Math 425 Sect 3 **Fall 2015**

Second Hour Exam

NAME ____

Answers may be expressed in terms of factorials, but not in terms of binomial or multinomial coefficients. Products do not need to be multiplied out, and fractions do not need to be expressed in lowest terms.

3 pts **1.** Let X be a random variable with expectation $E[X]$. Under what circumstances is it true that $E[aX + b] = aE[X] + b$?

3 pts **2.** If X is a random variable with $\text{Var}(X) = \sigma^2$, then $\text{Var}(aX + b) =$ ____________

3. Let X be an exponential random variable with parameter $\lambda > 0$. Then (for 4 pts each):

(a) $f_X(x) =$

(b) $F_X(x) =$

(c) $E[X] =$

(d) $\text{Var}(X) =$

4 pts (e) What do we mean when we say that X is 'memoryless'?

5 pts (f) Suppose that $a > 0$ and put $Y = aX$. Show that Y is an exponential random variable with parameter λ/a.

4. Suppose that X and Y are discrete random variables with joint mass function $p(x, y)$ as given below:

		$x = 1$	$x = 2$	$x = 3$
y	1	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{32}$
	2	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{16}$
	3	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{32}$

5 pts (a) Find the point masses $p_X(x)$ for $x = 1, 2, 3$.

5 pts (b) Find the point masses $p_Y(y)$ for $y = 1, 2, 3$.

5 pts (c) Are X and Y independent?

5. Let X be a standard normal variable (i.e., a normal variable with $\mu = 0$ and $\sigma = 1$).

5 pts (a) The density function of X is

$$f_X(x) =$$

5 pts (b) Derive a formula for $f''_X(x)$.

5 pts (c) Determine when $f_X(x)$ is concave upwards, and when it is concave down.

6. You enter a barbershop on the SW corner of Thompson and William, and you might find two barbers on duty, Matt and his dad Marty. Suppose that each of their haircuts lasts 15 minutes, and that they are not synchronized, so that the length of time one of them will take to finish is independent of the other. Let X denote the length of time, in minutes, that you need to wait for your haircut. (Assume that you are first in line.)

7pts (a) For real numbers a, $0 \le a \le 15$, find $P(X > a)$.

5pts (b) Let $f(x)$ denote the density function of X. Find a formula for $f(x)$.

5pts (c) Find $E[X]$.

7. Let X be a uniform random variable on $[0, 1]$, and put $Y = \sqrt{-2\log(1 - X)}$.

7 pts (a) For $a \ge 0$, derive a formula for $F_Y(a) = P(Y \le a)$.

3 pts (b) Deduce a formula for $f_Y(y)$.

8. Let X and Y be independent random variables that take only nonnegative integer values.

5 pts (a) Derive a formula for $P(X+Y=n)$ in terms of $p_X(i)$ and $p_Y(j)$ for suitable i and j.

7 pts (b) Now assume that X is Poisson with parameter $\lambda_1 > 0$, and that Y is Poisson with parameter $\lambda_2 > 0$. Show that $X+Y$ is Poisson $\lambda_1 + \lambda_2$.