Good luck!

**Problem 1** Find an equation for the line through the point (2, 1), which is perpendicular to the line $y = 5x - 3$.

Since the new line is perpendicular, we know $m = -1/5$. So the equation for the line should be

$$y = b - \frac{1}{5}x,$$

and we need to find $b$. Since the point (2, 1) is on the graph, we know

$$1 = b - \frac{1}{5}(2)$$

So $b = 7/5$, and so

$$y = \frac{7}{5} - \frac{1}{5}x$$

**Problem 2** Find $x$ when $6 \cdot 7^x = 4 \cdot 2^x$. Please give the exact answer, and also express it in decimal form with four significant digits.

$$6 \cdot 7^x = 4 \cdot 2^x$$

implies

$$\left(\frac{7}{2}\right)^x = \frac{4}{6} = \frac{2}{3}$$

Taking the natural logarithm on both sides we obtain:

$$\ln \left(\frac{2}{3}\right) = \ln \left(\left(\frac{7}{2}\right)^x\right) = x \ln \left(\frac{7}{2}\right),$$

which gives

$$x = \frac{\ln \left(\frac{2}{3}\right)}{\ln \left(\frac{7}{2}\right)} \approx -0.3237$$
Problem 3 (5 Points)
Find a formula the exponential function that passes through the points (1, 6) and (2, 18).

\[ y = y_0 a^x, \]

where \( y_0 \) and \( a \) are constants. They need to satisfy

\[
\begin{align*}
  y_0 a^1 &= 6 \\
  y_0 a^2 &= 18.
\end{align*}
\]

Diving the two equations gives:

\[
\frac{y_0 a^2}{y_0 a} = \frac{18}{6} = 3
\]

So \( a = 3 \). Also,

\[ 6 = y_0 a^1 = y_0 \cdot 3. \]

Then \( y_0 = 2 \).

Therefore

\[ y = 2 \cdot 3^x \]
**Problem 4**

A spherical balloon is growing with radius \( r = 3t + 1 \), in centimeters, for time \( t \) is seconds. Find a formula for the volume of the balloon as a function of time \( t \). Find the volume of the balloon at 3 seconds.

The volume of a balloon of radius \( r \) is given by

\[
V = \frac{4}{3} r^3.
\]

The radius as a function of time is \( r = 3t + 1 \). Composing the two functions, it gives

\[
V = \frac{4}{3} \pi (3t + 1)^3.
\]

Evaluating the function at \( t = 3 \) seconds, we obtain

\[
V(3) = \frac{4}{3} \pi 10^3 \approx 4188.79 \text{cm}^3
\]

**Problem 5** A photocopy machine can reduce copies to 80% of their original size. By copying an already reduced copy, further reductions can be made. Estimate the number of times in succession that a page must be copied to make the final copy less than 15% of the size of the original.

The size of the copy after \( t \) reductions is given by

\[
C(t) = C_0 (0.8)^t,
\]

where \( C_0 \) is the initial size. We want to find an integer \( t \) such that

\[
(0.80)^t \geq 0.15.
\]

Applying the natural logarithm to both sides it gives

\[
ln \left( (0.8)^t \right) = t \ln(0.8) \geq ln(0.5)
\]

Since \( ln(0.8) \) is negative, it gives

\[
t \geq \frac{ln(0.15)}{ln(0.8)} \approx 8.50
\]

So after \( t = 9 \) photocopies, the size of the final copy will be less than 15%. 