Math 115 — Practice for Exam 3

Generated November 30, 2015

Name: _____

Instructor: _____ Section Number: _____

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1. This exam has 3 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2015	3	8	acorns	11	
Winter 2015	3	7		10	
Winter 2015	3	1	driving	10	
Total				31	

Recommended time (based on points): 37 minutes

8. [11 points] Public opinion has swung against the King since his arrest. Elphaba has been travelling the Sovereign lands collecting donations of acorns to help launch an attack against the King. Let $P(x)$ be the total mass (in kg) of acorns that Elphaba has collected after she has travelled a total of x km. Let $Q(t)$ be Elphaba's velocity (in km/day) when she has been travelling for t days. You may assume that $Q(t)$ is continuous and always positive and that $P(x)$ is an increasing, differentiable function.
For each of questions (a) through (d) below, circle the <u>one</u> best answer. No points will be given for ambiguous or multiple answers.

a. [2 points] Circle the one equation below that best supports the following statement:
When Elphaba has travelled 100 km, she has collected approximately 3 kg less acorns than she will have collected when she has travelled 100.5 km.

i. $P'(100) = 6$

ii. $P'(100) = -3$

iii. $P'(100) = 1.5$

iv. $P'(100.5) = -6$

v. $P'(100.5) = 3$

vi. $P'(100.5) = -1.5$

b. [2 points] Which one of the following expressions is equal to the amount (in kg) by which Elphaba's collection of acorns increases over the course of the 50th km of her travels.

i. $P(50)$

ii. $P'(49)$

iii. $\displaystyle\int_{49}^{50} P(t)\,dt$

iv. $\displaystyle\int_{49}^{50} P'(x)\,dx$

c. [2 points] Which one of the following expressions is equal to the mass (in kg) of acorns that Elphaba collected during the 4th day of her travels?

i. $P'(4)$

ii. $P\left(\displaystyle\int_0^4 Q(t)\,dt\right) - P\left(\displaystyle\int_0^3 Q(t)\,dt\right)$

iii. $P(4) - P(3)$

iv. $P\left(\displaystyle\int_3^4 Q(t)\,dt\right)$

d. [2 points] Let m be a positive constant and let $R(t)$ be the antiderivative of $Q(t)$ such that $R(0) = 0$. Assuming that both $P(t)$ and $R(t)$ are invertible, which one of the following expressions is equal to the time (in days) it takes Elphaba to collect m kg of acorns?

i. $R(P(m))$

ii. $R^{-1}(P^{-1}(m))$

iii. $R(P^{-1}(m))$

iv. $P(R(m))$

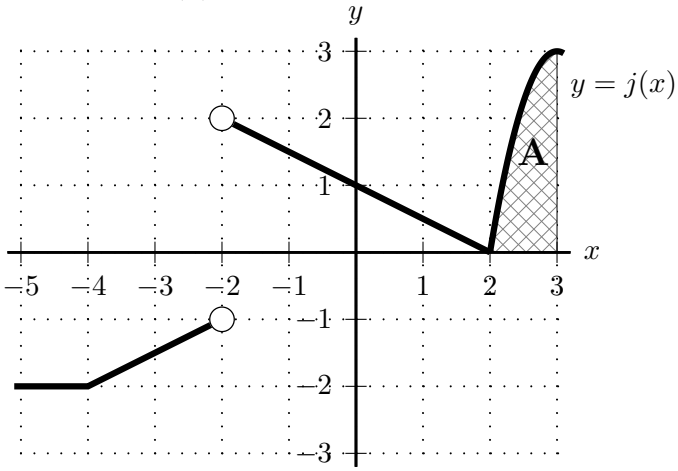
v. $P^{-1}(R^{-1}(m))$

vi. $P(R^{-1}(m))$

e. [3 points] Write an equation that expresses the following statement:
After Elphaba has been travelling for a total of 5 days, she has collected a total of 200 kg of acorns.

