Math 425 Sect 3 **Fall 2015**

The Birthday Paradox

We assume that birthdays fall evenly among the possible 365 days of the year. Then the probability that k people will have distinct birthdays is

$$\frac{365}{365}\,\frac{364}{365}\cdots\frac{365-k+1}{365} = \prod_{j=0}^{k-1}\left(1-\frac{j}{365}\right).$$

Since $1-\delta$ is approximately $e^{-\delta}$ when δ is small, we expect the above to be approximately $\exp(-k(k-1)/730)$. In the table below, we present the exactly-computed value, and our approximation to it.

k	$P(E_k)$	Approximation	k	$P(E_k)$	Approximation
1	1.0000000000	1.0000000000	26	0.4017591799	0.4104870248
2	0.9972602740	0.9972640236	27	0.3731407177	0.3822639901
3	0.9917958341	0.9918145070	28	0.3455385277	0.3550074734
4	0.9836440875	0.9836960163	29	0.3190314625	0.3287923880
5	0.9728644263	0.9729746406	30	0.2936837573	0.3036799823
6	0.9595375164	0.9597370960	31	0.2695453663	0.2797182023
7	0.9437642969	0.9440895580	32	0.2466524721	0.2569422031
8	0.9256647076	0.9261562438	33	0.2250281458	0.2353749882
9	0.9053761661	0.9060777707	34	0.2046831354	0.2150281561
10	0.8830518223	0.8840093220	35	0.1856167611	0.1959027364
11	0.8588586217	0.8601186523	36	0.1678178936	0.1779900913
12	0.8329752112	0.8345839700	37	0.1512659918	0.1612728651
13	0.8055897248	0.8075917322	38	0.1359321789	0.1457259615
14	0.7768974880	0.7793343910	39	0.1217803356	0.1313175313
15	0.7470986802	0.7500081297	40	0.1087681902	0.1180099541
16	0.7163959947	0.7198106244	41	0.0968483885	0.1057607996
17	0.6849923347	0.6889388665	42	0.0859695284	0.0945237541
18	0.6530885821	0.6575870803	43	0.0760771443	0.0842495035
19	0.6208814740	0.6259447624	44	0.0671146314	0.0748865616
20	0.5885616164	0.5941948725	45	0.0590241005	0.0663820383
21	0.5563116648	0.5625121946	46	0.0517471566	0.0586823407
22	0.5243046923	0.5310618892	47	0.0452255972	0.0517338053
23	0.4927027657	0.4999982478	48	0.0394020271	0.0454832588
24	0.4616557421	0.4694636606	49	0.0342203907	0.0398785056
25	0.4313002960	0.4395878005	50	0.0296264204	0.0348687459