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Preface

This is a preliminary release to students in Math 775 of material intended for a much larger work (roughly 25 chapters in two volumes). Readers are urged to contact the authors concerning any misprints that are detected. Other comments or suggestions for improvement will also be very welcome.

The object of this book is to introduce the interested student to the techniques, results, and terminology of multiplicative number theory. It is not intended that our discussion will always reach the research frontier. Rather, it is hoped that the material here will prepare the student for intelligent reading of the more advanced research literature.