Proceedings of the American Control Conference
Chicago, Illinois • June 2000

Modeling and Control of a Variable Valve Timing Engine

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Abstract

A cylinder-by-cylinder model of an experimental variable valve timing 4-cylinder engine has been developed. The model includes the cylinder and manifold mass, temperature, burned gas residual, and pressure dynamics, including combustion effects, as well as the valve actuator dynamics. The cylinder-by-cylinder model is used to obtain a cycle-averaged mapping between torque at a given engine speed and intake valve timing, which is suitable for future control design implementations.

1 Introduction

Recently, variable valve timing engines have attracted a lot of attention because of their ability to control valve events independent of crank shaft rotation, allowing for reduced pumping loss (work required to draw air into the cylinder under part-load operation), and increased torque performance over a wider range than conventional spark-ignition engine. Variable valve timing also allows control of internal exhaust gas recirculation (by control of the valve overlap), allowing for control of NOx emissions produced during combustion.

Several detailed studies have been performed to show the benefits of variable valve timing engines. [1] describes a mechanical variable valve timing system, and points to the possibility of controlling the air flow into the engine via the valve openings, thus eliminating the throttle and reducing pumping losses. In [2] and [3], varying the intake valve timing was shown to reduce pumping mean effective pressure while improving fuel economy and NOx emissions. [4] describes an electro-hydraulic valvetrain based engine and provides a good summary of the benefits of camless engines. [5] showed that by optimizing valve events at part-load conditions, volumetric efficiency, fuel economy, and NOx emissions could be improved. More recently, in [6] it was shown that using the flexibility of an electromechanical valvetrain to run in 2, 3, and 4 valve modes, as well as by using cylinder deactivation, fuel economy consumption could be improved over a wide range of operating conditions.

More recent attention has focused on the control-oriented modeling and the control of variable valve timing engines. [7] modeled the cylinder-by-cylinder breathing dynamics of a camless engine system, including higher frequency runner dynamics using the forced oscillator model. The model was then used to obtain a cycle-averaged model between valve duration and lift and mean-valued cylinder charge. In the companion paper, [7], the cycle-averaged model was used with the cylinder-by-cylinder model to develop and demonstrate a control approach to estimate cylinder charge online and use that information to update a feedforward map between charge demand and valve lift and duration. However, the model used in [7] and [8] did not include modeling of the exhaust process and the effects of combustion on cylinder pressure, and therefore the effects of combustion on the breathing process and torque produced. This paper attempts to to demonstrate a more complete cylinder-by-cylinder model that includes the exhaust valves, manifold, and breathing process, as well as the in-cylinder combustion.

2 In-Cylinder Dyanmics

The in-cylinder dynamics consist of the 4 states of cylinder pressure, temperature, mass, and burned gas residual fraction, as described in [9]. The cylinder pressure is obtained from the perfect gas law

$$P_{cyl}V_{cyl} = m_{cyl}RT_{cyl} \qquad (1)$$

Where P_{cyl} is the cylinder pressure, V_{cyl} is the cylinder volume, m_{cyl} is the cylinder mass, R is universal gas constant, and T_{cyl} is the cylinder temperature.

Differentiating (1) with respect to time, we obtain

$$\dot{P}_{cyl}V_{cyl} + P_{cyl}\dot{V}_{cyl} = \dot{m}_{cyl}RT_{cyl} + m_{cyl}\frac{dR}{dt}T_{cyl} + m_{cyl}R\dot{T}_{cyl} \quad (2)$$

$$\dot{P}_{cyl}V_{cyl} + P_{cyl}\dot{V}_{cyl} = \dot{m}_{cyl}RT_{cyl} + m_{cyl}\frac{dR}{dF_{cyl}}\frac{dF_{cyl}}{dt}T_{cyl} + m_{cyl}R\dot{T}_{cyl} \quad (3)$$

$$\dot{P}_{cyl}V_{cyl} + P_{cyl}\dot{V}_{cyl} = \dot{m}_{cyl}RT_{cyl} + m_{cyl}\frac{R_1 - R_2}{R}\dot{F}_{cyl}T_{cyl} + m_{cyl}R\dot{T}_{cyl} \quad (4)$$

