The Structure of Calculus: a Perspective from U(M)

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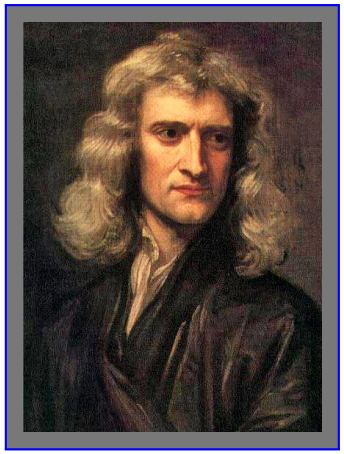
Talk Outline

- Context
  - What is Calculus?
  - Why?
- Implications: Education Research and Equity
- Calculus at Michigan
  - History
  - Current structure
  - Where we're going
- Sustaining Change and Program Assessment
- Conclusions

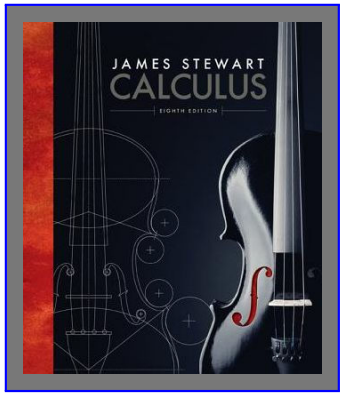
Leibnitz (portrait by Francke)



Newton (port. Kneller)

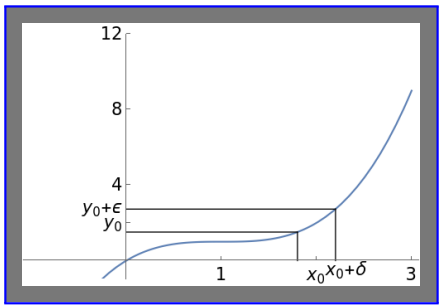
What is Calculus?

- In general
  - Calculus I = chapters 1–6
  - Calculus II = chapters 7–11
- And
  - *For every* $\epsilon > 0$, *find a* δ *for which. . .*
  - *Find* $\lim_{h \to 0} \dfrac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h}$.
  - *Evaluate* $\frac{d}{dx} \sin(e^{x^2-2} + x)$.
- In chalk, for an audience.



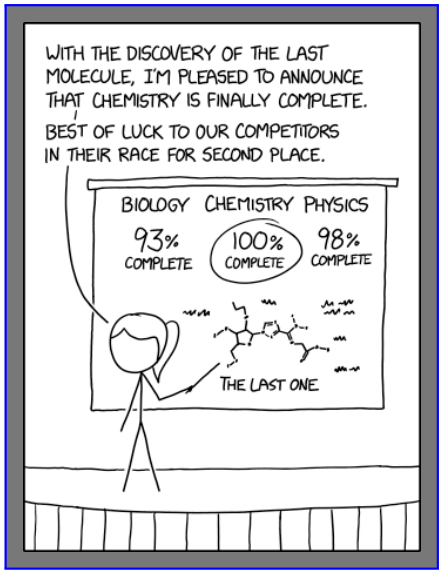
Stewart, Calculus


from YouTube, `9Jx6SntghhE`



And Why?

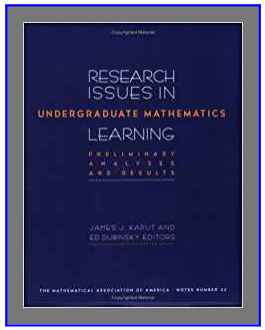
- Calculus is an Established Subject:
  - Newton and Leibnitz started this some 300 years ago.
  - Client disciplines have a well-defined set of needs.
- Claim:
  These allow us to focus on course pedagogy, structure, and emphasis in a way that is unique in the sciences.



xkcd.com/2552

. . . that is, we don't need to discuss content, sort of.

