1. Consider the matrix \( A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \). Apply the power method and the inverse power method to find the largest and smallest eigenvalues of \( A \), respectively, starting from the initial guess \( v^{(0)} = \frac{1}{\sqrt{3}} (1,1,1)^T \) in both cases. You may use MATLAB, but it is not necessary to turn in the code. For each method take six steps, \( k = 0 : 5 \), and present the results in a table with the following format.

<table>
<thead>
<tr>
<th>column 1: ( k ) - step index</th>
</tr>
</thead>
<tbody>
<tr>
<td>column 2: ( \lambda^{(k)} ) - approximate eigenvalue at step ( k )</td>
</tr>
<tr>
<td>column 3: (</td>
</tr>
<tr>
<td>column 4: ( \frac{</td>
</tr>
</tbody>
</table>

In columns 3 and 4, use \( \lambda = \lambda_1 \) for the power method and \( \lambda = \lambda_3 \) for the inverse power method, where the exact eigenvalues of \( A \) are \( \lambda_1, \lambda_2, \lambda_3 \) and they are ordered so that \( |\lambda_1| > |\lambda_2| > |\lambda_3| \). You may compute the exact eigenvalues using the characteristic polynomial or the `eig` command in MATLAB. In the table present at least eight significant digits (as in the example in class). Discuss the trends in each column. Do the results agree with the convergence theory presented in class? If not, explain.

2. Find the Taylor series for \( f(x) = \sin x \) about \( x = 0 \) up to the \( x^7 \)-term. Using MATLAB, plot the Taylor polynomials \( p_n(x) \) of degree \( n = 1, 3, 5, 7 \) (use `subplot` and put each case in a different subplot). Use the command `axis([-4*pi 4*pi -2 2])` to set the limits on the axes. In each case also plot the original function \( f(x) = \sin x \). Label each curve. We view the Taylor polynomial \( p_n(x) \) as an approximation to the original function \( f(x) \). For a given value of \( n \), is the approximation valid for all values of \( x \)? Does the approximation improve as the degree of the polynomial \( n \) increases?

**announcements**

1. The online teaching evaluations are available from Fri., Dec. 3. Please complete the evaluations - they provide valuable feedback from students to instructors.

2. The final exam is on Wed., Dec. 15, 1:30p-3:30p, in the usual classroom. The exam will cover the entire course. A review sheet with sample problems will be distributed soon. You may use a non-programmable calculator to do arithmetic, but to receive full credit you must show all intermediate steps. You may use two sheets of notes (e.g. two sides of one page, i.e. \( 2 \times 8.5\text{ in} \times 11\text{ in} = 187\text{ in}^2 \)). I will supply the exam booklets.