Where F_{cyl} is the fraction of burned gas in the cylinder and

$$R = F_{cyl}R_1 + (1 - F_{cyl})R_2 \quad (5)$$

Dividing the LHS of Equation (4) by $P_{cyl}V_{cyl}$ and the RHS by $m_{cyl}RT_{cyl}$, we have

$$\dot{P}_{cyl} = [\frac{\dot{m}_{cyl}}{m_{cyl}} + \frac{\dot{T}_{cyl}}{T_{cyl}} + \frac{R_1 - R_2}{R}\dot{F}_{cyl} - \frac{\dot{V}_{cyl}}{V_{cyl}}]P_{cyl} \quad (6)$$

The cylinder mass rate of change from conservation of mass, assuming the convention that flow rate is positive into the control volume, is

$$\dot{m}_{cyl} = \dot{m}_{in} + \dot{m}_{ex} \quad (7)$$

Where $\dot{m}_{in}$ and $\dot{m}_{ex}$ are the mass flow rates through the intake and exhaust valves, respectively. The flow through the valves can be modeled as flow through an orifice as follows:

$$\dot{m} = A_{eff}d(P_1, P_2) \quad (8)$$

where A_{eff} is the effective flow areas of the orifice and P_1 and P_2 are the upstream and downstream pressures and d is the differential pressure constant.

The cylinder temperature, from the 1st Law of Thermodynamics as described in [9], is given by

$$\dot{E} = \dot{Q}_w - \dot{W} + \dot{m}_{in}h_{in} + \dot{m}_{ex}h_{ex} + \dot{Q}_{ch} = \dot{Q}_w - P_{cyl}\dot{V}_{cyl} + \dot{m}_{in}h_{in} + \dot{m}_{ex}h_{ex} + \dot{Q}_{ch} \quad (9)$$

Where $\dot{E}$ is the total energy in the system $\dot{Q}_w$ is the rate of heat transfer through the cylinder wall, $\dot{W}$ is the rate of work done on the piston, h_{in} and h_{ex} are the enthalpy of the flow through the intake and exhaust flows, and $\dot{Q}_{ch}$ is the combustion heat release rate, given by

$$\dot{Q}_{ch} = \frac{dm_b}{dt}Q_{LHV} \quad (10)$$

Here, Q_{LHV} is the lower heating value of the fuel, which is a measure of the energy of the fuel, and can be found in fuel property tables. m_b is the mass of the burned fuel, which is given as the product of the mass fraction burned, x_b and the injected fuel, m_{if}

$$m_b = x_b m_{if} \quad (11)$$

And therefore

$$\frac{dm_b}{dt} = \dot{x}_b m_{if} + x_b \dot{m}_{if} \quad (12)$$

One common method of obtaining the mass fraction burned, x_b, is to use an empirically fit function of mass fraction burned versus crank angle, θ, such as the Wiebe function described in [9], given as

$$x_b = 1 - exp[-a(\frac{\theta - \theta_0}{\Delta\theta})^{m+1}] \quad (13)$$

Where θ_0 is the crank angle at the start of combustion, $\Delta\theta$ is the total combustion duration, and a and m are correlation parameters.

Noting that the total energy is equal to $d(m_{cyl}u)/dt$, and the internal energy, u, is a function of temperature and burned gas fraction, then

$$\dot{E} = \frac{d(m_{cyl}u)}{dt} = \dot{m}_{cyl}u + m_{cyl}\frac{du}{dt} = \dot{m}_{cyl}u + m_{cyl}(\frac{\partial u}{\partial F_{cyl_1}}\frac{\partial F_{cyl_1}}{\partial t} + \frac{\partial u}{\partial T_{cyl}}\frac{\partial T_{cyl}}{\partial t}) \quad (14)$$

