# Appendix for "Two-Sample Instrumental Variables Estimators"

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This note summarizes the Monte Carlo experiments mentioned in footnote 4 of Inoue and Solon (2006). The data-generating process is

$$y_{1i} = \beta x_{1i} + \varepsilon_{1i}, \tag{1}$$

$$x_{1i} = \Pi z_{1i} + \eta_{1i}, \tag{2}$$

$$x_{2i} = \Pi z_{2i} + \eta_{2i}, \tag{3}$$

for $i = 1, 2, ..., n$, where $x_{1i}$ and $x_{2i}$ are scalar endogenous variables, $z_{1i}$ and $z_{2i}$ are $q$-dimensional vectors of instruments, $\Pi$ is a $1 \times q$ vector of parameters, and

$$\begin{bmatrix} \varepsilon_{1i} \\ \eta_{1i} \end{bmatrix} \sim N\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \right)$$

and $\eta_{2i} \sim N(0, 1)$ are independent. We use $\beta \in \{0, 1, 5\}$, $n_1 = n_2 = n \in \{100, 500, 1000\}$, $q \in \{1, 3\}$, $\Pi$ is set so that the population first-stage $R^2$ is 0.25, 0.5 or 0.75, and the number of Monte Carlo replications is set to 10000.

The main findings include:

- As expected, the bias of the two-sample IV estimators decreases as the sample size grows while other model parameters are fixed.
- As our asymptotic theory predicts, the TSIV estimator exhibits larger variabilities in terms of median absolute value and standard deviation, resulting in larger root mean squared error.
- As in the one-sample case, the TSLIML estimator is less vulnerable to the finite-sample bias associated with weak instruments.

**Table 1. Finite-Sample Properties of Two-Sample IV Estimators ($q = 1$)**

| $n_1$ | | | TSIV | | | | TS2SLS=TSLIML | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n_2$ | $\beta$ | $R^2$ | MB | MAD | SD | RMSE | MB | MAD | SD | RMSE |
| 100 | 0 | 0.25 | 0.004 | 0.119 | 0.190 | 0.190 | 0.004 | 0.120 | 0.186 | 0.186 |
| 100 | 0 | 0.50 | 0.002 | 0.068 | 0.105 | 0.105 | 0.002 | 0.069 | 0.103 | 0.104 |
| 100 | 0 | 0.75 | 0.001 | 0.039 | 0.060 | 0.060 | 0.001 | 0.040 | 0.059 | 0.059 |
| 100 | 1 | 0.25 | -0.003 | 0.266 | 0.453 | 0.457 | -0.002 | 0.236 | 0.384 | 0.386 |
| 100 | 1 | 0.50 | -0.003 | 0.190 | 0.302 | 0.304 | -0.001 | 0.136 | 0.209 | 0.210 |
| 100 | 1 | 0.75 | -0.003 | 0.154 | 0.243 | 0.245 | -0.001 | 0.078 | 0.119 | 0.119 |
| 100 | 5 | 0.25 | 0.007 | 1.095 | 1.884 | 1.906 | -0.011 | 0.901 | 1.463 | 1.473 |
| 100 | 5 | 0.50 | -0.007 | 0.826 | 1.341 | 1.351 | -0.007 | 0.518 | 0.786 | 0.788 |
| 100 | 5 | 0.75 | -0.001 | 0.731 | 1.150 | 1.156 | -0.004 | 0.300 | 0.444 | 0.445 |
| 500 | 0 | 0.25 | 0.001 | 0.052 | 0.079 | 0.079 | 0.001 | 0.052 | 0.078 | 0.078 |
| 500 | 0 | 0.50 | 0.000 | 0.030 | 0.045 | 0.045 | 0.000 | 0.030 | 0.045 | 0.045 |
| 500 | 0 | 0.75 | 0.000 | 0.017 | 0.026 | 0.026 | 0.000 | 0.017 | 0.026 | 0.026 |
| 500 | 1 | 0.25 | 0.001 | 0.120 | 0.181 | 0.182 | 0.001 | 0.106 | 0.157 | 0.157 |
| 500 | 1 | 0.50 | 0.001 | 0.085 | 0.127 | 0.127 | 0.001 | 0.061 | 0.090 | 0.090 |
| 500 | 1 | 0.75 | 0.001 | 0.069 | 0.103 | 0.103 | 0.000 | 0.035 | 0.052 | 0.052 |
| 500 | 5 | 0.25 | 0.002 | 0.488 | 0.742 | 0.744 | -0.003 | 0.391 | 0.589 | 0.590 |
| 500 | 5 | 0.50 | 0.005 | 0.371 | 0.558 | 0.559 | -0.002 | 0.226 | 0.336 | 0.336 |
| 500 | 5 | 0.75 | 0.004 | 0.321 | 0.483 | 0.484 | -0.001 | 0.131 | 0.193 | 0.193 |
| 1000 | 0 | 0.25 | 0.001 | 0.037 | 0.055 | 0.055 | 0.001 | 0.037 | 0.054 | 0.054 |
| 1000 | 0 | 0.50 | 0.001 | 0.021 | 0.031 | 0.031 | 0.001 | 0.021 | 0.031 | 0.031 |
| 1000 | 0 | 0.75 | 0.000 | 0.012 | 0.018 | 0.018 | 0.000 | 0.012 | 0.018 | 0.018 |
| 1000 | 1 | 0.25 | 0.001 | 0.085 | 0.127 | 0.127 | 0.003 | 0.074 | 0.109 | 0.109 |
| 1000 | 1 | 0.50 | 0.001 | 0.060 | 0.090 | 0.090 | 0.002 | 0.042 | 0.063 | 0.063 |
| 1000 | 1 | 0.75 | -0.000 | 0.049 | 0.074 | 0.074 | 0.001 | 0.025 | 0.036 | 0.036 |
| 1000 | 5 | 0.25 | -0.000 | 0.346 | 0.525 | 0.526 | 0.007 | 0.277 | 0.412 | 0.413 |
| 1000 | 5 | 0.50 | -0.001 | 0.266 | 0.400 | 0.401 | 0.004 | 0.159 | 0.237 | 0.237 |
| 1000 | 5 | 0.75 | -0.003 | 0.233 | 0.350 | 0.350 | 0.002 | 0.092 | 0.136 | 0.136 |

*Note*: MB stands for median bias, MAD for median absolute deviation, SD for standard deviation and RMSE for root mean squared error.

