Price and Time to Sale Dynamics in the Housing Market: the Role of Incomplete Information[1]

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Abstract

A model of the house-selling process in which sellers possess incomplete information regarding the state of the housing market generates the observed negative correlation between prices and time on market. This result persists even if sellers employ real estate agents with complete information, provided agents' and sellers' incentives are not perfectly aligned. Comparing self-reported homeowner perceptions of house prices to market prices suggests that homeowners' perceptions lag actual price movements. High perceptions relative to actual prices are associated with lower sales volumes. Price misperceptions can explain over one-fifth of the within-state variation in sales volumes from 2000 to 2010.

1 Introduction

A stylized fact of the market for existing homes holds that there is a positive correlation between sales prices and sales volumes and a negative correlation between sales prices and time on market, or the average time houses take to sell. Figure 1 illustrates this pattern using monthly data for the United States from January 2000 to December 2010.[1] The top panel shows the log of the CoreLogic House Price Index for single family detached homes, the middle panel illustrates log single family home sales, and the bottom panel displays months' supply of single family homes for sale (months' supply), a proxy for time on market.[2] The correlation between prices and sales volumes in this period was 0.80 while the correlation between prices and months' supply was -0.19; both correlations are statistically significant at the 5-percent level.

Although directly observed time on market data are not available at a national level, studies on the city level find a similar pattern when considering time on market directly. For instance, Genesove and Mayer (2001) document that in the Boston condominium market, fewer than 30 percent of listed units sold within 180 days during 1992, the trough of a housing bust. In 1997, after the market had recovered, more than 60 percent of new listings sold within 180 days. Miller and Sklarz (1986) document similar trends in Hawaii and Salt Lake City. Stein (1995) shows that a 10 percent drop in house prices was associated with approximately 1.6 million fewer sales nationally in the years 1986 to 1992, when annual U.S.

[1]Sales volume and months' supply data are from the National Association of Realtors and price data is from CoreLogic.

[2]Months' supply is the ratio of the number of homes listed as being for sale at the end of the month divided by the number of sales that month. It is used as a proxy for the average time on market because nationally representative data for time on market is unavailable. All series have had a linear time trend removed, and prices were deflated using the CPI.

home sales ranged between three and four million units. Ortalo-Magne and Rady (1998) show the same qualitative pattern holds in the United Kingdom. Genesove and Mayer (1997) describe this pattern as "one of the most distinctive and puzzling macro features of the market for existing homes."

This paper offers a stylized model to explain that pattern. The model has three key features. First, sellers have incomplete information regarding conditions in the housing market. Second, sellers face idiosyncratic variation in the offers they receive. Third, misalignment between sellers' and real estate agents' (hereafter simply called agents) incentives prevents agents from completely conveying their knowledge of market conditions to sellers.

The first assumption stems naturally from most households' infrequent participation in the housing market. As Case and Shiller (2003) note, "Buyers and sellers in the housing market are overwhelmingly amateurs, who have little experience with trading. High transactions costs, moral hazard problems, and government subsidization of owner-occupied homes have kept professional speculators out of the market." Furthermore, the idiosyncratic nature of housing units causes difficulty in inferring the market value of an individual house from recent sales prices of other units. Anenberg (2013) estimates that California home sellers in the period 2007 to 2009 had substantial uncertainty about their home's value, with a standard deviation of beliefs prior to beginning the sales process equal to 7 percent of the average sales price.

Idiosyncratic variation in the offers a seller receives may arise from differences in the match quality between potential buyers and the house, variations in buyers' eagerness to transact quickly, or other factors. Although there is limited data concerning the distributions of offers sellers receive, Merlo and Ortalo-Magne (2004) document variation in offers for a given house both from different prospective buyers and within negotiations between a given buyer and seller pair. The existence of idiosyncratic variation in offers seems well-accepted in the theoretical literature. For instance, Haurin (1988) discusses the optimal decision rule for sellers facing particular offer distributions.

Finally, recent research by Levitt and Syverson (2002, 2008) shows that agents' typical compensation structure leads to misalignment between agents' and sellers' incentives. They document that, empirically, agents seem to encourage clients to sell their homes more quickly than would be optimal for a fully informed seller. The model in this paper demonstrates that sellers' and agents' incentives are misaligned, expected time on market will be negatively correlated with house prices even if sellers employ agents with perfect information regarding the state of the housing market.

