Michigan Calculus: Structure and Outcomes

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March 06, 2019

Calculus at Michigan: Outline

- Outline
- History
  - Context
- Michigan Calculus
  - Active Learning
  - Conceptual Focus, Assessment
  - Structure
  - Instructor Support
- Observations on Change and Sustaining It
- Assessment and Evidence of Success
  - Observations on Active Learning
- A Small Footnote
- Any Conclusions That Might Be Appropriate

The Origins of Michigan Calculus

- 1990: Dept chair called by dean and regent
  - *Graduating seniors "universally" named calculus as their worst experience at UM.*
    –Don Lewis
- Calculus Reform: NSF funded over 350 projects between 1987 and 1995, including at Michigan.
  - Driving motivation: calculators?, but concepts and active learning soon took over.
  - Pilot 1991–Full Implementation 1994/5

Don Lewis, 1926–2015

New Wave Calculus

- Michigan's Calculus Reform:
  - Reduced class sizes from 35–45 to 24.
  - Adopted a reformed textbook.
  - Instituted use of graphing calculators.
  - Implemented cooperative learning both in and out of the classroom.
  - Extensively revised and extended the new instructor training program.
- By 2000, program continued (but class sizes were up to 32).

calc class, 90s

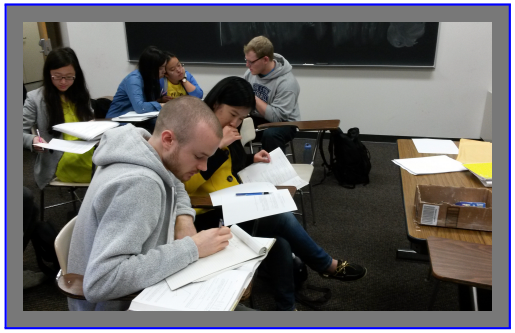
Michigan Calculus 25 Years On

- Michigan Calculus is the Michigan Introductory Program:
  - Math 105, Data, Functions, & Graphs,
  - Math 115 & 116, Calculus I & II.
- Defining characteristics.
  - Active learning in classes of 32 (until 2015) or 18 (since).
  - Conceptual focus and assessment.
    - And Mastery/Gateway assessment.
  - Structure and Instructor support: common lesson plans, online homework, exams, grading, course scale, instructor support.
  - Student Support.
    - Placement, and mid-course drop-back course (math 115 $\rightarrow$ 110).
    - Math Lab.

ma105 class, 10s

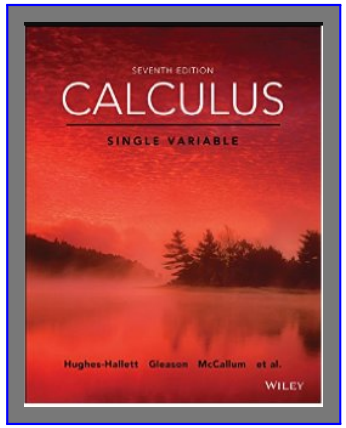
Departmental Context

- The department: ∼63 T/TT faculty, ∼70 post-docs, ∼135 graduate students, over 600 undergrad majors.
  - Math 105 Instructors: mostly grad students.
  - Math 115, 116: grad stu/post-docs; some lecturers, T/TT faculty.
- Multifaceted undergraduate program: five 2-year introductory sequences (std, 4 honors); IBL center; clubs. . .
- Support structures: placement, advising, tutoring, math lab, computer labs and testing, DHSP. . .
- Instructor support: training, lesson plans, class visits. . .



Course Context

- Calculus I (math 115)
  - Hughes Hallett textbook: functions through introduction to antiderivatives.
  - Including: Conceptual limits, quadratic approximation, mean value theorem.
  - Excluding: Algebraic limits, hyperbolic functions, L'Hôpital's rule, parametric equations.



