Math 116 - 005 Name: Solutions

Derivatives review: due Jan. 4

1. Calculate the derivatives with respect to x of the functions below. Show your work. (Assume the variables y, μ, σ, π, e are constant.)

(a) $f(x) = e^{-\ln(5x)}$.

$f(x) = \frac{1}{e^{\ln(5x)}} = \frac{1}{5x}$,

$f'(x) = -\frac{1}{5x^2}$

(b) $h(x) = e^{\cos^2 x}$.

$h'(x) = e^{\cos^2 x} \cdot (-2\cos x \sin x)$

(Chain Rule)

(c) $g(x) = e^{\cos^2 x} e^{\sin^2 x}$.

$g(x) = e^{\cos^2 x + \sin^2 x} = e^1$. constant !

$g'(x) = 0$

(d) $t(x) = \tan(x + y)$.

$t'(x) = \sec^2(x+y)$

(Chain rule: $\frac{d}{dx}(x+y) = 1$)

(e) $r(x) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$.

$$r'(x) = \frac{\sec^2 x\,(1 - \tan x \tan y) + \sec^2 x \tan y\,(\tan x + \tan y)}{(1 - \tan x \tan y)^2}$$

OR

use trig. identity $r(x) = \tan(x+y) \Rightarrow r'(x) = \sec^2(x+y)$

(f) $s(x) = \dfrac{x^{29} + \sin(x)}{x^{29} + \sin(x) + 1}$.

$$s(x) = \frac{x^{29} + \sin(x) + 1}{x^{29} + \sin(x) + 1} - \frac{1}{x^{29} + \sin(x) + 1} = 1 - \frac{1}{x^{29} + \sin(x) + 1}$$

$$s'(x) = \frac{29x^{28} + \cos(x)}{(x^{29} + \sin(x) + 1)^2}$$

(g) $n(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$. ($n(x)$ = *normal distribution* with mean μ and variation σ^2.)

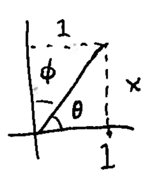
$$n'(x) = -\frac{1}{\sigma\sqrt{2\pi}} \cdot \frac{(x-\mu)}{\sigma^2} \cdot e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

(h) $p(x) = \arctan(x) + \arctan(\frac{1}{x})$.

$$p'(x) = \frac{1}{1+x^2} + \frac{1}{1 + 1/x^2} \cdot \left(-\frac{1}{x^2}\right) = \frac{1}{1+x^2} - \frac{1}{x^2+1} = 0$$

OR

using trigonometry,



$\tan\theta = x \Rightarrow \arctan(x) + \arctan\left(\frac{1}{x}\right)$

$\tan\varphi = \frac{1}{x} \quad = \theta + \varphi = \frac{\pi}{2}$,

constant!

$\Rightarrow p'(x) = 0$