**Table 2. Finite-Sample Properties of Two-Sample IV Estimators ($q = 3$)**

| $n_1$ | | | TSIV | | | | TS2SLS | | | | TSLIML | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n_2$ | $\beta$ | $R^2$ | MB | MAD | SD | RMSE | MB | MAD | SD | RMSE | MB | MAD | SD | RMSE |
| 100 | 0 | .25 | -.003 | .111 | .179 | .179 | -.003 | .114 | .178 | .178 | -.003 | .120 | .193 | .193 |
| 100 | 0 | .50 | -.002 | .066 | .102 | .102 | -.002 | .067 | .102 | .102 | -.002 | .068 | .103 | .103 |
| 100 | 0 | .75 | -.001 | .039 | .059 | .059 | -.001 | .039 | .058 | .058 | -.001 | .040 | .059 | .059 |
| 100 | 1 | .25 | -.109 | .281 | .422 | .425 | -.069 | .244 | .361 | .362 | -.020 | .247 | .390 | .391 |
| 100 | 1 | .50 | -.065 | .201 | .295 | .297 | -.026 | .143 | .207 | .207 | -.010 | .142 | .211 | .211 |
| 100 | 1 | .75 | -.049 | .164 | .240 | .241 | -.010 | .082 | .119 | .119 | -.006 | .082 | .119 | .119 |
| 100 | 5 | .25 | -.491 | 1.166 | 1.732 | 1.749 | -.328 | .911 | 1.347 | 1.357 | -.056 | .895 | 1.420 | 1.423 |
| 100 | 5 | .50 | -.295 | .887 | 1.308 | 1.315 | -.124 | .526 | .775 | .777 | -.029 | .519 | .785 | .785 |
| 100 | 5 | .75 | -.216 | .774 | 1.135 | 1.139 | -.043 | .303 | .447 | .447 | -.013 | .299 | .446 | .446 |
| 500 | 0 | .25 | .003 | .052 | .078 | .078 | .002 | .052 | .078 | .078 | .002 | .052 | .078 | .078 |
| 500 | 0 | .50 | .001 | .030 | .045 | .045 | .001 | .030 | .045 | .045 | .001 | .030 | .045 | .045 |
| 500 | 0 | .75 | .001 | .017 | .026 | .026 | .001 | .018 | .026 | .026 | .001 | .018 | .026 | .026 |
| 500 | 1 | .25 | -.019 | .121 | .178 | .179 | -.011 | .103 | .154 | .155 | -.002 | .105 | .156 | .157 |
| 500 | 1 | .50 | -.013 | .086 | .127 | .127 | -.004 | .060 | .089 | .089 | -.001 | .060 | .090 | .090 |
| 500 | 1 | .75 | -.012 | .071 | .105 | .105 | -.001 | .035 | .051 | .051 | -.000 | .035 | .052 | .052 |
| 500 | 5 | .25 | -.094 | .496 | .731 | .733 | -.065 | .394 | .579 | .580 | -.014 | .386 | .586 | .586 |
| 500 | 5 | .50 | -.062 | .381 | .562 | .563 | -.021 | .227 | .335 | .335 | -.005 | .225 | .336 | .336 |
| 500 | 5 | .75 | -.059 | .333 | .494 | .495 | -.008 | .131 | .193 | .193 | -.002 | .130 | .193 | .193 |
| 1000 | 0 | .25 | .001 | .036 | .053 | .053 | .001 | .036 | .054 | .054 | .001 | .036 | .054 | .054 |
| 1000 | 0 | .50 | .000 | .021 | .031 | .031 | .000 | .021 | .031 | .031 | .000 | .021 | .031 | .031 |
| 1000 | 0 | .75 | .000 | .012 | .018 | .018 | .000 | .012 | .018 | .018 | .000 | .012 | .018 | .018 |
| 1000 | 1 | .25 | -.010 | .085 | .124 | .124 | -.006 | .074 | .107 | .107 | -.000 | .074 | .107 | .107 |
| 1000 | 1 | .50 | -.006 | .060 | .089 | .089 | -.002 | .043 | .062 | .062 | .000 | .043 | .062 | .062 |
| 1000 | 1 | .75 | -.006 | .049 | .073 | .073 | -.001 | .025 | .036 | .036 | -.000 | .025 | .036 | .036 |
| 1000 | 5 | .25 | -.055 | .342 | .510 | .511 | -.029 | .273 | .400 | .400 | -.002 | .275 | .402 | .403 |
| 1000 | 5 | .50 | -.038 | .265 | .394 | .394 | -.010 | .158 | .231 | .231 | -.000 | .158 | .231 | .231 |
| 1000 | 5 | .75 | -.032 | .231 | .347 | .347 | -.004 | .091 | .133 | .133 | -.000 | .091 | .133 | .133 |

*Note*: MB stands for median bias, MAD for median absolute deviation, SD for standard deviation and RMSE for root mean squared error.