- Calculus II (math 116)
  - Hughes Hallett textbook: definite integrals through sequences, series and differential equations.
  - Including: Partial fractions, L'Hôpital's rule, polar coordinates and parametric equations.
  - Excluding: Tables of integrals, Euler's method.
- Calculus III (math 215)...

is rather different

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ma115 class, 10s

Active Learning

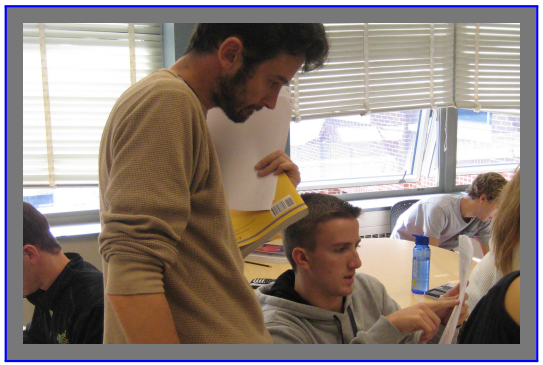
Sample class:

10:10–10:15am	Group work on introductory problem
10:15–10:20am	Announcements
10:20–10:30am	Summary of group work solutions
10:30–10:40am	Mini-lecture on new material
10:40–11:10am	Group work on new material
11:10–11:20am	Discussion of solution that group wrote on board
11:20–11:25am	Group discussion
11:25–11:30am	Summary of remaining group work

Total:

group work: $\sim$40 min

lecture: $\sim$40 min



Paul Kessenich, math 115(?)

Lesson Plans

Math 115 - Lesson 3: *Section 1.3 - New Functions From Old*

Notices		**Assignments**	
		READ:	Section 1.4
REMINDERS:	Team HW due date and time (beginning of class)	DO:	WeBWorK 1.3
		DUE:	
ANNOUNCE:	Date for upcoming quiz over course material		

Suggested Lesson Plan: [Time is shown as number of minutes after the hour or 1/2-hour]

[10 - 25] Give a short quiz on the Student Guide and/or the reading for today's class, if you have indicated you will do so. This need not be long or difficult, just enough to determine if they have actually read the guide and are doing the section reading before class. Announce the date and sections to be covered for an upcoming in-class quiz over the course material (an actual math quiz!).

[25 - 35] Discuss the quiz immediately after it is collected. Make certain that students understand the course grading policy (and that YOU do, too–ask if you are unsure), the fact that this course will require a minimum of 8 hours of outside of class work, etc.
Take a couple of minutes to make sure students are dong the web homework and meeting with their teams.
Clear up any lingering "course administrivia" questions.

Note: In this lesson, there are several good opportunities for students to work together at the blackboards. And colored chalk is extremely useful for this section.

[35 - 55] It's very important for students to recognize basic "manipulations" (transformations) of the functions in their library. They should be comfortable with the following facts and know how to use them:

- $f(x) + k$ and $f(x + k)$ represent vertical and horizontal shifts of $f(x)$, respectively.
  (They should know the direction and magnitude of the shifts as well.)

Lesson Plans and Book Problems

[35-55] It's very important for students to recognize basic "manipulations" (transformations) of the functions in their library. They should be comfortable with the following facts and know how to use them:

- $f(x) + k$ and $f(x + k)$ represent vertical and horizontal shifts of $f(x)$, respectively. (They should know the direction and magnitude of the shifts as well.)
- $-f(x)$ and $f(-x)$ represent vertical and horizontal "flips."

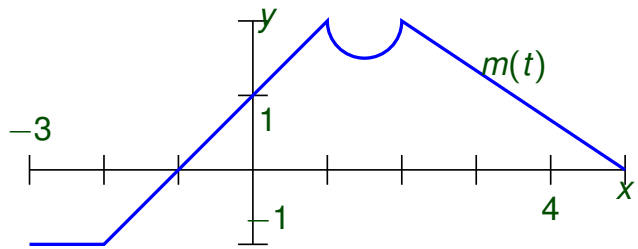
⋮

You can use the Rule of Four to demonstrate these.

- Starting with a table of values for some function $f(x)$, have students make new tables of $f(x + 1)$, $f(x) + 1$, $-f(x)$, $f(-x)$, $2f(x)$, etc.

⋮

A function like $f(x) = x^2 - 4x + 7$ can be a good example to use. . . Alternatively, **§1.3 #4–7 (page 26)** are good problems.



Use the figure to graph the functions

4. $n(t) = m(t) + 2$; **5**. $p(t) = m(t - 1)$;

6. $k(t) = m(t + 1.5)$;

7. $w(t) = m(t - 0.5) - 2.5$

More Comments About the Book

- Students are expected to read the book.
  - Lesson Plans reinforce this.
  - And Reading Homework is available in calculus I and II.

