

MATH 116 — PRACTICE FOR EXAM 2

Generated March 21, 2019

NAME: _____

INSTRUCTOR: _____ SECTION NUMBER: _____

1. This exam has 8 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2015	1	8	shark	9	
Winter 2015	1	5	money pool	11	
Fall 2013	3	4		9	
Fall 2017	2	9		12	
Fall 2017	2	2		9	
Winter 2014	2	10		12	
Fall 2018	2	5	bus	10	
Winter 2018	2	9	cult	11	
Total				83	

Recommended time (based on points): 77 minutes

9. [12 points] For each of parts **a** through **c** below, determine the radius of convergence of the power series. Show your work carefully.

a. [3 points]
$$\sum_{n=1}^{\infty} \frac{e}{n!} (x-1)^n$$

Answer: radius of convergence = _____

b. [3 points]
$$5(x+\pi) + 5 \cdot 4(x+\pi)^2 + 5 \cdot 9(x+\pi)^3 + 5 \cdot 16(x+\pi)^4 + \dots$$

Answer: radius of convergence = _____

c. [3 points]
$$\sum_{n=0}^{\infty} \frac{\pi}{8^n} (x+2)^{3n}$$

Answer: radius of convergence = _____

- d. [3 points] Consider the power series $\sum_{j=0}^{\infty} C_j (x-5)^j$, where each C_j is a constant. Suppose this power series

- converges when $x = 2$ and
- diverges when $x = 12$.

Based on this information, which of the following values **could** be equal to the radius of convergence of the power series? Circle all possibilities from the list below.

- | | | | | |
|----|----|----|---------------|---|
| 0 | 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | NONE OF THESE | |

4. [9 points]

Determine if each of the following sequences is increasing, decreasing or neither, and whether it converges or diverges. If the sequence converges, identify the limit. Circle all your answers. No justification is required.

a. [3 points] $a_n = \int_1^{n^3} \frac{1}{(x^2 + 1)^{\frac{1}{5}}} dx.$

Converges to _____

Diverges.

Increasing

Decreasing

Neither.

b. [3 points] $b_n = \sum_{k=0}^n \frac{(-1)^k}{(2k + 1)!}.$

Converges to _____

Diverges.

Increasing

Decreasing

Neither.

c. [3 points] $c_n = \cos(a^n)$, where $0 < a < 1$.

Converges to _____

Diverges.

Increasing

Decreasing

Neither.

2. [9 points] Consider the graph of $y = e^{-14x}$ for $x \geq 0$.

- a. [3 points] Let \mathcal{R} be the region in the first quadrant between the graph of $y = e^{-14x}$ and the x -axis. Which of the following improper integrals best expresses the volume of the solid that is obtained by rotating \mathcal{R} around the x -axis?

Circle one:

$$\int_0^{\infty} \pi e^{-14x} dx \quad \int_0^{\infty} x e^{-14x} dx \quad \int_0^{\infty} x e^{-28x} dx \quad \int_0^{\infty} \pi e^{-28x} dx$$

$$\frac{\pi}{7} \int_0^1 \ln(y) dy \quad \frac{\pi}{14} \int_0^1 y \ln(y) dy \quad \frac{1}{14} \int_0^1 y \ln(y^2) dy$$

- b. [6 points] Determine whether the improper integral you circled in part a converges or diverges.

- If the integral converges, circle “converges”, find its exact value (i.e. no decimal approximations), and write the exact value on the answer blank provided.
- If the integral diverges, circle “diverges” and justify your answer.

In either case, **you must show all your work carefully using correct notation.** Any direct evaluation of integrals must be done **without using a calculator.**

Converges to _____

Diverges

10. [12 points] Suppose that $g(x)$ and $h(x)$ are positive continuous functions on the interval $(0, \infty)$ with the following properties:

- $\int_1^\infty g(x) dx$ converges.
- $\int_0^1 g(x) dx$ diverges.
- $e^{-x} \leq h(x) \leq \frac{1}{x}$ for all x in $(0, \infty)$.

For each of the following questions, circle the correct answer.

a. [2 points] Does the integral $\int_1^\infty h(x)^2 dx$ converge?

Converge

Diverge

Cannot determine

b. [2 points] Does the integral $\int_0^1 h(x) dx$ converge?

Converge

Diverge

Cannot determine

c. [2 points] Does the integral $\int_1^\infty h(1/x) dx$ converge?

Converge

Diverge

Cannot determine

d. [2 points] Does the integral $\int_0^1 g(x)h(x) dx$ converge?

Converge

Diverge

Cannot determine

e. [2 points] Does the integral $\int_1^\infty g(x)h(x) dx$ converge?

Converge

Diverge

Cannot determine

f. [2 points] Does the integral $\int_1^\infty e^x g(e^x) dx$ converge?

