4. [13 points]

a. [6 points] A cylindrical tank with height 8 m and radius of 8 m is standing on one of its circular ends. The tank is initially empty. Water is added at a rate of 2 m^3 /min. A valve at the bottom of the tank releases water at a rate proportional to the water's depth (proportionality constant $= k$). Let $V(t)$ be the volume of the water in the tank at time t, and $h(t)$ be the depth of the water at time t.

i. Find a formula for $V(t)$ in terms of $h(t)$. $V(t) =$ _____

ii. Find the differential equation satisfied by $V(t)$. Include the appropriate initial conditions.

Differential equation: _____ Initial condition: _____

b. [7 points] Let $M(t)$ be the balance in dollars in a bank account t years after the initial deposit. The function $M(t)$ satisfies the differential equation

$$\frac{dM}{dt} = \frac{1}{100}M - a.$$

where a is a positive constant. Find a formula for $M(t)$ if the initial deposit is $1,000$ dollars. Your answer may depend on a.

3. [15 points] A model for cell growth states that the volume $V(t)$ (in mm^3) of a cell at time t (in days) satisfies the differential equation

$$\frac{dV}{dt} = 2e^{-t}V.$$

a. [2 points] Find the equilibrium solutions of this equation.

b. [8 points] Solve the differential equation. The initial volume of the cell is V_0 mm^3. Your answer should contain V_0.

c. [3 points] How long does it take a cell to double its initial size?

d. [2 points] What happens to the value of the volume of the cell in the long run?

4. [11 points] A tank initially has 27 m^3 of water. At $t = 0$ (t in minutes), a pump takes water out of the tank. Let $V(t)$ be the volume of water (in m^3) in the tank t minutes after the pump was activated. Suppose the function $V(t)$ satisfies the differential equation

$$\frac{dV}{dt} = kV^{\frac{1}{3}}$$

where k is a constant.

a. [2 points] Is k positive or negative? What are the units of k?

b. [7 points] Find a formula for $V(t)$. Your formula must contain only the constant k and the variable t.

c. [2 points] How long does it take for the tank to empty? Your answer may contain the constant k.

7. [14 points] You want to open a savings account to deposit 1000 dollars. Three banks offer the following options:

a. [3 points] Bank A offers its clients a savings account that earns 1.5% per year compounded annually. Define the sequence A_n to be the amount of money in the savings account n years after you deposit your 1000 dollars. Find a formula for A_n.

b. [7 points] Bank B offers its clients a savings account that earns 2% per year compounded annually. At the end of each year, after the bank deposits the interest you earned, it withdraws a 1 dollar service fee from the account. Define the sequence B_n to be the amount of money, right after the service fee deduction, in the savings account n years after you deposit your 1000 dollars. Find B_1, B_2, B_3 and a **closed form** formula for B_n.

c. [4 points] Bank C offers its clients a savings account that earns interest continuously at a rate of 1.5% of the current balance per year. At the same time, the bank withdraws a service fee from the account at a rate of 1 dollar per year continuously. Let $M(t)$ be the amount of money in the savings account t years after you deposit your 1000 dollars. Write the differential equation satisfied by $M(t)$. Include initial conditions.