Reading the textbook

In this course, it is absolutely essential that you read the textbook sections as assigned, before they are covered in class. Your experience with previous math courses may make this seem unlikely; many students are used to being able to do each homework problem by scanning for the appropriately similar example in the textbook (or their notes) to use as a template. This is a reasonable way to learn to do a very specific sort of problem, but does not give you the conceptual understanding necessary to solve similar problems with even small changes, which is what we ultimately want you to be able to do.

Your instructor will be counting on you to read the text, since they may not be lecturing very much, and will be assuming you have spent some time thinking about the material before you work with it in class. Further, not every small point on which you may be tested will be covered by in-class examples.

Intro Program Student Guide

For which of the following choices of $f(x)$ and $g(x)$ is it true that $f(g(x)) = \dfrac{1}{(x-3)^2}$?

Check all that are true.

- ☐ **A.** $f(x) = \frac{1}{x}, g(x) = (x-3)^2$
- ☐ **B.** $f(x) = \frac{1}{x^2}, g(x) = x - 3$
- ☐ **C.** $f(x) = x, g(x) = \frac{1}{(x-3)^2}$
- ☐ **D.** $f(x) = \frac{1}{x-3}, g(x) = x^2$
- ☐ **E.** $f(x) = (x-3)^2, g(x) = \frac{1}{x}$

Sample Reading HW Question, Text §1.3

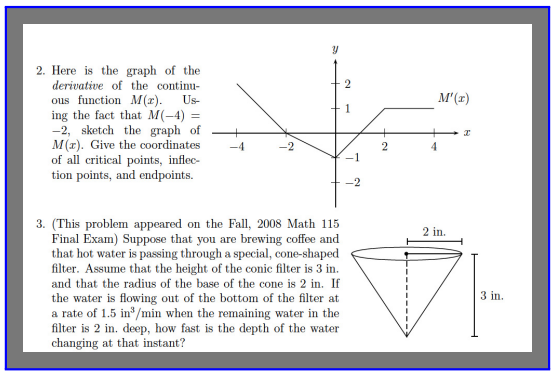
Some Observations

- Class environment and structure matter.
  - Classroom furniture and layout have a dramatic impact on class function.
  - 80 minutes is a good class length.
    - Our classes meet 3$\times$ weekly, 80 min each.
  - Class sizes of 18–24 are possibly optimal (but 32 works too).

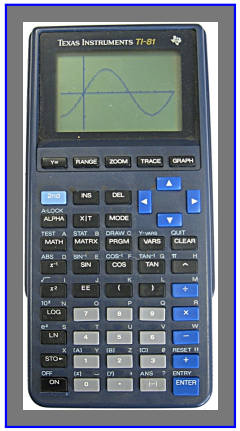
Intro Program classroom

Course Focus

- A Conceptual Focus characterizes the entire course.
  - Throughout, the Rule of Four is emphasized. *. . . see the Student Guide*
- Exams and most homework are highly conceptual
- Calculators and notecards are allowed (in general).
- Skills (e.g., derivatives, integrals) are assessed with gateway tests.


2. Here is the graph of the *derivative* of the continuous function $M(x)$. Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$. Give the coordinates of all critical points, inflection points, and endpoints.

3. (This problem appeared on the Fall, 2008 Math 115 Final Exam) Suppose that you are brewing coffee and that hot water is passing through a special, cone-shaped filter. Assume that the height of the conic filter is 3 in. and that the radius of the base of the cone is 2 in. If the water is flowing out of the bottom of the filter at a rate of 1.5 in^3/min when the remaining water in the filter is 2 in. deep, how fast is the depth of the water changing at that instant?



The Student Guide

Course structure

You should expect this course to be different from other math courses you have taken. Many math courses are taught in a very formulaic way: students are given an example of a particular kind of problem and shown a step-by-step approach to solving such a problem, spend time practicing these steps on twenty or so nearly identical problems, and then repeat with the next skill. Our courses do not work that way, which can be disconcerting to students who arrive expecting more of the same. Here are some of the key features emphasized in our courses.