To test the theory, I construct a measure of homeowner perceptions of housing market conditions using data from the American Community Survey. I compare that measure to market-based house price indices to create a measure of homeowner misperceptions of market conditions. Homeowner perceptions of price movements track actual market conditions imperfectly. The standard deviation of the baseline misperceptions index, which tracks homeowners' perceived relative to actual price appreciation since 2000, is nearly 7 percent in the period from 2001 to 2010.[3] Homeowner perceptions of price appreciation that are high relative to actual price appreciation are associated with lower sales volumes at both the state and Metropolitan Statistical Area (MSA) levels. A simple fixed effects regression of sales volumes on the misperceptions index explains 22 percent of the time series variation in sales volumes within states and 19 percent of the variation within MSAs. A one percent increase

[3]The index is normalized to have a value of zero in the year 2000.

in homeowner perceptions of prices relative to market prices predicts 1.2 percent lower sales volumes in my baseline specification. Furthermore, variation in the misperceptions index is related to volatility and persistence in prices and a measure of the difficulty of inferring house prices from observable characteristics.

A substantial literature attempts to explain the correlations between prices, sales volumes, and time on market. A number of papers adopt the search and matching framework for the housing market originally developed by Wheaton (1990). Krainer (2001) presents a model in which high service flows from housing imply high prices and a high opportunity cost to failing to transact in the housing market, generating the observed correlation between prices and time on market. Piazzesi and Schneider (2009) document the existence of a cluster of buyers with irrationally optimistic beliefs about future prices, and show that changes in the size of this group can generate comovement in prices and sales volumes. Diaz and Jerez (2013) show that a competitive search equilibrium improves the search and matching framework's ability to match the observed correlations relative to a random matching framework. Albrecht et al. (2007) model a process in which sellers become more anxious to sell the longer the sales process takes, and therefore accept lower offers when a house has sat on the market for a long time. Anenberg (2013) studies seller uncertainty regarding demand conditions, and shows that price depreciation predicts longer time on market in the Los Angeles area during the recent housing crash. However, Magnus (2010) and Ehrlich (2012) show that it can be difficult to match the observed correlations in dynamic stochastic general equilibrium search and matching models with endogenous housing supply.

Other authors have studied the question outside of the search and matching framework. Stein (1995) presents a model in which down payment requirements and credit constraints generate these correlations. Genesove and Mayer (1997) find empirical support for this mechanism. Genesove and Mayer (2001) argue that prospect theory can explain the correlations if sellers use the price they paid for their house as a reference point to evaluate offers. They provide evidence that this reference point influences seller behavior. Lazear (2010) argues that sellers find it optimal to accept a lower probability of sale when demand and prices fall, consistent with the theory of monopoly pricing.

Outside of the housing market, the model in this paper resembles the model of Lucas (1972), in which agents' inability to distinguish between aggregate and idiosyncratic shocks generates a short-run correlation between output and the price level.

The paper proceeds as follows. Section 2 presents a model of the house seller's problem. Two cases are considered, one in which the seller has no assistance in the decision problem, and one in which a real estate agent advises the seller. Section 3 analyzes one of the model's implications empirically. The section measures homeowner misperceptions of price movements in the housing market, and examines the association between those misperceptions and sales volumes. The section also studies the determinants of house price misperceptions. Section 4 concludes.

2 The Model

I consider a simplified model of the home-selling process in which a seller has a single house to sell, and attempts to maximize the sales price. The seller receives one offer per period from an exogenous distribution. The seller cannot negotiate with buyers in the model, so the seller's decision problem is whether to accept or reject the offer received each period.[4] I consider the

[4] An offer cannot be recalled after it is rejected.

situation in which the seller makes decisions without assistance, and the situation in which a real estate agent advises the seller. For clarity, I will use female pronouns to refer to the seller and male pronouns to refer to the agent. The agent in the model has complete information regarding the state of the housing market. However, the agent's incentives are potentially misaligned from the seller's. The body of the paper examines a two-period model, which is extended to multiple periods in the appendix. In both cases, the seller must accept the final period offer if the house has not sold by that time.

2.1 The Offer Distribution

Offers are the sum of an 'aggregate demand' component, z, which is constant across periods, and an idiosyncratic component, x_t, which is identically and independently distributed across periods. For simplicity, both offer components are distributed uniformly: $z \sim U[z_L, z_H]$, and $x_t \sim U[x_L, x_H]$. z and x_t are independently distributed. Denoting the period t offer as ψ_t,

$$\psi_t = z + x_t \tag{1}$$

The distribution analyzed here is chosen primarily for illustration rather than for realism. The essential feature of the distribution is the presence both of aggregate and of idiosyncratic variation in the offers.