Noting the following identities for internal energy

$$u = F_{cyl}u_1 + (1 - F_{cyl})u_2 \quad (15)$$

And the specific heat, c_v, which is defined as

$$c_v = \frac{du}{dT} \quad (16)$$

Then, Equation (14) becomes

$$\dot{E} = \dot{m}_{cyl}u + m_{cyl}c_v\dot{T}_{cyl} + m_{cyl}(u_1 - u_2)\dot{F}_{cyl_1} \quad (17)$$

Setting Equation (9) equal to (17) and solving for $\dot{T}_{cyl}$

$$m_{cyl}c_v\dot{T}_{cyl} = \dot{Q}_w - P_{cyl}\dot{V}_{cyl} + \dot{m}_{in}h_{in} + \dot{m}_{ex}h_{ex} + \dot{Q}_{ch} - \dot{m}_{cyl}u - m_{cyl}(u_1 - u_2)\dot{F}_{cyl_1} \quad (18)$$

The dynamics of the burned gas fraction in the cylinder are described as

$$\frac{dm_{cyl}F_{cyl}}{dt} = \dot{m}_{cyl}F_{cyl} + m_{cyl}\dot{F}_{cyl} = \dot{m}_{in}F_{i\leftrightarrow cyl} + \dot{m}_{ex}F_{cyl\leftrightarrow e} \quad (19)$$

$$+ min((m_{cyl}(1 - F_{cyl}))_{SOC}, (m_{if}AFR)_{SOC})\dot{x}_b$$

Where the function $min(.)$ evaluates to $(m_{if}AFR)$ at the start of combustion (SOC) if the mixture is lean or $(m_{cyl}(1-F_{cyl}))$ at SOC, if the mixture is stoichiometric or rich. This is done since only the part of the mixture that is stochiometric will burn completely, i.e, excess fuel is not burned and if the fuel is lean, only the portion of unburned gases in the cylinder stoichiometrically proportional to the fuel will burn. Also, $\dot{m}_{in}F_{i\leftrightarrow cyl}$ and $\dot{m}_{ex}F_{cyl\leftrightarrow e}$ are mass flow rate of the burned gases across the intake and exhaust valves, respectively. If the flow of burned gases to the cylinder changes direction, the sign of the mass flow is automatically taken care of but the burned fraction of the flow should be that of the manifold if flow is into the cylinder, or that of the cylinder if the flow is out of the cylinder, this is accomplished by letting the fraction, $\dot{m}_{in}F_{i\leftrightarrow cyl}$, be defined as

$$F_{i\leftrightarrow cyl} = \begin{cases} F_i & \text{if} \quad \dot{m}_{in} > 0 \\ F_{cyl} & \text{if} \quad \dot{m}_{in} \leq 0 \end{cases} \tag{20}$$

And

$$F_{cyl\leftrightarrow e} = \begin{cases} F_e & \text{if} \quad \dot{m}_{ex} > 0 \\ F_{cyl} & \text{if} \quad \dot{m}_{ex} \leq 0 \end{cases} \tag{21}$$

Where F_i and F_e are the intake and exhaust manifold fraction, respectively.

Solving Equation (19) for $\dot{F}_{cyl}$, the rate of change of burned fraction in the cylinder can be obtained from

$$\begin{aligned} m_{cyl}\dot{F}_{cyl_1} &= \dot{m}_{in}F_{i\leftrightarrow cyl} + \dot{m}_{ex}F_{cyl\leftrightarrow e} - \dot{m}_{cyl}F_{cyl} \\ &+ min((m_{cyl}(1-F_{cyl}))_{SOC}, (m_{if}AFR)_{SOC})\dot{x}_b \end{aligned} \tag{22}$$

3 Manifold Dynamics

The manifold dynamics also consist of 4 states: manifold pressure, temperature, mass, and burned gas residual fraction, analogous to the cylinder dynamics. The manifold pressure is obtained from

$$\dot{P}_i V_i = \dot{m}_i R T_i + m_i \frac{R_1 - R_2}{R} \dot{F}_i + m_i R \dot{T}_i \tag{23}$$

Where P_i, m_i, V_i, T_i, and F_i, are the intake manifold pressure, mass, volume, temperature, and fraction.

