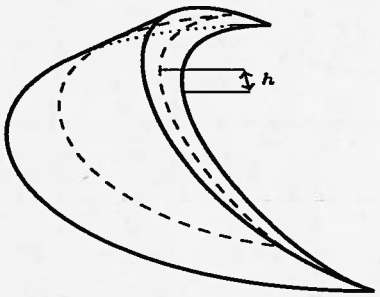
2. [13 points] An ice cream cone has a height of 15 centimeters and the diameter of the top is 5 centimeters. The cone is filled with soft-serve ice cream such that the ice cream completely fills the cone, but does not exceed the top of the cone. The ice cream has a constant density of 2 grams per cubic centimeter.

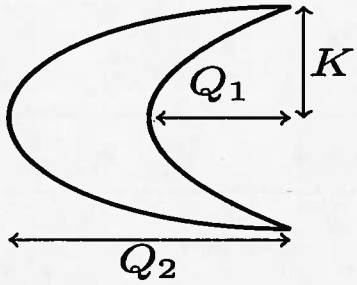
a. [5 points] Write an expression for the approximate mass of ice cream contained in a circular cross-sectional slice that is located h_i centimeters from the the bottom tip of the cone and has depth Δh centimeters . Your answer may be in terms of h_i and Δh. Don't forget to include units.

b. [4 points] Set up a definite integral that can be used to determine the EXACT total mass of ice cream that is filling the cone, then solve for this exact value. Include appropriate units in your answer.

c. [4 points] At what height above the tip of the cone is the center of mass of the ice cream? Give an EXACT answer, show all work, and include appropriate units.

8. [12 points] Sand dunes come in many shapes. *Barchan* dunes, which have the shape shown on the left, are studied extensively by geomorphologists. Horizontal cross-sections of these dunes are crescent-shaped (the dashed line encloses one such cross-section), and can be approximated as the shape on the right. The area of this shape is given by the formula $A_h = K(\frac{\pi}{2}Q_2 - \frac{4}{3}Q_1)$.





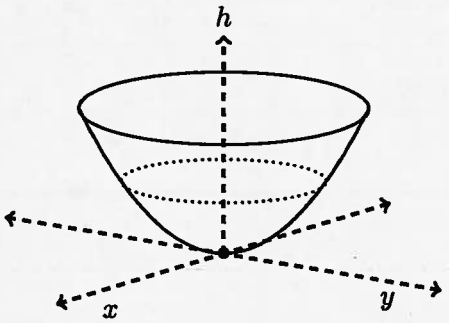
You are studying a barchan dune of 10 meters height, for which the values of Q_1, Q_2, and K vary with respect to the height h (in meters) of the cross-section according to the functions $Q_1(h) = 10-h$, $Q_2(h) = 20-2h$, $K(h) = 100-h^2$. The density of sand in the dune is $\delta = 1600$ kilograms per cubic meter.

a. [5 points] Write an expression for the volume of one slice of sand dune h meters above the ground and Δh meters thick.

b. [5 points] Write a definite integral that represents the total mass of sand in the dune. You do not need to evaluate this integral.

c. [2 points] Write an expression (involving integrals) for the height of the center of mass of the sand dune. You do not need to evaluate this integral.

10. [10 points] Martin is having a party to celebrate the beginning of spring and he is serving punch out of a parabolic punch bowl. The bowl is sitting on a table (the xy-plane) as depicted in the figure below. At a height h above the table, the cross section of the bowl perpendicular to the h-axis is a circle with equation, $h = 4x^2 + 4y^2$. The punch bowl is 1 meter tall. Assume the units of x, y, and h are in meters and the density of the punch is 1200 kg/m^3. Recall the gravitational constant is $g = 9.8$ m/s^2.



a. [5 points] Write an expression for the mass of a slice of punch of thickness Δh meters at a height h meters above the table.

b. [5 points] Assuming the bowl is filled with punch up to a height of $h = 1/2$, write an integral which gives the amount of work needed to lift all of the punch over the rim of the bowl. Do not evaluate your integral.

8. [9 points] Sally, the marine scientist, is reeling in a large shark she caught onto her boat. The edge of her boat lies 5 meters above the water as shown in the figure below. The total length of the sharking line is 30 meters. The shark weighs 500 newtons in water, and her sharking line weighs 30 newtons per meter out of water, and 10 newtons per meter in water. The figure below depicts this situation - the sharking line is the thick dark line and the boat is shaded. Write an expression which gives the work Sally does pulling the shark's snout to the surface of the water.