Converge

Diverge

Cannot determine

5. [10 points] You are at a bus stop waiting for a bus to arrive. The cumulative distribution function for the time, in minutes, a passenger will wait for the next bus to arrive is given by

$$P(t) = \begin{cases} 0 & t \leq 0 \\ 1 - e^{-0.05t} & t > 0. \end{cases}$$

- a. [3 points] What is the median amount of time that a passenger has to wait for a bus to arrive? Provide an **exact** answer. Remember to show all your work.

Answer: Median = _____

You decide that you are going to take the 2nd bus that arrives. It can be shown that the number of minutes a passenger has to wait for 2 buses to arrive has probability density function

$$q(t) = \begin{cases} 0 & t \leq 0 \\ Cte^{-0.05t} & t > 0 \end{cases}$$

for some constant C .

- b. [5 points] Find the value of C . **Show all your work using correct notation.** Any evaluation of integrals must be done **without a calculator**.

Answer: $C =$ _____

- c. [2 points] Write an expression (possibly involving one or more integrals) for the mean number of minutes it takes for 2 buses to arrive. (You do not need to evaluate your expression.)

Answer: Mean = _____

9. [11 points] Leight Vloss had trouble paying his rent so he started a cult. His followers believe that Leight receives holy messages from “The Great Consciousness” hiding in the internet. Each month Leight recruits D new followers and loses 20% of the followers he had in the previous month to disillusionment and other cults. That is, the number of followers Leight has after n months is described by the recursive formula

$$F_n = D + .8F_{n-1}$$

- a. [4 points] Supposing that Leight has 0 followers the moment he gets the idea to start a cult, which is to say that $F_0 = 0$, compute the number of followers he has in the first three months. Your answer may be in terms of D .

Answer: $F_1 =$ _____

$F_2 =$ _____

$F_3 =$ _____

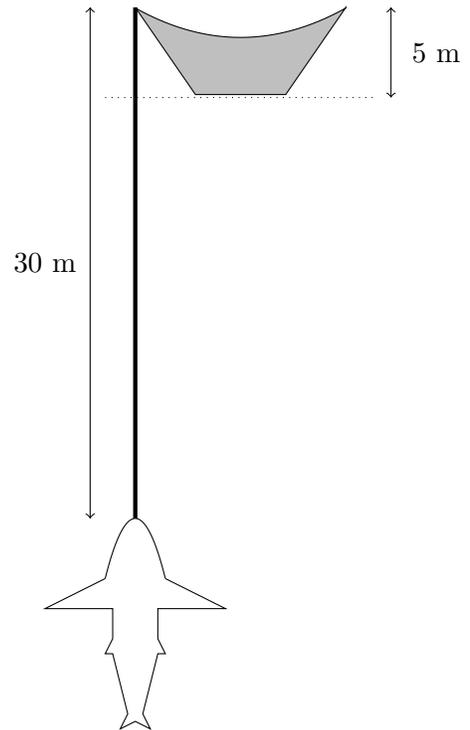
- b. [4 points] Find a closed form expression for F_n , the number of followers Leight has after n months of channelling the spirit of the internet.

Answer: $F_n =$ _____

- c. [3 points] Leight finds he needs the number of followers to tend to 1000 in the long run to ensure he can make rent each month. What’s the fewest number of followers D that Leight Vloss needs to recruit each month to make sure he can pay rent?

Answer: _____

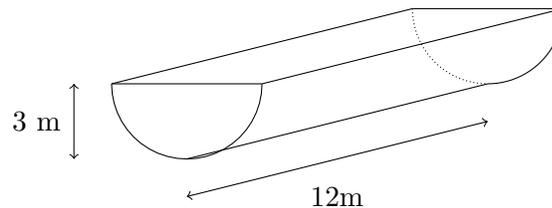
8. [9 points] Sally, the marine scientist, is reeling in a large shark she caught onto her boat. The edge of her boat lies 5 meters above the water as shown in the figure below. The total length of the sharking line is 30 meters. The shark weighs 500 newtons in water, and her sharking line weighs 30 newtons per meter out of water, and 10 newtons per meter in water. The figure below depicts this situation - the sharking line is the thick dark line and the boat is shaded. Write an expression which gives the work Sally does pulling the shark's snout to the surface of the water.



5. [11 points] Calvin Currency is making a large cash donation in \$100 bills. Before making the donation, he decides to fill an empty pool with the money. The pool is a half cylinder with radius 3 meters and length 12 meters as shown below. After an afternoon of diving into his pool of money and swimming around, the distribution of bills in the pool becomes nonuniform and so the density of money in the pool is given by

$$\delta(y) = 30,000 \sqrt{\frac{10}{\pi}} e^{-y^2},$$

measured in bills per m^3 , where y is height in meters measured from the bottom of the pool. Recall the gravitational constant is $g = 9.8 \text{ m/s}^2$



- a. [5 points] Write a definite integral which gives the volume of the pool.
- b. [2 points] Write a definite integral which gives the value of the money in the pool, in dollars.
- c. [4 points] Write a definite integral which gives the amount of work done in lifting the money out of the pool if each bill has mass 0.001 kg.