Education Research: Pedagogy



MAA Notes 33, 1991

- We understand learning better (than ever?) now:
  - Active learning improves student understanding and disposition, *[Laursen et al., 2014]*
  - Increases student performance (0.47σ, more on concept inventories) and decreases DWF rates 45%, *[Freeman, et al., 2014]*
  - Especially for underrepresented groups. *[Laursen, et al., 2011]*
- Our community is clear on this: "*A wealth of research has provided clear evidence that active learning results in better student performance and retention. . . we call on [faculty and policy makers]. . . to ensure that effective active learning is incorporated into post-secondary mathematics classrooms.*" *[CBMS Statement, 2016]*

Education and Equity

- Active Learning can address retention and inclusion issues:
  - Mindset and math disposition, stereotype threat, and inclusion of underrepresented groups. *[Dweck, 2007; Steele, 2010; Seymour & Hewitt, 1997]*

> *Central to equity-focused teaching is the cultivation of a learning environment in which all students have equal access to learning and feel valued and supported in their learning. —UM CRLT*

- Equity-focused teaching is intentional, and systemic: it is a guiding intent, not one pedagogy or curriculum.
- What we know matters:
  - Academic Belonging: *Feelings of belonging correlate strongly with learning.*
  - Transparency: *Clear expectations improve students' learning and persistence.*
  - Structured Interactions: *Promote a sense of acceptance in the community.*

The Origin of Michigan Calculus

- Local and national challenges with calculus: failure to generate proficiency and promote retention in STEM fields, especially Math, and college-level mathematics serving as a bottleneck not a pump. *[Steen, 1987]*
- 1990: Dept chair called by dean and regent
  - *Graduating seniors "universally" named calculus as their worst experience at UM.*
    –Don Lewis

Don Lewis, 1926–2015

- Calculus Reform: NSF funded over 350 projects between 1987 and 1995, incl. at Michigan.
  - Learning context: *"This is a time for establishing, no matter how tentatively, [frameworks]. . . that can help. . . guide further work."* *[Kaput & Dubinsky, 1994]*

New Wave Calculus, 1990s

- Michigan's Calculus Reform:
  - Reduced class sizes from 35–45 to 24.
  - Adopted a reformed textbook.
  - Instituted use of graphing calculators.
  - Implemented cooperative learning both in and out of the classroom.
  - Extensively revised and extended the new instructor training program.
- By 2000, program continued (but class sizes were up to 32).

calc class, 90s

Michigan Calculus, 2010s

- Michigan Calculus is
  - Active learning in classes of 32 (until 2015) or 18 (since).
  - Conceptual focus and assessment.
    - And Mastery/Gateway assessment.
  - Structural Support
    - Of Instructors:
      training, lesson plans, online homework, exams, grading, course scale, instructor support.
    - Of Students:
      Placement, and mid-course drop-back course (math 115 $\rightarrow$ 110).
      Math Learning Center *(Math Lab)*.

math 115 class, 10s

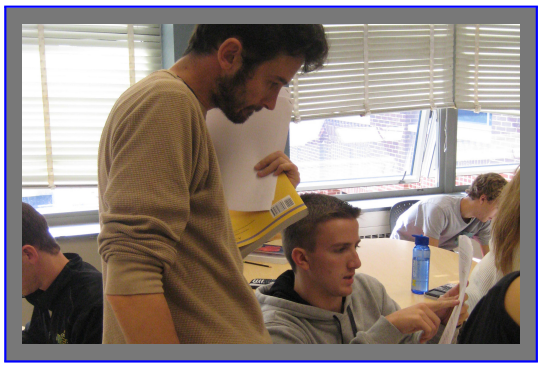
Active Learning

Sample class:

10:10–10:15am	Group work on introductory problem
10:15–10:20am	Announcements
10:20–10:30am	Summary of group work solutions
10:30–10:40am	Mini-lecture on new material
10:40–11:10am	Group work on new material
11:10–11:20am	Discussion of solution that group wrote on board
11:20–11:25am	Group discussion
11:25–11:30am	Summary of remaining group work

Total: group work: $\sim$40 min; lecture: $\sim$30 min

Mason Hall 2449



Paul Kessenich, math 115(?)

