

4. [15 points] For this problem, m is a differentiable function with $m'(x) > 0$ for all x . The following table gives some values of m .

x	0	1	2	3	4	5	6	7	8
$m(x)$	0	2	3	4	6	9	10	11	12

- a. [3 points] What is the average value of $m'(x)$ on $[1, 7]$?
- b. [3 points] Use a left Riemann sum with 3 subdivisions to estimate $\int_2^8 m(x) dx$. Write out each term of your sum. Is this an overestimate or underestimate?
- c. [3 points] Use a midpoint sum with 3 subdivisions to estimate $\int_0^{12} m^{-1}(y) dy$. Write out each term of your sum.
- d. [6 points] Consider the region bounded by the y -axis, the line $y = 12$ and the curve $y = m(x)$. Write an integral that gives the volume of the solid obtained by rotating this region about the y -axis. Use a right Riemann sum with 2 subdivisions to estimate your integral. Write out each term of your sum.

1. [10 points] Indicate if each of the following statements are true or false by circling the correct answer. **Justify your answers.**

a. [2 points] If $F(x)$ is an antiderivative of an even function $f(x)$, then $F(x)$ must also be an even function.

True False

b. [2 points] If $G(x)$ is an antiderivative of $g(x)$ and $(G(x) - F(x))' = 0$, then $F(x)$ is an antiderivative of $g(x)$.

True False

c. [2 points] Let $f(t) = bt + ct^2$ with $b > 0$ and $c > 0$, then $\text{Left}(n) \leq \int_0^{10} f(t) dt$ for all n .

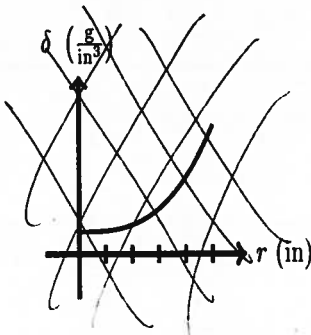
True False

d. [2 points] The average of an even function $f(x)$ over the interval $[-a, a]$ is equal to twice its average over the interval $[0, a]$.

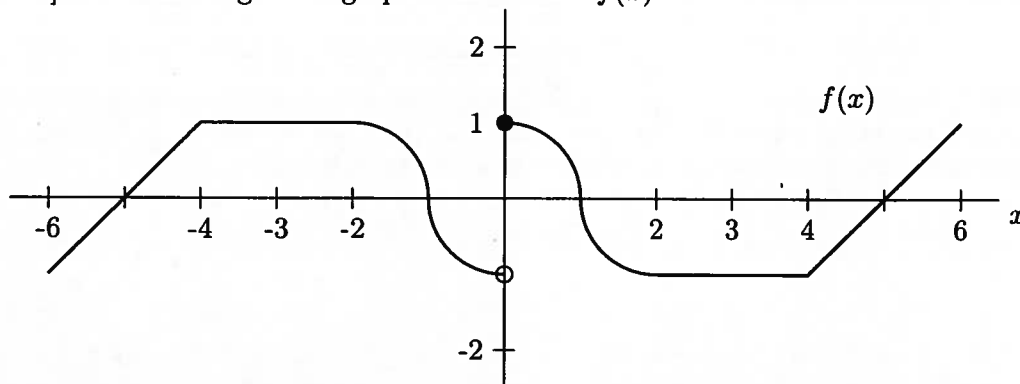
True False

e. [2 points] The density δ of a circular porcelain dinner plate depends on the distance r from the center of the plate. The relationship between δ and r is shown in the graph below. The center of mass of this plate is located near the edge of the plate.

True False



2. [12 points] The following is the graph of a function $f(x)$.

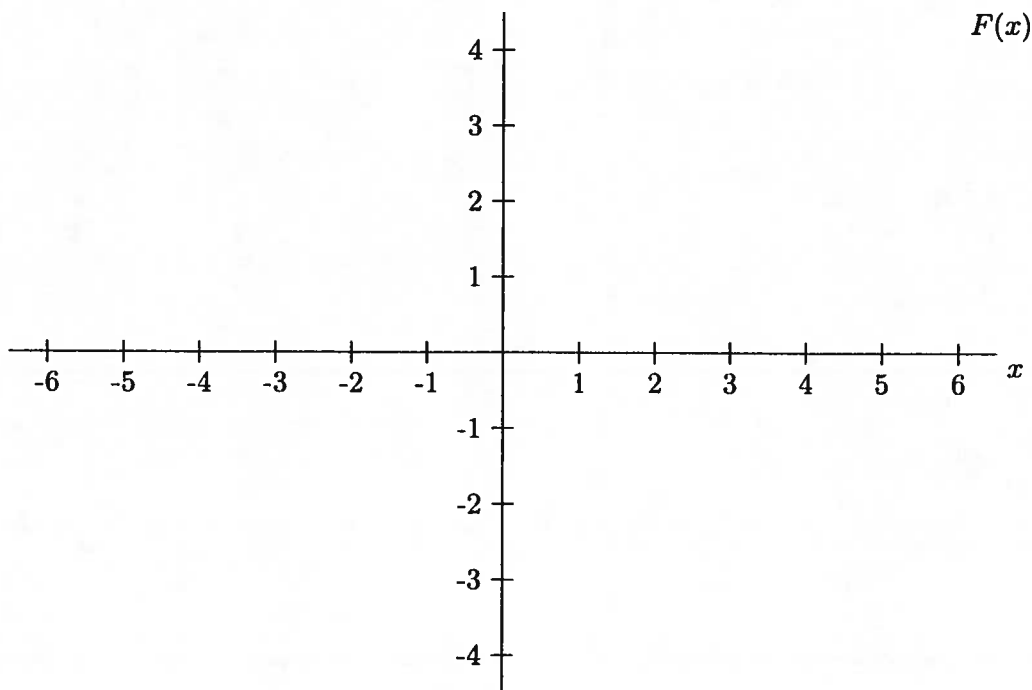


Note that the graph of $f(x)$ is a quarter of a circle on each of the intervals $[-2, -1]$, $[-1, 0]$, $[0, 1]$, $[1, 2]$ and linear on each of the intervals $[-6, -4]$, $[-4, -2]$, $[2, 4]$, $[4, 6]$.

Let $F(x)$ be a function satisfying:

- $F(0) = 0$.
- $F'(x) = f(x)$ for $-6 < x < 0$ and $0 < x < 6$.

Carefully sketch a graph of $F(x)$ using the axes provided below. If there are features of $F(x)$ that are difficult for you to draw, indicate these on your graph. Label the x - and y -coordinates of the points on your graph of F at $x = -3$, $x = 1$ and $x = 5$.



6. [10 points] The distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Consider the curve described by $y = \sqrt{3x^2 - 3}$, over the domain $2 \leq x \leq 4$. What is the average distance of the points on this curve to the point $(2, 0)$?