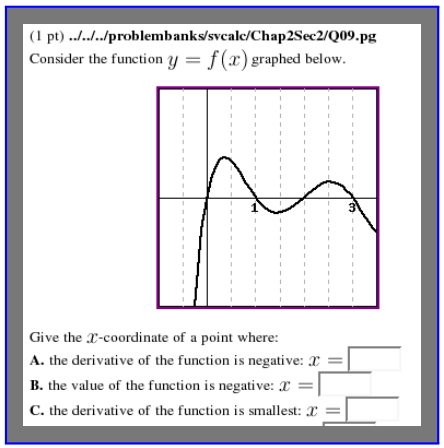
Conceptual understanding. We want students to learn not only how to "get the right answer", but also how to choose an appropriate approach, why the approach works, how to decide whether your answer is correct or even reasonable, and what your answer means.	**Collaboration.** You'll be expected to work with classmates during class and on homework. Working in groups gives you the opportunity to share ideas and see different approaches. Explaining mathematics to your classmates will also improve your own understanding of the material.
Interpretations. You'll be asked to think about mathematical topics in a variety of real-world contexts, to explain what the math means in that context, and to translate between mathematical expressions and English sentences.	**Multiple representations.** When you hear the word "function", you likely think of a symbolic representation, like $f(x) = 5x-2$. In these courses, we'll also be representing functions graphically, numerically (with tables of data), and

Assessment

- Uniform Exams (3)
  - highly conceptual,
  - allow calculators and notecards.
- Team Homework (7–10)
  - teams of four students,
  - with designated roles: manager, reporter, scribe, clarifier;
  - require solutions written in full sentences;
  - (also) highly conceptual.
- Web Homework ($\approx$ daily)
  - largely drawn from the textbook
- Gateway testing (1-2/course)
  - Test basic or essential skills,
  - Allow unlimited practice,
  - Taken for credit in a proctored lab.

Exam: A wind turbine, spinning counterclockwise at a constant rate… At exactly 1pm, a blade is pointing straight toward the ground. Find a formula for the height of the blade above the ground.

Team HW: Coulomb's law describes electrical force, $F(d)$ between two electrically charged objects. What is a reasonable domain for $F(d)$? Why should it be invertible? Fit a formula to data.

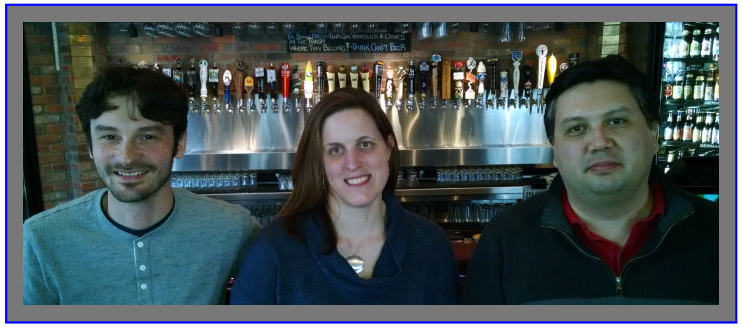


web homework

Gateway: Find the derivative of $S(r) = r\cos(r^2 + 1) - \pi$.

Structure and Coordination

- Courses are very Uniform
  - Syllabus
  - Daily Schedule
    - 4–6 weekly class schedules.
  - Assessment
    - all but quizzes, which are by section.
    - And exam grading: no instructor grades their section, teams grade each problem.
    - Final grading scale: is set by the coordinator, with some leeway for instructors to change grades.
  - Pedagogy
    - strong emphasis on active, group learning.
- All of these are managed by a Course Coordinator for each course.
  - One course teaching release.

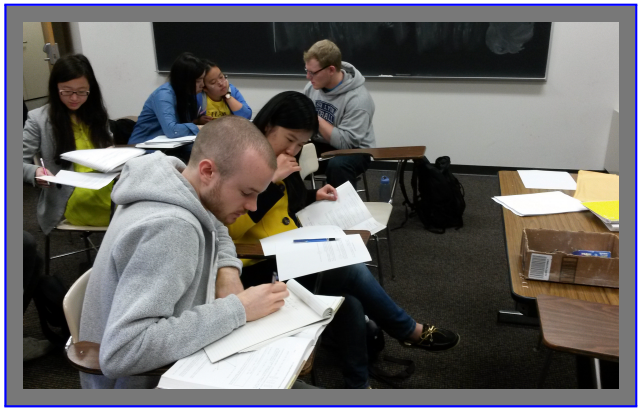


Paul Kessenich, Angela Kubena, Fernando Carreon

. . . or two, or three

Other Coordination Models

- Math 105, 115, and 116 have this coordination model (our course before calculus, calculus I, and calculus II).
- Math 215, 216 are coordinated but less uniform (calculus III, differential equations).
  - Coordinator (0.5 course release)
  - Lecture/lab format (100–120/25–30 students).
  - Uniform syllabus, exams, labs, homework assignments.
  - Grading may be more or less uniform.
- Math 217 has had several coordination models (linear algebra with proof)
  - Syllabus, exams, homework, worksheets determined by instructors collaboratively.
  - Or, by a coordinator, with input from instructors.