Conceptual Focus

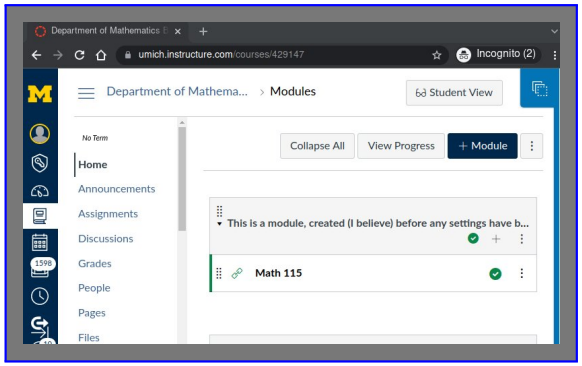
- Text (Hughes-Hallett)
- Uniform Exams (2–3)
  - highly conceptual,
  - allow calculators and notecards.
- Team Homework ($\sim$5)
  - teams of four students,
  - with designated roles: manager, reporter, editor, clarifier;
  - require solutions written in full sentences;
  - (also) highly conceptual.
- Mastery Assessments...

Exam: A wind turbine, spinning counterclockwise at a constant rate... At exactly 1pm, a blade is pointing straight toward the ground. Find a formula for the height of the blade above the ground.

Team HW: Coulomb's law describes electrical force, $F(d)$ between two electrically charged objects. What is a reasonable domain for $F(d)$? Why should it be invertible? Fit a formula to data.

Structure

- We run at a large scale: fall 2021
  students: 550 (pre-calc), 1900 (calc 1), 670 (calc 2)
  sections: 28 (pre-calc), 96 (calc 1), 38 (calc 2)
  instructors: 24 (pre-calc), 74 (calc 1), 30 (calc 2)
- All courses have Coordinators (fall: 1–2 faculty, 1 grad stu; winter 1 faculty, 0–1 grad stu), who set:
  - Daily schedules, Syllabus
  - Assessment (exams, team hw, web homework, masteries) and grading (exams, team hw)
  - (Partial) Lesson plans, instructors guide
  - Active learning expectation
  - Canvas Blueprint
- Instructors manage
  - Class (group work, etc.)
  - Quizzes
  - Grading of quizzes and team homework



Canvas Blueprint

Structural Support of Instruction

- Course structures
  - Class schedules
  - Suggested problems, lesson plans
  - Grading rubrics and meetings
- Instructor support structures
  - Training (one week, end of summer)
  - Course meetings (up to weekly)
  - Class visits (all new instructors), Midterm evaluations
  - Open/informal Mentoring

Training session, pre-pandemic

Structural Support of Students

- Placement
  - Multi-factor assessment: test, SAT/ACT, HS GPA
  - Advising in summer registration
  - Focused: we seek only to place into calculus or our course before
- Drop-back course
  - After first calculus midterm
  - Drop into a self-paced course before calculus
  - Replaces calculus on transcript
- Math Learning Center (Math Lab)
  - Walk-in tutoring
  - Open 40 hours a week
  - Staffed by undergraduate tutors and instructors

U(M) Math Learning Center

Calculus Reform, v.2.0b

- Concerns: differential success rates, equity of high-stakes exams.
  - Grade penalty relative to expectation (Grade $-$ GPAO):
    - Black women: -1; Black men: -0.8; White women: -0.75; White men: -0.6; Asian men: -0.45
  - Lower grade penalties correlate with higher privilege
- Historical assessment model:
  - 95%: three exams
  - 5%: web homework
  - Gateway: letter grade drop if not passed
- New assessment model:
  - 50%: two exams (no comprehensive final)
  - 38%: four mastery assessments (including derivative gateway)
  - 6%: web homework (including prepwork)
  - 6%: team homework and quizzes

