Math 115 — Practice for Exam 3

Generated December 3, 2015

Name: _____

Instructor: _____ Section Number: _____

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1. This exam has 6 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2013	3	4	braking	6	
Fall 2011	3	8	Twitter	12	
Winter 2010	3	4		12	
Winter 2013	3	7		11	
Winter 2010	3	7		10	
Fall 2012	3	8		11	
Total				62	

Recommended time (based on points): 74 minutes

3. (continued)

d. [4 points] Give a practical interpretation of the formula

$$g'(3.5) = -3500$$

that begins with
"If Eddie and Laura decrease the price of the soup from \$3.50 per liter to \$3.40 per liter ..."

4. [6 points] A car, initially going 100 feet per second, brakes at a constant rate (constant negative acceleration), coming to a stop in 8 seconds. Let t be the time in seconds after the car started to brake.

a. [3 points] Sketch a graph of the velocity of the car from $t = 0$ to $t = 8$, being sure to include labels.

b. [3 points] Exactly how far does the car travel? Make it clear how you obtained your answer.

8. [12 points] Below is a table of values for the function $t(y)$ which gives the number of tweets per day, in millions, on the social media website Twitter, y years after January 1, 2007. For this problem assume $t(y)$ is an increasing function.

year y	0	1	2	3	4
millions of tweets per day $t(y)$	0.005	0.3	2.5	35	50

a. [4 points] Using the table, estimate the expression

$$365 \int_1^4 t(y)dy$$

using a left-hand Riemann sum. Please write all of the terms in the sum for full credit.

b. [4 points] Give a practical interpretation of the expression $365 \int_1^4 t(y)dy$.

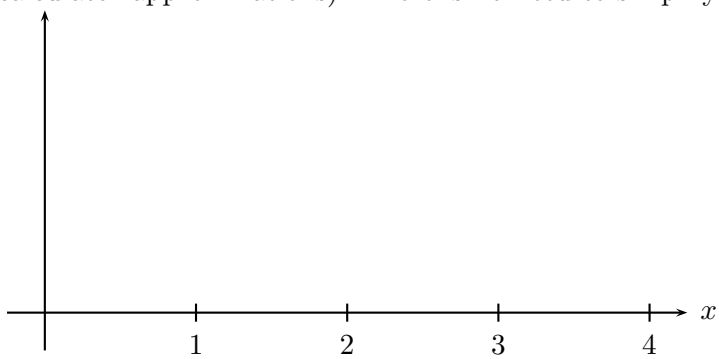
c. [4 points] Suppose $T(y)$ is the total number of tweets, in millions, y years after January 1, 2007. If $T(3) = 9797$, estimate the total number of tweets between January 1, 2007 and January 1, 2011. Indicate what method you use to obtain your estimate and be sure to show your work.

4. [12 points]

a. [6 points] Using 4 equal subdivisions, find a Riemann sum which is an underestimate for

$$\int_2^4 \ln(x)dx.$$

Sketch a graphical representation of your Riemann sum on the axes below, and write "LHS" or "RHS" next to your figure to indicate whether you are using a left-hand sum or a right-hand sum. Write out the terms of the Riemann sum using exact values (no calculator approximations). There is no need to simplify the sum.



$\int_2^4 \ln(x)dx \approx$ _____

b. [3 points]

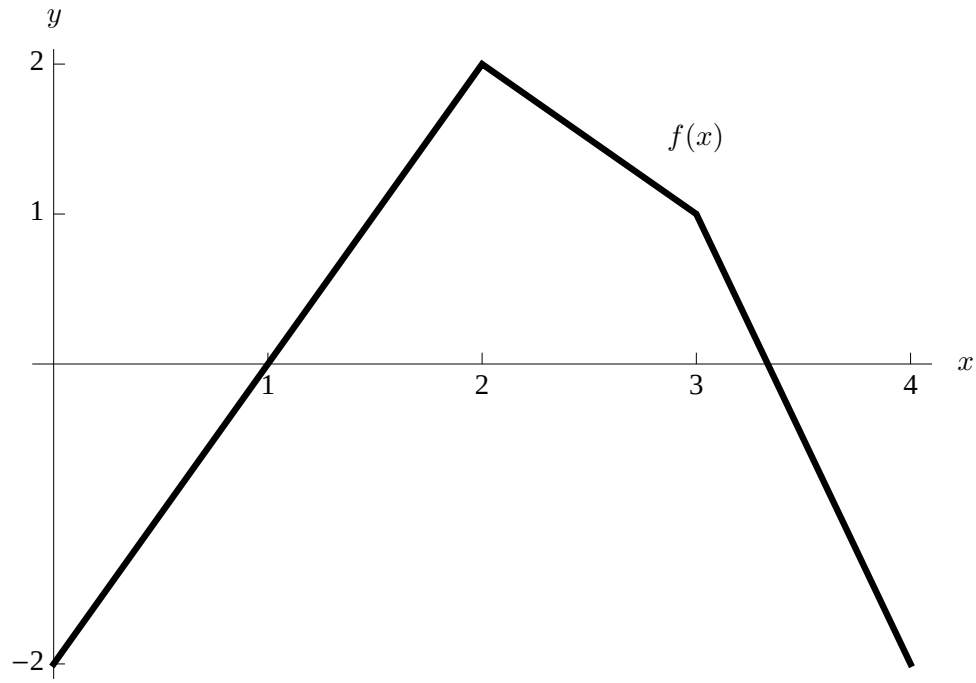
Show that $\int \ln(x)dx = x\ln(x) - x + C$, where C is a constant.