Dividing the LHS of Equation (23) by P_iV_i and the RHS by m_iRT_i

$$\dot{P}_i = [\frac{\dot{m}_i}{m_i} + \frac{\dot{T}_i}{T_i} + \frac{R_1 - R_2}{R}\dot{F}_i]P_i \tag{24}$$

The intake manifold mass rate of change from conservation of mass is

$$\dot{m}_i = \dot{m}_{throttle} + \sum_i \dot{m}_{in_i} \tag{25}$$

Where $\dot{m}_{throttle}$ is the mass flow rate through the throttle, and i is the cylinder index. For the manifold temperature state, the manifold volume is constant and heat transfer through the manifold walls are neglected

$$\begin{aligned} m_i c_v \dot{T}_i &= \dot{m}_{throttle}h_{throttle} + \dot{m}_{in}h_{in} \\ &-\dot{m}_i u - m_i(u_1 - u_2)\dot{F}_i \end{aligned} \tag{26}$$

The burned gas fraction rate of change in the intake manifold is given by

$$\begin{aligned} \frac{dm_iF_i}{dt} = \dot{m}_iF_i + m_i\dot{F}_i &= \dot{m}_{throttle}F_{throttle\leftrightarrow i} \\ &+\dot{m}_{in}F_{i\leftrightarrow cyl} \end{aligned} \tag{27}$$

Where

$$F_{throttle\leftrightarrow i} = \begin{cases} F_{throttle} & \text{if} \quad \dot{m}_{throttle} > 0 \\ F_i & \text{if} \quad \dot{m}_{throttle} \leq 0 \end{cases} \tag{28}$$

We will assume $F_{throttle} = 0$, since the intake air contains no burned component. Solving Equation (27) for $\dot{F}_{i_1}$

$$m_i\dot{F}_i = \dot{m}_{throttle}F_{throttle\leftrightarrow i} + \dot{m}_{in}F_{i\leftrightarrow cyl} - \dot{m}_iF_i \tag{29}$$

The exhaust manifold is analogous to the intake manifold, with analogous states, P_e, m_e, V_e, T_e, and F_e, which are the exhaust manifold pressure, mass, volume, temperature, and fraction, except that flow is through the exhaust pipe instead of the throttle.

The cylinder mass rate of change from conservation of mass is

$$\dot{m}_e = \dot{m}_{ex} + \dot{m}_{epipe} \tag{30}$$

The burned gas fraction rate of change in the exhaust manifold is given by

$$m_e\dot{F}_e = \dot{m}_{ex}F_{cyl\leftrightarrow e} + \dot{m}_{epipe}F_{e\leftrightarrow epipe} - \dot{m}_eF_{e_1} \tag{31}$$

Where

$$F_{e\leftrightarrow epipe} = \begin{cases} F_{epipe} & \text{if} \quad \dot{m}_{epipe} > 0 \\ F_e & \text{if} \quad \dot{m}_{epipe} \leq 0 \end{cases} \tag{32}$$

4 Rotational Dynamics

The cylinder pressure acting against the piston creates a force. The force is composed of an inertial component and a component due to pressure acting on the piston area and is given by

$$F_{piston} = F_{press} + F_{inertial} \tag{33}$$

Or, more explicitly,

$$F_{piston} = (P_{cyl} - P_{atm})\frac{\pi B^2}{4} - (m_{eff})A \tag{34}$$

Where P_{atm} is atmospheric pressure, B is the cylinder bore, m_{eff} is the effective mass, and A is the piston acceleration, which can be obtained from

$$A = R\omega^2[cos\theta + \frac{\lambda(cos2\theta + \lambda^2 sin^4\theta)}{(1-\lambda^2 sin^2\theta)^{1/2}}] \quad (35)$$
$$+R\dot{\omega}sin\theta[1 + \frac{\lambda cos\theta}{(1-\lambda^2 sin^2\theta)^{1/2}}]$$

where R is the crank radius, and λ is the ratio of crank radius to connecting road length, and ω and $\dot{\omega}$ are the angular velocity and acceleration, respectively.

