Math 115 — Practice for Exam 3

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Name: SOLUTIONS

Instructor: ______________________ Section Number: __________

1. This exam has 4 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2009	3	6	traffic	12	
Fall 2007	3	2		7	
Winter 2007	3	5		16	
Fall 2004	3	5	eggnog	13	
Total				48	

Recommended time (based on points): 58 minutes

6. [12 points] The rate $q(t)$ at which cars passed through the intersection of Main Street and Huron after a football game is presented in the table below.

t (in minutes after the game ended)	0	20	40	60	80	100	120
$q(t)$ (in cars per minute)	10	15	19	21	20	17	13

a. [4 points] What is the meaning of $\int_0^{120} q(t)\,dt$? Using a left Riemann sum and $n = 6$, estimate $\int_0^{120} q(t)\,dt$. (Write out the terms of your sum.)

Solution: The expression $\int_0^{120} q(t)\,dt$ gives the total number of cars that passed through the intersection in the first two hours after the game. A left-hand approximation with 6 subdivisions is given by

$$(20)(10 + 15 + 19 + 21 + 20 + 17) = 2040 \text{ cars} .$$

b. [2 points] Write an expression for the average rate at which cars passed through the intersection for the first two hours after the game ended.

Solution: The average rate at which cars passed through the intersection during this time period is given by $\frac{1}{120}\int_0^{120} q(t)\,dt$.

c. [3 points] Estimate $q'(30)$.

Solution: The best estimate we can get from the table is

$$q'(30) \approx \frac{19 - 15}{40 - 20} = 0.2 \text{ cars per minute per minute.}$$

d. [3 points] If $Q(t)$ denotes the total number of cars that have passed through the intersection t minutes after the game ended, find and interpret $Q'(60)$.

Solution: We can read $Q'(60)$ from the table. We have $Q'(60) = 21$ and indicates that one hour after the game, approximately 21 additional cars would pass through the intersection in the next minute.

2. (7 points) Use a Riemann Sum with 4 equal subdivisions to find a ***lower*** estimate for

$$\int_0^2 e^x + 1\,dx.$$

Clearly indicate whether you are using a left-hand sum or a right-hand sum, and show all intermediate calculations. Show your answer to three decimal places (or in exact form).

The function is increasing, therefore the Left Sum is the lower sum.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$e^x + 1$	2	$e^{1/2} + 1$	$e + 1$	$e^{3/2} + 1$	$e^2 + 1$
$e^x + 1$	2	2.6487	3.7183	5.4817	8.3891

Left Sum

$$LHS_{(4)} = (0.5)(2) + (0.5)(e^{1/2} + 1) + (0.5)(e + 1) + (0.5)(e^{3/2} + 1) = 6.9243$$

3. (7 points) Let $f(x) = \cos(x) + bx$ and $g(x) = x^2 - x$. Find the value of b such that $f(x) > g(x)$ on $[0, 1]$ and the area between the curves from $x = 0$ to $x = 1$ is equal to 1.

$$1 = \int_0^1 \cos x + bx - (x^2 - x)dx$$

$$1 = \sin(1) - \frac{1}{3} + \frac{b+1}{2}$$

$$b = \frac{5}{3} - 2\sin(1) = -0.0162753$$

[Note that with this value of b, $f(x) > g(x)$ on $[0, 1]$–but, with the set-up of the problem as indicated above, we are assuming that $f(x) > g(x)$ on the interval.]

5. (16 points) Use the information given in the table below to calculate the indicated values. If a value cannot be determined, state explicitly what is missing. Assume that f and f' are continuous, and that the table is reflective of the behavior of f.

x	0	3	6	9	12
$f(x)$	30	20	13	8	5
$f'(x)$	-4	-3	-2	-1.5	-.5

Determine the following and show your work (3 points each):

(a) an approximate value for $f(3.1)$ using a local linearization

$f(3.1) \approx f(3) + f'(3)(3.1-3) \approx 20 + (-3)(0.1) = 19.7$

(b) a left-hand sum with 4 subdivisions to approximate $\int_0^{12} f(x)dx$

$\text{LHS}_{(4)} = (f(0) + f(3) + f(6) + f(9))(3) = 213$

(c) the least number of subdivisions necessary to assure that the left- and right-hand approximations of $\int_0^{12} f(x)dx$ agree to within 1 unit

If $|RHS - LHS| \leq 1$, then $|f(12) - f(0)|\Delta x = 25\Delta x \leq 1$. Thus, $25\left(\frac{12-0}{n}\right) \leq 1 \Rightarrow$ $(25)(12) \leq n$. This implies we need at least 300 subdivisions.

(d) $\int_3^{12} f'(x)dx$

From the FTofC, we know $\int_3^{12} f'(x)dx = f(12) - f(3) = 5 - 20 = -15$

Explain your answers to the following (2 points each):

(e) Do you expect your approximation for $f(3.1)$ from part (a) to be an overestimate or an underestimate?

If the table is representative of the behavior of the function f, then $f''(3) > 0$ which imples that f is concave up at 3. Thus we expect the approximation to be an underestimate.

(f) Do you expect your left-hand approximation from part (b) to be an overestimate or an underestimate?

If the table is representative of the behavior of f, then f is decreasing, thus the left-hand sum is an overestimate.

5. (4+6+3 points) Your uncle Harry absolutely LOVES eggnog around the holidays. The rate at which he drinks it at your family holiday party is given by the function $r(t)$ where t is measured in hours and $r(t)$ is in liters/hour. Suppose $t = 0$ corresponds to 6 pm when the party begins.

(a) Write a definite integral that represents the total amount of eggnog uncle Harry consumes between 8 pm and 2 am the next morning.

$$\int_2^8 r(t)\,dt$$

(b) If Uncle Harry's rate of eggnog drinking is given by $r(t) = e^{-t} + 1$, use a left hand sum with three (3) subdivisions to estimate the amount of nog Harry drinks in the first four hours of the party. Show all of your work.

$$r(0)\frac{4}{3} + r\left(\frac{4}{3}\right)\frac{4}{3} + r\left(\frac{8}{3}\right)\frac{4}{3} = 2 \cdot \frac{4}{3} + (e^{-\frac{4}{3}} + 1)\frac{4}{3} + (e^{-\frac{8}{3}} + 1)\frac{4}{3}.$$

(c) Should your estimate in part (b) be an underestimate or an overestimate? Explain.

It is an overestimate because the function $r(t)$ is a decreasing function.