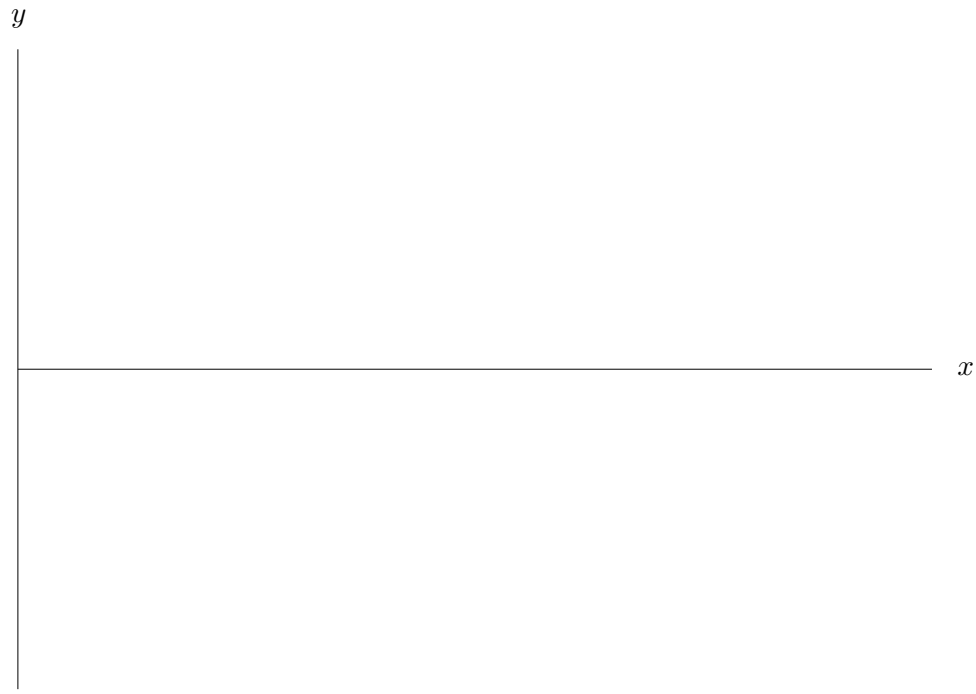
c. [3 points]

Using part (b), find the exact value of the integral $\int_2^4 \ln(x)dx$.

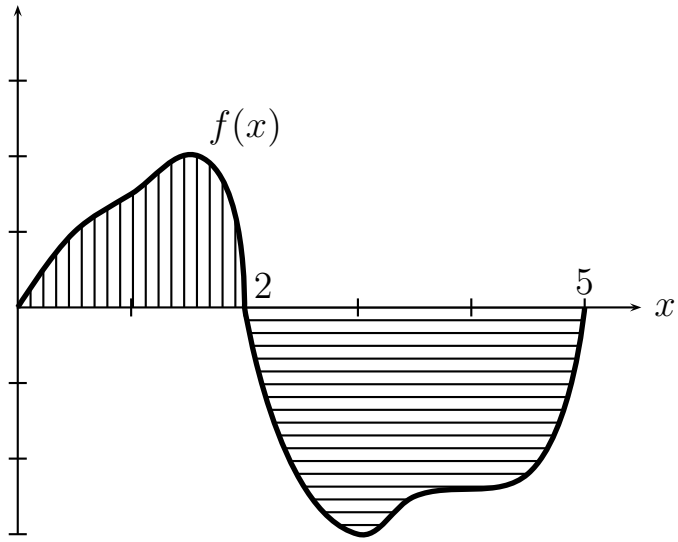
7. [11 points] Below is a picture of the function $f(x)$, which is piecewise linear. On the axes provided, sketch an *antiderivative*, $F(x)$, of the function $f(x)$, with $F(2) = -1$. To receive full credit:

- Label the points on the graph of $F(x)$ where $x = 0, 1, 2, 3, 4$, including y-coordinates.
- Be sure the concavity and local extrema on the graph of $F(x)$ are clear.





7. [10 points] Suppose that f is an even function. A portion of f is graphed below.



The area of the shaded region between $x = 0$ and $x = 2$ (with vertical stripes) is 3 units, and the area of the shaded region between $x = 2$ and $x = 5$ (with horizontal stripes) is 8 units. Find exact values for each of the following integrals. If it is not possible to find the exact value, write "insufficient information".

a. [2 points]

$$\int_{-2}^{2} f(x)dx$$

b. [2 points]

$$\int_{0}^{5} |f(x)|dx$$

c. [2 points]

$$\int_{0}^{1} f(2t)dt$$

d. [2 points]

$$\int_{5}^{8} f(t-3)dt$$

e. [2 points]

$$\int_{5}^{2} 9f(z)dz$$

8. [11 points] Let $W(t)$ be the temperature, in degrees Fahrenheit, of a cake t minutes after it is put in the oven. Assume $W(10) = 220$.

a. [3 points] Give a practical interpretation of the statement $\int_5^{10} W'(t)dt = 120$.

b. [3 points] Give a practical interpretation of the statement $\frac{1}{2}\int_3^5 W(t)dt = 80$.

c. [3 points] Write a single mathematical equation describing the following statement: The average temperature of the cake over the first five minutes in the oven is the same as its temperature after three minutes in the oven.

d. [2 points] Assuming all of the above statements in (a)-(c) are true, what will the temperature of the cake be five minutes after it is put in the oven?