Problem 1 (5 points)

Thomas Gross is a researcher in the Department of Cellular, Molecular and Developmental Biology here at Michigan; you may soon also know him as the guy playing the harmonica and washboard outside the UGLi (the Undergraduate Library).

A few years back, the Michigan Daily did some investigative reporting and discovered the following facts: The amount of time $G(d)$, in minutes, that Mr. Gross plays is a linear function of $d$ (here $d$ refers to Fahrenheit degrees). Reporters for the daily observed that Mr. Gross played for two hours and 15 minutes when the average daily temperature was $92^\circ F$ and that he played for one hour when the average daily temperature was $32^\circ F$.

(1) Find a formula for $G(d)$ as a function of $d$ when $t \geq 0$.

The slope is given by $m = \frac{(135-60)\text{min}}{92-32} = 1.25 \text{ min/}^\circ F$. The y-intercept can be found by evaluating the linear function at $d = 32^\circ F$.

$60\text{min} = b + (1.25 \text{ min/}^\circ F) \cdot 32^\circ F = b + 40\text{min} \rightarrow b = 20\text{min}$

Therefore

$G(d) = 20\text{min} + (1.25 \text{ min/}^\circ F) \cdot d$

(2) Calculate and interpret the slope of the graph of $G(d)$. Include units.

$m = 1.25 \text{ min/}^\circ F$ means that Mr. Gross plays 1.25 extra minutes every time the daily average temperature increases by $1^\circ F$.

(3) Calculate and interpret $G(0)$. Include units.

$G(0) = 20 \text{ min}$ means that Mr. Gross plays 20 minutes when the average temperature is $0^\circ F$. 

Date: September 15, 2010.
(4) What is the average daily temperature on a day when Mr. Gross plays for 2 hours? Include units.

We need to solve the equation

\[ 120 \text{ min} = 20 \text{ min} + (1.25 \text{ min/}°\text{F})d, \]

which solution is

\[ d = \frac{120 \text{ min} - 20 \text{ min}}{1.25 \text{ min/}°\text{F}} = 80°\text{F} \]

Problem 2 (4 Points)

The figure below shows the graph of the function \( g(x) \).

(1) Estimate \( \frac{g(6) - g(4)}{6 - 4} \)
$\frac{g(6) - g(4)}{6 - 4} = \frac{1 - 2}{6 - 4} = -\frac{1}{2}$

(2) The ratio in part (1) is the slope of a line segment joining two points in the graph. Sketch this line segment on the graph.

See graph

(3) Estimate the rate of change for this function over the interval $[-4, 4]$ ($a = -4$ and $b = 4$).

The rate of change is given by

$$\frac{g(4) - g(-4)}{4 - (-4)} = \frac{2 - (-2)}{4 - (-4)} = \frac{4}{8} = \frac{1}{2}$$

(4) On the graph, sketch the line segment whose slope is given by the ratio in part (c).

See the graph

**Problem 3 (5 Points)**

For the following statements, decide whether they are true or false. If the statement is true, give a reason why. If it is false, provide an example where it is not true.

(1) A function must be defined by a formula.

False. Rule of four

(2) If $f$ is a decreasing function, then the average rate of change of $f$ on any interval is negative.

True. We can see it clearly in the figure below (Figure 2).

(3) The average rate of change of $f(x) = 10 - x^2$ between $x = 1$ and $x = 2$ is the ratio $\frac{10 - 2^2 - 10 - 1^2}{2 - 1}$.

False. It is

$$\frac{10 - 2^2 - 10 + 1^2}{2 - 1}$$
(4) The following table demonstrates the relationship between two quantities $P$ and $Q$

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>5</td>
<td>12</td>
<td>0</td>
<td>12</td>
<td>1</td>
</tr>
</tbody>
</table>

This table shows that $P$ is a function of $Q$ and that $Q$ is a function of $P$.

False. The vertical line test says $Q$ is a function of $P$, but $P$ is not a function of $Q$.

**Problem 4 (2 Points)**

You are looking at the graph of $y$, a function of $x$.

(1) What is the maximum number of times that the graph can intersect the $y$-axis? Explain.

One, because of the vertical line test

(2) Can the graph intersect the $x$-axis an infinite number of times? Explain

Yes. The constant function $f(x) = 0$ is an example