Math 217, c. fall 2015

Support

- Course Coordinators, Introductory Program Directors, Department Administration *(and me)*
- Training Week
  - All new graduate students and post-docs.
  - Week-long training program.
- Lesson Plans
  - For all of math 105, first third of math 115.
- Weekly Course Meetings
- Class visits
  - For all new instructors, At least once in first semester.
- Midterm Evaluations

Scissors congruence training session

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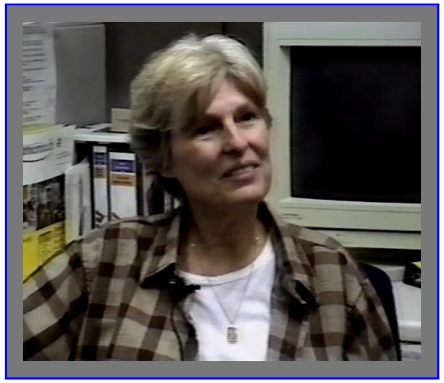
Looking Ahead: Sustaining Change

- Building Internal Bridges
  - *"We got everything in place so that it was a done deal."* –Pat Shure
  - *"... the level of internal skepticism and outright opposition that I expected to find, while present, is much less than I expected."* –Wayne Roberts
- Administrative Backing and Champions
  - *"After a few complaints... any unsympathetic department chair or dean might have quickly squelched the new program without a fair trial. Fortunately, we had full support from both."* –Mort Brown
  - Department Admin Support: Don Lewis, Al Taylor, Mel Hochster
  - Internal Champions: Pat Shure, Mort Brown, Karen Rhea, Stephen DeBacker



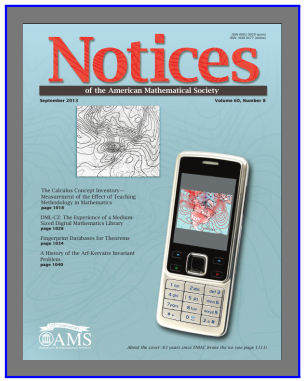
Pat Shure

Mort Brown

Assessment and Evidence of Success

"Never mistake activity for achievement." –John Wooden

- Assessment may be Formative
  - Characteristic of successful programs: Regular collection and use of local data to improve.
    - Formal and Informal feedback mechanisms: program meetings, training planning and assessment, Whitaker grant, placement analysis
- and Summative
  - 1990s: Site visits.
    - 1995: Wayne Roberts, Sharon Ross, Jeff Eiseman
      "The positive things I had heard are in fact true; indeed, the depth of activity. . . go[es] well beyond what I knew about it."
  - 2000s: Calculus Concept Inventory.
    - 2008: Pre-/Post- test of calculus concepts, used at many institutions
      Average normed gain over all sections was as good as the best seen before. And two standard deviations above the existing average.
  - 2010s: The Calculus Study. We are the obvious large midwestern university.

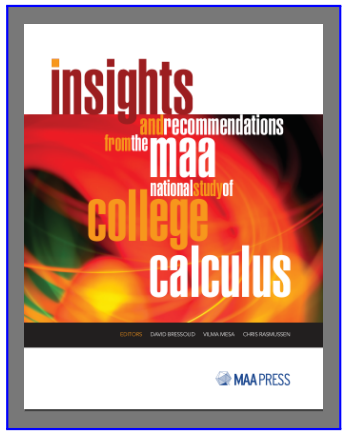


Notices, 2013-08

Characteristics of Successful Programs in Calculus

"Our survey revealed that Calculus I, as taught in our colleges and universities, is extremely efficient at lowering student confidence, enjoyment of mathematics, and desire to continue in a field that requires further mathematics. The institutions we selected bucked this trend."

–Bressoud & Rasmussen



[Bressoud, Mesa, Rasmussen; 2015]

- Local Data. Regular collection and use of local data to guide program modifications as part of continual improvement efforts.
- Placement. Effective procedures for placing students appropriately into their first Precalculus to Calculus II (P2C2) course (both initial placement and re-placing students after the term begins).
- Coordination System. A coordination system for instruction that (i) makes use of a uniform textbook and assessments and (ii) goes beyond uniform curricular elements to include regular P2C2 instructor meetings in development of de facto communities of practice.

