

## Appendix C

### Conway's Doomsday Rule

The rule referred to is a calculational procedure or mnemonic for determining the day of the week from the date. For simplicity we restrict ourselves to the 20<sup>th</sup> and 21<sup>st</sup> centuries, although one can also apply the method to earlier times.

We associate each day of the week with one of the numbers 0 through 6, as follows:

TABLE 1. Day to number pairing.

Sunday	=	Noneday	=	0
Monday	=	Oneday	=	1
Tuesday	=	Twosday	=	2
Wednesday	=	Threesday	=	3
Thursday	=	Foursday	=	4
Friday	=	Fivesday	=	5
Saturday	=	Sixerday	=	6

In any given year, there is one particular day of the week that is known as the Doomsday for that year. Our first task is to determine which day this is. In 1900, Doomsday came on a Wednesday (= 3). Since  $365 \equiv 1 \pmod{7}$ , it follows that Doomsday moves forward one day per year, except that it moves forward by 2 days in a leap year. Thus in year  $1900 + n$ ,  $0 \leq n < 100$ , Doomsday occurred on day  $3 + n + [n/4]$ . Here  $[x]$  denotes the integer part of  $x$ , which is to say the largest integer not exceeding  $x$ . When  $n$  is large, this involves a moderate amount of arithmetic. To shorten this, we note that in any span of twelve consecutive years (within a given century) there are precisely 3 leap years, and hence Doomsday moves forward by  $12 + 3 \equiv 1 \pmod{7}$  days. Thus we say,

#### TWELVE YEARS IS BUT A DAY

For example, we might compute that for 1957, Doomsday came on day  $3 + 57 + [57/4] = 74 \equiv 4 \pmod{7}$ , which is to say on a Thursday. Alternatively, we could observe that  $57 = 4 \cdot 12 + 9$ , and hence that Doomsday came on day  $3 + 4 + 9 + [9/4] = 18 \equiv 4 \pmod{7}$ . In the year 2000, Doomsday came on a Tuesday. Hence, in 2007, Doomsday is on day  $2 + 7 + [7/4] \equiv 3$ , which is to say on Wednesday.

Now that we know which day of the week is Doomsday, we need to associate Doomsday with a particular date in each month. This runs as follows:

TABLE 2. A Doomsday for each month.

1 January	31/32	7 July	11
2 February	28/29	8 August	8
3 March	7	9 September	5
4 April	4	10 October	10
5 May	9	11 November	7
6 June	6	12 December	12

Of course there is no such thing as January 32, but we understand this date to refer to February 1. For even numbered months after February, the Doomsday is simply the number of the month. For odd numbered months after January, it is the number of the month + 4 if the month is long (31 days), and the number of the month - 4 if the month is short (30 days). Here March, May and July are long, while September and November are short. An alternative way of remembering this pattern is by reciting,

I WORK 9 TO 5 AT THE SEVEN ELEVEN

That is, May (the fifth month) is associated with 9 and September (the ninth month) with 5, while July (the seventh month) is associated with 11 and November (the eleventh month) is associated with 7. This leaves March, but one can think of Doomsday in March as March 0, which is to say February 28/29. Once we know the day of the week of one particular date in a month, we can recover the day of the week of any other date in that month.

### Exercises

1. Count the number of days from January 31/32 to February 28/29, also the number of days from then to March 7, from then to April 4, and so on, to December 12, and verify that all these counts are multiples of 7.
2. Count the number of days from December 12 to the next January 31, and verify that this number is one more than a multiple of 7.
3. What is Doomsday in 2007?
4. In 2007, Easter fell on April 15. What day of the week was that? (Don't be a Smart Alec: Do the math!)

5. John Kennedy was shot on 22 November, 1963. What day of the week was that?
6. On February 3, 1959, Buddy Holly, The Big Bopper, and Richie Valens were passengers in a small plane that crashed, killing all aboard. (The song *American Pie* by Don McLean is based on this event.) So what day of the week was it, the day the music died?

*But February made me shiver, with every paper I'd deliver.  
Bad news on the doorstep—I couldn't take one more step.  
I can't remember if I cried, when I read about his widowed bride,  
But something touched me deep inside, the day the music died.*

*So bye, bye miss American pie.  
Drove my chevy to the levee, but the levee was dry.  
And them good old boys were drinkin' whiskey and rye,  
Singin', "This'll be the day that I die,  
This'll be the day that I die."*