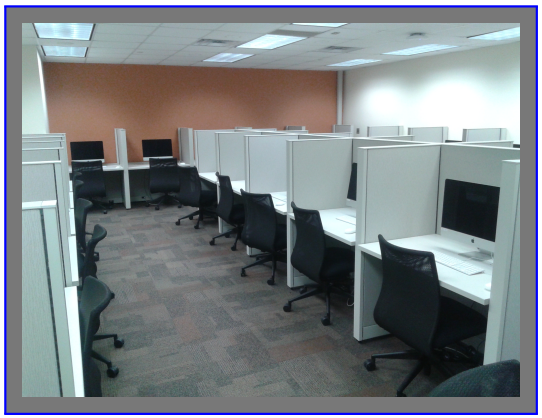
Uniform exam

. . . with significant support from CRLT's FCI program and the College.

Mastery Assessment

- Function Concepts Mastery (8%)
  - Linear functions, Exponential functions, Solving equations with exponentials, Inverse functions and compositions, Sinusoidal functions, Polynomial and rational functions, Interpretation and applications
- Derivative Procedures Mastery (5%)
- Integral Concepts Mastery (10%)
  - Finding distance from velocity, Approximating definite integrals, Integrals graphically, FTC: translating rates to change, FTC: applications, FTC: conceptual understanding, Properties of integrals
- Final Mastery (15%)
  - 10 problems
- Plus: Guaranteed course scale

original mastery lab



new mastery lab

Equity in Instruction

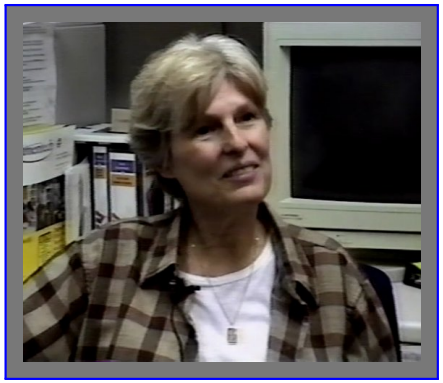
Key Challenge: equity-focused instruction in a program with 130 instructors.

- Training
  - CRLT workshop on equity-focused instruction for our new graduate students and post-docs
  - Updated training week, focus on equity
- Course Structure
  - Lesson plans, course design to promote principles of equity
  - Pedagogical structure (small class sizes, active learning) promote community and inclusion in the classroom
  - Central structure facilitates transparency in expectations and information transfer to students
  - Team homework seeks to promote structured interactions between students

CRLT: GSI support

Sustaining Change over 30 years

- Building Internal Bridges *Wayne Roberts, site visit*
  - *"... the level of internal skepticism and outright opposition that I expected to find, while present, is much less than I expected."*
  - *"It is perhaps a mistake to mention [the] supportive climate [in the department] as the last item... [identifying] the reasons [for the program's success]... This is the school that has drawn national attention for what they have been able to do* ***as a department****."*
- Administrative Backing and Champions
  - *"After a few complaints... any unsympathetic dept chair or dean might have quickly squelched the new program without a fair trial. Fortunately, we had full support from both."* –Mort Brown
  - Department Admin Support: Don Lewis, Al Taylor, Mel Hochster, Tony Bloch
  - Internal Champions: Pat Shure, Mort Brown, Karen Rhea, Stephen DeBacker



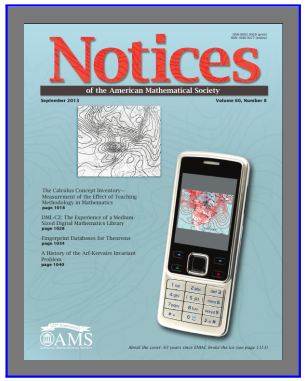
Pat Shure

Mort Brown

Program Assessment and Evidence of Success

> *"Never mistake activity for achievement."* –John Wooden

- We are moderately assessed
  - 1990s: Site visits
    - 1995: Wayne Roberts, Sharon Ross, Jeff Eiseman
      "The positive things I had heard are in fact true. . . "
  - 2000s: Calculus Concept Inventory
    - 2008: Pre-/Post- test of calculus concepts, used at many institutions
      Average normed gain over all sections was as good as the best seen before. And two standard deviations above the existing average.