The net indicated torque for each piston, T_i is equal to the product of the tangential component of the force and the crank radius, so

$$T_i = F_{piston}[sin\theta + \frac{\lambda sin\theta cos\theta}{(1-\lambda^2 sin^2\theta)^{1/2}}]R \quad (36)$$

The angular acceleration is then given by

$$\dot{\omega} = \frac{1}{I_{eff}}(\sum T_i - T_{mf} - T_a - T_{load}) \quad (37)$$

Where T_{mf} is the mechanical friction loss torque, T_a is the essential accessory loss torque, T_{load} is the load torque, and I_{eff} is the effective inertia.

5 Simulation Results

A 24 state model was implemented in Simulink. A block diagram of the model is shown in Figure 1. The control inputs to the model are change in valve command, spark, and fuel injection duration from a nominal value. For example, intake valve closing (IVC) nominal was chosen as bottom dead center before combustion (crank angle = -180 degrees, using the convention that combustion top dead center is 0 degrees). So a change in IVC of 20 degrees would be a delay in intake valve closing to -160 degrees.

Experimental data was only available at low speeds and load conditions, because of the durability of the experimental engine. Peak pressure was chosen as the model correlation parameter because of its direct impact on net indicated engine torque. Comparison of experimental and model peak pressures are shown in Figure 2 and 3.

6 Control Architecture

A proposed control architecture is shown in Figure 4. Although the cylinder-by-cylinder model predicts instantaneous torque and cylinder pressure at a given crank angle, the control can only affect the plant on a per-cycle basis. Therefore, for controller design, a relationship is required between valve timing and net indicated torque on a cycle average basis. The cylinder-by-cylinder model is used to obtain this relationship. The map is then used in combination with the valve actuator dynamics, rotational dynamics, and torque loss models for design and evaluation of the controller.

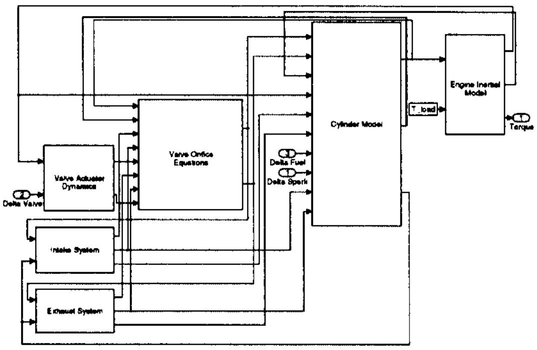

Figure 1: Variable Valve Timing Engine Model

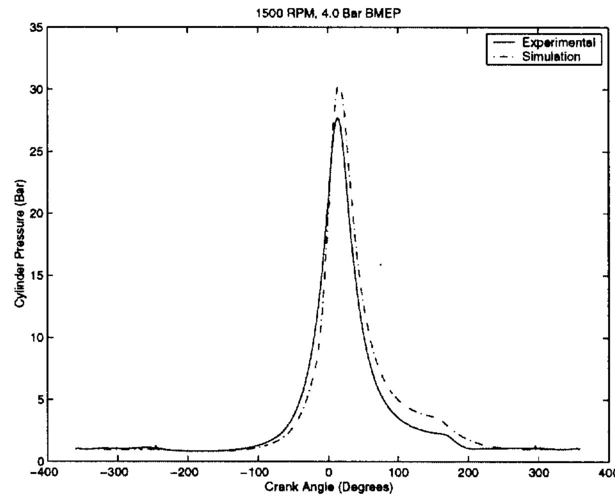

Figure 2: Peak Pressure Comparison at 650 RPM and 1.5 BMEP

Because the throttle position and air inlet to the cylinder is no longer controlled by driver input, a nonlinear map between pedal position and desired torque demand is typically used. This demand map requires the work of experienced vehicle calibrators, and for the purpose of this study is assumed to already exist. The net indicated torque demand is then used as the desired signal to be tracked. Because the control is only available on a per-cyle basis, the average net indicated torque per cycle, rather than the instantaneous torque is required. The cycle averaged torque is obtained as by integrating the net integrated torque over the engine

