^3 - 0\cdot e^0) - (e^3 - 1) = \boxed{2e^3 + 1}$

(e) $\int \frac{1}{\sqrt{1-x}}dx$ $= -\int \frac{1}{\sqrt{s}}ds = -\int s^{-1/2}ds = -2s^{1/2} + C = \boxed{-2\sqrt{1-x} + C}$

$s = 1-x$

$ds = -dx$

(f) $\int \frac{1}{x^2-1}dx = \int \left(\frac{A}{x-1} + \frac{B}{x+1}\right)dx = A\ln|x-1| + B\ln|x+1| + C = \boxed{\frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C}$

(Partial Fractions)

where $1 = A(x+1) + B(x-1)$

@ $x=-1$: $1 = A\cdot 0 + B\cdot(-2) \Rightarrow B = -\frac{1}{2}$

@ $x=1$: $1 = A\cdot(2) + B\cdot 0 \Rightarrow A = \frac{1}{2}$

(g) $\int \frac{1}{x^2+1}dx. = \boxed{\arctan(x) + C}$

(memorize this one)

(h) $\int_0^{\pi/3} \frac{\sin(x)}{\cos(x)}dx = -\int_{\cos 0}^{\cos \pi/3} \frac{dz}{z} = -\ln|z|\Big|_1^{1/2} = -\ln(1/2) + \underbrace{\ln(1)}_{0} = \boxed{\ln(2)}$

$z = \cos x$

$dz = -\sin x\, dx$

(i) $\int \sin^3(x)\cos^{2n}(x)dx$ where n is a positive integer ($n = 1, 2, 3, \ldots$)
(Hint: see Example 4 on p. 361)

$= \int \sin(x)(1-\cos^2 x)\cos^{2n}(x)\,dx = -\int (1-u^2)u^{2n}\,du = -\int (u^{2n} - u^{2n+2})\,du$

$u = \cos x$

$du = -\sin x\, dx$

$= -\frac{1}{2n+1}u^{2n+1} + \frac{1}{2n+3}u^{2n+3} + C = \boxed{-\frac{\cos^{2n+1}(x)}{2n+1} + \frac{\cos^{2n+3}(x)}{2n+3} + C}$

(j) $\int (\sin^4(x) + 2\sin^2(x)\cos^2(x) + \cos^4(x))dx$ (Hint: there is a quick way to do this)

$= \int (\sin^2 x + \cos^2 x)^2\,dx = \int 1^2\,dx = \boxed{x + C}$