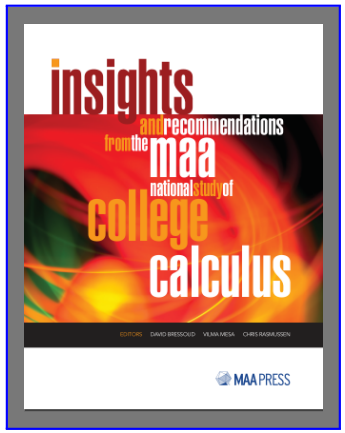
Notices, Aug 2013

  - 2000s: Study of Calculus II
    - Standard Calculus II produced similar outcomes to honors
  - 2010s: The Calculus Study. We are the obvious large midwestern university
  - 2018: DFW Rate
    - In general, under 15% (fall 18: DF $=$ 6%)

Indirect Evaluation

"Our survey revealed that Calculus I, as taught in our colleges and universities, is extremely efficient at lowering student confidence, enjoyment of mathematics, and desire to continue in a field that requires further mathematics. The institutions we selected bucked this trend."

–Bressoud & Rasmussen

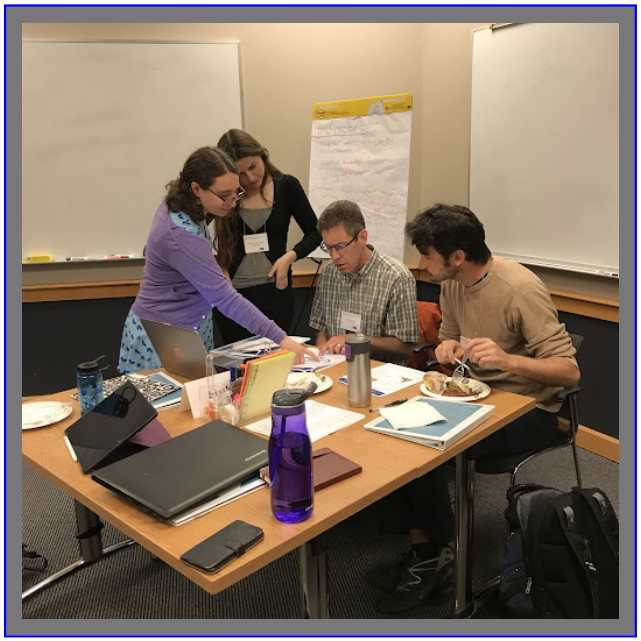


MAA, 2015

- Effective student Placement procedures
- A Coordination System with a uniform textbook and assessments, and regular P2C2 instructor meetings developing de facto communities of practice
- Course Content that challenges and engages students
- Active, Student-Centered Pedagogy
- GTA Preparation and Development
- Student Support Services, including tutoring centers

University Pastoral?

- This is all true.
- But it takes a lot of work.
  - Recognition to: Hanna Bennett, Paul Kessenich, Angela Kubena, Beth Wolf
  - *It is difficult to overstate the amount of work that these efforts can entail.*



Course design workshop, summer 2019

Conclusions

There are many models for successful, effective, and equity-focused calculus instruction.

And we may claim that Michigan's model is one of these.

East Hall

- Research argues for
  - Active learning
  - Systemic work to promote
    - Academic Belonging
    - Transparency
    - Structured Interactions
- Structured Course Models can promote all of these things:
  - Expectation and support of effective pedagogy and active learning
  - Structures and instructional support of equity-focused instruction
- And may support (Formative) Assessment of the program