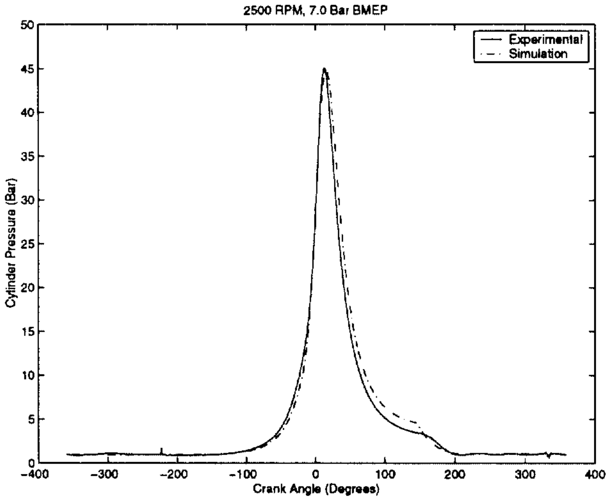

Figure 3: Peak Pressure Comparison at 2500 RPM and 7.0 BMEP

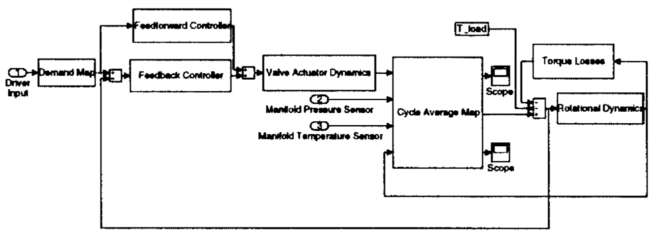

Figure 4: Control Approach

cycle.

$$T_{ave} = \frac{1}{\theta_{cycle}} \int_{cycle} (\sum_i T_i) d\theta \qquad (38)$$

Where i is the cylinder index. The relationship between valve timing and torque and air charge was obtained be sweeping valve position to obtain the steady-state, cycle-averaged map. The torque output at 2500 RPM is shown in Figure 5. Because of it's dominant effect on volumetric efficiency and load control, only intake valve timing will be varied.

7 Conclusions

A cylinder-by-cylinder model of 4-cylinder vararible valve timing engine has been developed and correlated for peak pressure at several engine speed and load conditions. The model has been used to gain insight into the variable valve timing engine and has been used to generate cycle-averaged mapping between IVC timing and speed-load conditions that will be necessary for control system design.

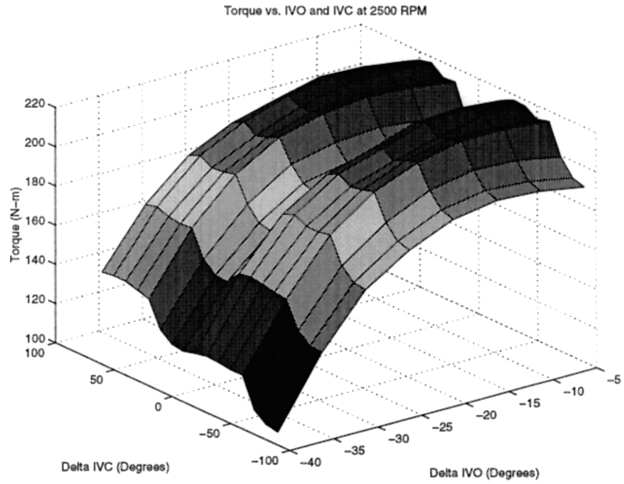

Figure 5: Torque vs. IVO and IVC at 2500 RPM

Acknowledgment

The authors would like to thank İbrahim Haskara for his help with the solver for the thermodynamic properties and help debugging, and Brett Collins for the contribution of the friction/inertia model.

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