

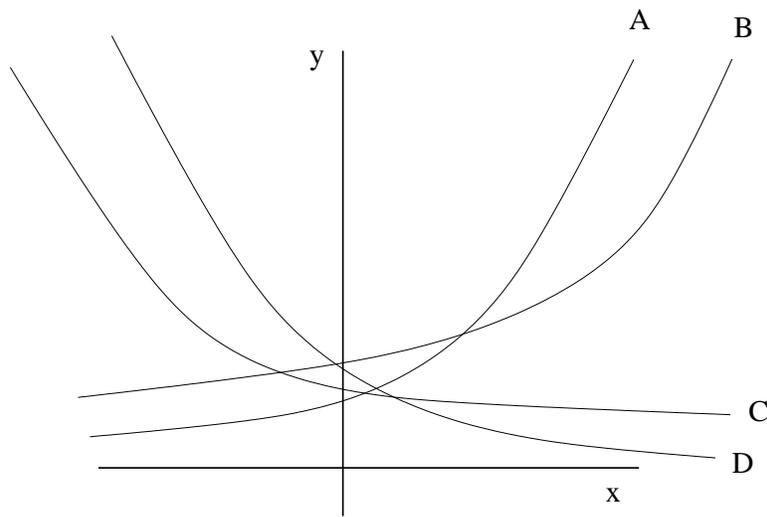
Name:

MATH 105 - SEC 001, FALL 2010. QUIZ 4
TIME LIMIT: 20 MINUTES

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Good luck!

Problem 1



Assuming that the equations for A , B , C and D above can be written in the form $y = a(b)^t$, use the graphs to answer the following questions.

(1) Which curve has the equation with the SMALLEST value of a ?

A

(2) Which curve has the equation with the SMALLEST value of b ?

D

Problem 2 During the summer, the undergraduate population of Ann Arbor drops dramatically and stays low until the start of August, when some students start moving in. The following table gives some values for $P(d)$, the undergraduate population of Ann Arbor on the d^{th} day of August.

d	5	14	23	31
$P(d)$	12,166	17,317	24,647	33,731

- (1) Could $P(d)$ be modeled by a linear function? To receive credit on this question, you must provide correct, specific reasons for your answer.

The average rate of change in each interval $[5, 14]$, $[14, 23]$, $[23, 31]$ is given by
 $\frac{17317 - 12166}{9} = 572.3$, $\frac{24647 - 17317}{9} = 814.454$, $\frac{33731 - 24647}{8} = 1135.5$,
respectively. Since the average rate of change is not constant, $P(d)$ cannot be modeled by a linear function.

- (2) Does the table indicate that $P(d)$ is concave up, concave down, or neither? To receive credit on this question, you must provide correct, specific reasons for your answer.

We can see that the rate of change are increasing, so the table indicates that $P(d)$ is concave up.

Problem 3

The number of asthma sufferers in the world was about 84 millions in 1990 and 130 millions in 2001. Let N represent the number of asthma sufferers (in millions) worldwide t years after 1990.

- (1) Write N as a linear function of t . What is the slope? What does it tell you about asthma sufferers?

$N(t)$ is the number of asthma sufferers at year t . Assuming the function is linear, we have $N(t)$ is of the form

$$N(t) = b + m t, \text{ and need to find } b \text{ and } m$$

The slope is computed as

$$m = \frac{130\text{million} - 84\text{million}}{11\text{years}} = 4.18\text{millions/year}.$$

and the y-intercept is N at $t = 0$ (in 1990), so $b = 84\text{million}$. Finally

$$N(t) = 84\text{millions} + (4.18\text{million/year})t$$

The slope $m = 4.18\text{million/year}$ tells me that the number of asthma sufferers is increasing by 4.18 millions every year.

- (2) Write N as an exponential function of t . What is the growth factor? What does it tell you about asthma sufferers?

Assuming the function $N(t)$ is exponential, then $N(t)$ is of the form $N(t) = a(b)^t$. The initial value a is clearly $a = 84$ millions. To compute b , we notice that 2001 is 11 years after 1990, and so

$$84\text{million } b^{11} = 130\text{million}, \text{ and so}$$

$$b = \left(\frac{130}{84}\right)^{\frac{1}{11}} \approx 1.0405, \text{ which gives}$$

$$N(t) = 84\text{million } (1.0405)^t. \text{ DON'T FORGET UNITS}$$

The growth factor $b = 1.0405$ tells us the number of asthma sufferers increases by 4.05 % every year, according to the exponential model.

Problem 4

The population of a colony of rabbits grows exponentially. The colony begins with 10 rabbits; five years later there are 340 rabbits.

- (1) Give a formula for the population of the colony of rabbits as a function of time.

Since the population of rabbits grows exponentially, the population of rabbits, $P(t)$, at time t , has the form

$$P(t) = a(b)^t$$

The initial value is, according to the problem, $a = 10$ rabbits (**DON'T FORGET UNITS**). To find b , notice that for $t = 5$ years,

$$340 \text{ rabbits} = 10 \text{ rabbits} b^5, \text{ and so}$$

$$b = \left(\frac{340 \text{ rabbits}}{10 \text{ rabbits}} \right)^{\frac{1}{5}} = (34)^{1/5} \approx 2.024$$

Therefore

$$P(t) = 10 \text{ rabbits} (2.024)^t$$

- (2) Use a graph to estimate how long it takes for the population of the colony to reach 1000 rabbits.

Since **WE CANNOT USE LOGS IN MIDTERM 1**, we plot two curves, the curve $P(t) = 10 (2.024)^t$ and the constant function $Q = 1000$ in the graphing calculator. We find the intersection of the two curves, which gives

$$t = 6.53 \text{ years}$$