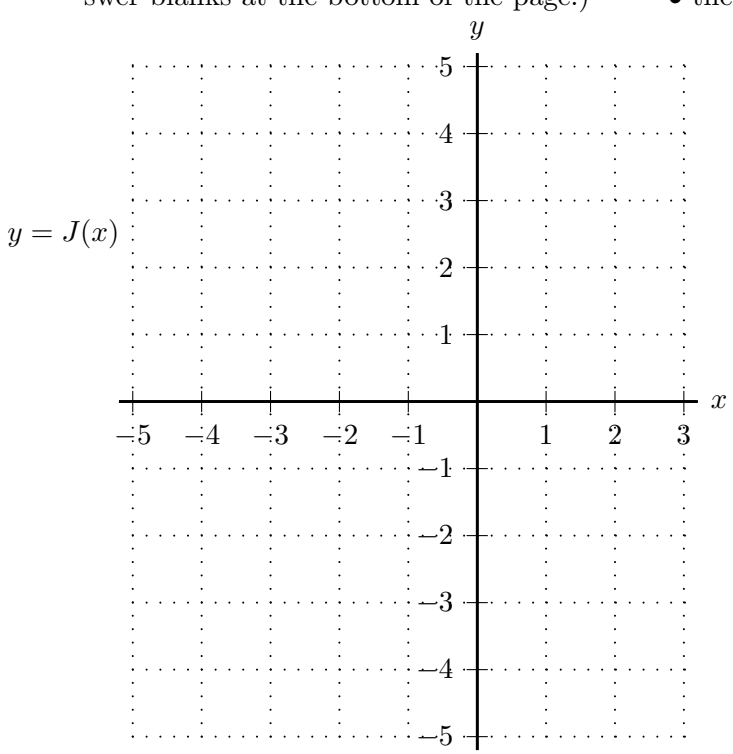
Answer: _____

7. [10 points] The graph of a function $j(x)$ is shown below. The shaded region A has area 2.



On the axes provided below, sketch a well-labeled graph of an antiderivative of $J(x)$ of $j(x)$ that is defined and continuous on the interval $-5 \le x \le 3$ and that satisfies $J(0) = 1$.
Be sure that you pay close attention to each of the following:

- the value of $J(x)$ at each of its critical points and inflection points
  (Be sure to also write this data in the answer blanks at the bottom of the page.)
- where J is/is not differentiable
- where J is increasing/decreasing/constant
- the concavity of the graph of $y = J(x)$

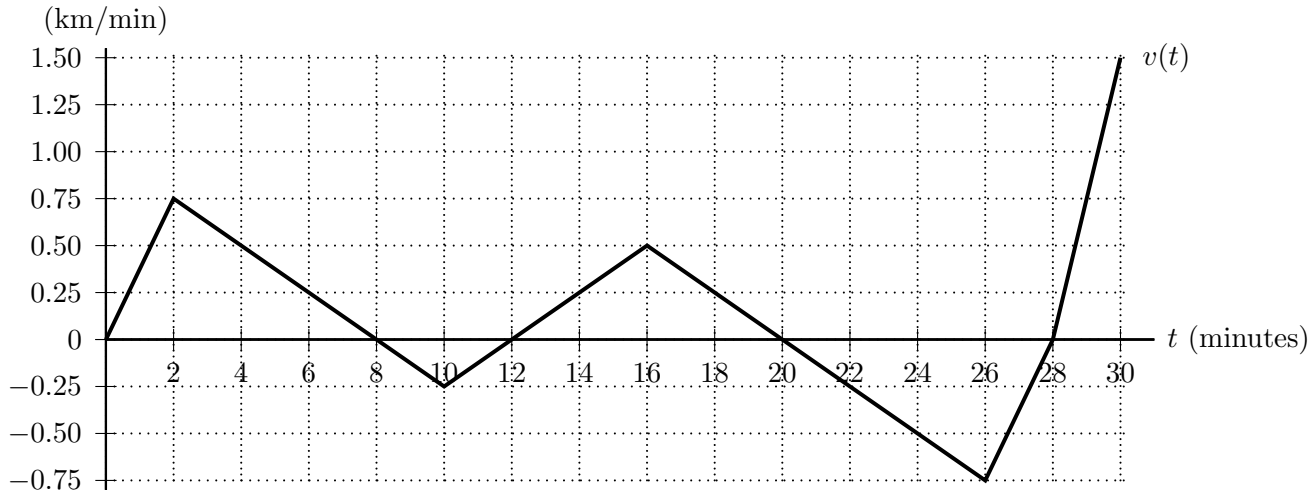


On the answer blanks below, write both the x– **and** y–coordinates of all critical points and all inflection points of $J(x)$. Write NONE if $J(x)$ has no such points.

Both coordinates of all critical points: _____

Both coordinates of all inflection points: _____

1. [10 points] Unfortunately, Sebastian left the King's castle but never made it to Adam's manor because the brakes on his car were sabotaged. Sebastian was driving on a straight road between the King's castle and Adam's manor when he found himself unable to brake and racing down a hill. Let $v(t)$ be Sebastian's velocity (in kilometers per minute) t minutes after he left the King's castle. Note that $v(t)$ is positive when Sebastian is traveling towards Adam's manor. Sebastian suspected he was being followed so he occasionally backtracked.
Sebastian crashed 30 minutes into his journey. A graph of $v(t)$ is given below.



a. [3 points] How far from the King's castle was Sebastian 12 minutes into his journey? *Include units.*

Answer: _____

b. [2 points] What was Sebastian's average velocity during the first 12 minutes of his journey?

Answer: _____

c. [2 points] Of the four times below, circle the one at which Sebastian's acceleration was the greatest (i.e. most positive).

$t = 6$ $t = 13$ $t = 20$ $t = 27$

d. [3 points] In the interval $0 \leq t \leq 30$ when was Sebastian the closest to the King's castle? When was he the furthest from the King's castle?

Answer: Sebastian was the closest to the King's castle at $t =$ _____.

Sebastian was the furthest from the King's castle at $t =$ _____.