Characteristics, cont.

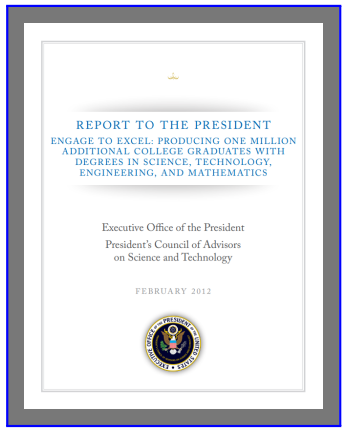
- **Course Content**. Course content that challenges and engages students with mathematics.
- **Active Pedagogy**. The use and support of student-centered pedagogies, including active learning strategies.
- **GTA Preparation and Development**. Robust teaching development programs for teaching assistants.
- **Student Support Service**. Proactive student support services (e.g., tutoring centers, services for first-generation students) that foster students' academic and social integration.

U(M) Math Learning Center

What Matters

- The Calculus Study suggests that there is no one characterization of successful calculus instruction
- Active Learning seems important, however. *(but what is passive learning?)*
  - It results in Improved student understanding and disposition. *[Laursen et al., 2014]*
  - Increased student performance and decreased DWF rates, especially for underrepresented groups. *[Freeman, et al., 2014]; Laursen, et al., 2011]*
- It may also help address retention and inclusion issues:
  - Mindset and math disposition,
  - Stereotype threat, and
  - Inclusion of underrepresented groups.

[Dweck, 2007; Steele, 2010; Seymour & Hewitt, 1997]



PCAST Report, 2012

A Rose By Any Other Name

- But what is Inquiry Based Learning (IBL), or Active Learning?
  - The *Academy of Inquiry Based Learning* (AIBL) describes it as:
    - *a form of active learning in which students are given a carefully scaffolded sequence of mathematical tasks and are asked to solve and make sense of them, working individually or in groups.*
    - And it has core principles:
      - deep engagement in rich mathematical activities, and
      - opportunities to collaborate with peers.

[inquirybasedlearning.org]

Karen Smith, U(M) Math Club

A Name By Any Other Rose

- Other IBL in Michigan Math
  - Math 217: Linear Algebra
    (for majors; also our intro to proof).
    - Lecture/group work
  - Math 217: Linear Algebra
    (for majors; also our intro to proof).
    - Worksheets/"real" IBL.
  - Math 285: Honors Calculus III
    - Lecture/IBL work one day a week.
    - Similar: Math 215, 216 (Calc III, Differential Equations)

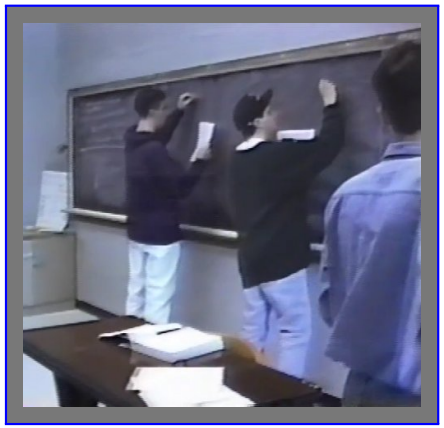
Math 215 Lab, c.2014

Footnote: What About Calculus III

- Math 215, Calculus III
  - Is one of two courses in the Department taught in a large lecture (100–120), with an associated lab (25–30).
  - Is taught from Stewart, covering vectors through the divergence theorem.
  - Has uniform schedule, syllabus, assessment, grading(?).
  - . . . and "IBL" in the labs.
    - With some evidence of success.
    - And some continued evolution.

Conclusions, Such As They Are

- Michigan Calculus has a somewhat remarkable 25 year record of:
  - Teaching calculus effectively,
  - With a really huge number of transient, inexperienced, instructors with a very broad range of backgrounds.



calculus class, 1990s

- Some of its character, including structure and training and instructor support are driven by its instructional team.
  - Though this doesn't necessarily invalidate their significance.
- Many of its defining characteristics, including conceptual focus, high expectations, and active learning,
  - Are consistent with our understanding of characteristics of good teaching

    [Chickering & Gamson, 1991]
  - And with extensive research on effective (active) mathematics pedagogy.

    [Laursen, et al., 2014; Freeman, et al., 2014]