Math 115 — Practice for Exam 3

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Name: ____

Instructor: ____________________ Section Number: __________

1. This exam has 5 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2003	2	4	bell curve	12	
Winter 2001	2	9		10	
Fall 2001	3	7		6	
Winter 2005	2	5		14	
Fall 2008	2	2	peak oil	12	
Total				54	

Recommended time (based on points): 50 minutes

(4.) (12 points) Consider the function:

$$f(x) = e^{\frac{-(ax)^2}{2}}, \qquad \text{for } a \text{ a positive constant.}$$

The graph of $y = f(x)$ is the (in)famous "bell curve," which occurs frequently in statistics, and occasionally in heated political debates as well.

(a) Compute $f''(x)$. Show your work.

(b) For which value of a does the function f have an inflection point at $x = 3$?

(9.) (10 pts) Determine a, b, and c so that the graph of the function $f(x) = x^3 + ax^2 + bx + c$ has
a local maximum at $x = -2$,
a local minimum at $x = 1$,
and passes through the point (0,2). [Show your work.]

7. (6 pts) In this problem we will investigate the family of functions

$$f(x) = a\ln(x) - bx.$$

Calculate values of a and b which cause such a function to have a critical point at $(2, 1)$.

5. (14 points) A family of functions is given by $r(x) = \frac{a}{x}e^{bx}$ for a, b, and $x > 0$.

(a) For what values of a and b does the graph of r have a local minimum at the point $(4, 5)$? Show your work and **all supporting evidence** that your function satisfies the given properties.

(b) Write an explicit formula for $r(x)$. Circle your answer.

(c) Is the graph of r concave up or down for $x > 0$? Explain using arguments based on calculus–not only from a graph.

2. In 1956, Marion Hubbert began a series of papers predicting that the United States' oil production would peak and then decline. Although he was criticized at the time, Hubbert's prediction was remarkably accurate. He modeled the annual oil production $P(t)$, in billions of barrels of oil, over time t, in years, as the *derivative* of the *logistic function* $Q(t)$ given below—*i.e.*, $Q'(t) = P(t)$. The function P is measured in years since the middle of 1910.

   The function $Q(t)$ is given by

$$Q(t) = \frac{Q_0}{1 + ae^{-bt}}\text{ , where } a, b, Q_0 > 0. \tag{1}$$

   For your convenience, the first and second derivatives of $Q(t)$ are given as well:

$$Q'(t) = -\frac{Q_0}{(1 + ae^{-bt})^2}\left(-abe^{-bt}\right) = \frac{abQ_0e^{-bt}}{(1 + ae^{-bt})^2},$$

   and

$$Q''(t) = \frac{ab^2Q_0e^{-bt}}{(1 + ae^{-bt})^3}\left[ae^{-bt} - 1\right].$$

   (a) (2 points) Interpret, in the context of this problem, $P'(56)$.

   (b) (6 points) Determine the year of maximum annual production t_{max}. Your answer may involve all or some of the constants a, b, Q_0.

   (c) (2 points) Find the maximum annual production $P(t_{max})$. Again, your answer may involve all or some of the constants a, b, Q_0.

   (d) (2 points) In his 1962 paper, Hubbert studied the available data on oil production to date and concluded that $a = 46.8$, $b = 0.0687$, and $Q_0 = 170$ Bb (billion barrels). Using your results from part (b), when would Hubbert's curve predict the peak in US oil production? (The actual peak occurred in 1964.)