Math 425 Section 6 **Fall 2015**

Poker Probabilities

Let P_0 denote the probability of being dealt a bust (i.e., a useless hand). There are $\binom{13}{5}$ ways of choosing the values of the 5 cards, except that they must not be consecutive, which can be done in 10 ways, assuming that an ace is considered to be adjacent both to king and to 2. There are 4^5 ways of choosing the suits of these cards, but they must not be all in the same suit, so there are $4^5 - 4$ ways of choosing the suits. Thus

$$P_0 = \frac{\left(\binom{13}{5} - 10\right)(4^5 - 4)}{\binom{52}{5}} = \frac{1277}{2548} = 0.5011773940\ldots$$

Let P_1 denote the probability of being dealt a pair. Then

$$P_1 = \frac{\binom{13}{1}\binom{4}{2}\binom{12}{3}4^3}{\binom{52}{5}} = \frac{352}{833} = 0.4225690296\ldots$$

Let P_2 denote the probability of being dealt two pairs. Then

$$P_2 = \frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{44}{1}}{\binom{52}{5}} = \frac{198}{4165} = 0.0475390156\ldots$$

Let P_3 denote the probability of being dealt a three of a kind. Then

$$P_3 = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}4^2}{\binom{52}{5}} = \frac{88}{4165} = 0.0211284514\ldots$$

Let P_4 denote the probability of being dealt a straight (5 cards with consecutive values). There are 10 possibilities for the values, if one assumes that aces can be high or low. The cards can be of arbitrary suits, except that they must not all be of the same suit. Hence

$$P_4 = \frac{10(4^5 - 4)}{\binom{52}{5}} = \frac{5}{1274} = 0.0039246468\ldots$$

Let P_5 denote the probability of being dealt a flush (5 cards of the same suit, but not with consecutive values). There are $\binom{13}{5}$ ways of choosing 5 distinct values. Of these, there are 10 ways in which the values are consecutive. Thus the values can be chosen in $\binom{13}{5} - 10$ ways. There are 4 possible suits, so

$$P_5 = \frac{\left(\binom{13}{5} - 10\right)4}{\binom{52}{5}} = \frac{3 \cdot 71}{2 \cdot 5 \cdot 7^2 \cdot 13 \cdot 17} = \frac{1277}{649740} = 0.0019654015\ldots$$

Let P_6 denote the probability of being dealt a full house. Then

$$P_6 = \frac{\binom{13}{1}\binom{12}{1}\binom{4}{3}\binom{4}{2}}{\binom{52}{5}} = \frac{6}{4165} = 0.0014405762\ldots$$

Let P_7 denote the probability of being dealt four of a kind. Then

$$P_7 = \frac{\binom{13}{1}\binom{12}{1}\binom{4}{1}}{\binom{52}{5}} = \frac{1}{4165} = 0.0002400960\ldots$$

Let P_8 denote the probability of being dealt a straight flush that is not a royal flush (cards with consecutive values, all in one suit, but not values $10 - A$). There are 9 allowed straights and 4 possible suits, so

$$P_8 = \frac{9 \cdot 4}{\binom{52}{5}} = \frac{3}{216580} = 0.00001385\ldots$$

Finally, let P_9 denote the probability of being dealt a royal flush (5 cards of the same suit, with values $10 - A$). The values are specified, and there are 4 possible suits, so

$$P_9 = \frac{4}{\binom{52}{5}} = \frac{1}{649740} = 0.0000015391\ldots$$

It may be verified that $P_0 + \cdots + P_9 = 1$.