2.2 The Seller's Problem with no Agent

The seller is risk-neutral, perfectly patient, and does not bear any cost of leaving her home on the market. Therefore, the seller's goal is simply to obtain the highest possible price.

Accordingly, she will accept the period 1 offer, ψ_1, if and only if it is greater than or equal to the expectation of the period 2 offer, $E[\psi_2]$. The seller cannot observe the state of market demand z or the idiosyncratic component of the offer x_t directly. Instead, she must infer z using Bayes' Theorem:

$$f_Z(z|\Psi = \psi) = \frac{f_{Z,\Psi}(z, \psi)}{f_\Psi(\psi)} = \frac{f_\Psi(\psi|z)f_Z(z)}{f_\Psi(\psi)}$$

Define $\tilde{z}_{L,1} = \max(z_L, \psi_1 - x_H)$ and $\tilde{z}_{H,1} = \min(z_H, \psi_1 - x_L)$. Then the seller's posterior belief about the distribution of z conditional on ψ_1 is $z \sim U[\tilde{z}_{L,1}, \tilde{z}_{H,1}]$.[5] It is straightforward to calculate the seller's expectation of ψ_2 from this belief. The seller's conditional expectation function, $E[\psi_2|\psi_1]$, is plotted in Figure 2. The top panel plots the case in which $x_H - x_L \geq z_H - z_L$, and the second panel plots the case in which $z_H - z_L > x_H - x_L$. Figure 2 shows that the seller's expectation of ψ_2 is a weakly increasing function of ψ_1. This result implies the negative correlation between expected prices and time on market formalized in proposition 1.

Proposition 1: When the seller follows the optimal policy, the expected time on market in the model with no agent is weakly decreasing in z while the expected sales price is strictly increasing in z.

Figure 3 illustrates the expected time on market and expected sales price in the model with no agent for the two cases concerning the relative variances of z and x_t. If the variance of the idiosyncratic component of the offer is greater than the variance of aggregate demand, then the expected time to sale will be strictly decreasing in z. If the variance of aggregate demand is greater, the expected time to sale will be constant for very low and very high

[5] Derivations and proofs of the propositons are located in the appendix.

values of aggregate demand, and falling for intermediate values of z. Therefore, expected time on market will be negatively correlated with expected prices.

This simple model illustrates the essential mechanism by which incomplete information generates a negative correlation between the strength of housing demand and expected time on market. When aggregate demand is higher than expected, an offer with an average idiosyncratic component will appear ‘strong’, so the seller will rationally accept it. Therefore, sellers are likely to sell quickly and at a high price when demand is strong. When aggregate demand is weaker than expected, an offer with an average idiosyncratic component will appear ‘weak’, so the seller will rationally reject it. Therefore, sellers are likely to sell slowly and at a low price when demand is low. In contrast, if sellers could perfectly observe the state of aggregate demand z, they would base their decisions solely on the idiosyncratic component of the offer, x_1. In that case, the model would predict zero correlation between prices and time on market.

2.3 The Seller’s Problem with an Agent

Home sellers commonly employ real estate agents who are better informed than they are about housing market conditions. The interaction between sellers and agents complicates analysis of the seller’s decision problem. However, even if agents possess perfect information regarding market conditions, incentive misalignment between the seller and the agent can prevent the agent from credibly conveying that knowledge to the seller.

This section develops a model in which the seller must employ an agent to sell her house. The model assumes a particular contract between the seller and the agent, in which the agent receives a fixed fraction α of the sales price. If there is no sale in period 1, the agent must pay a flow cost c_a at the beginning of period 2 in order to market the house. Following Levitt and Syverson (2002), the agent communicates with the seller only by recommending whether to accept or reject the offer after it has been received. Both the seller and the agent are assumed to know the full structure of the model.

To fix notation, denote the agent's recommendation about the time t offer as ξ_t, with $\xi_t = 0$ if the agent recommends 'reject' and $\xi_t = 1$ if he recommends 'accept'. Let $\tilde{f}(\psi_2|\psi_1, \xi_1)$ be the seller's posterior belief about the distribution of ψ_2 conditional on the first period offer, ψ_1, and the agent's recommendation, ξ_1. Finally, call the seller's period 1 policy function $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$, with $\gamma_1 = 0$ indicating that the seller rejects the period 1 offer and $\gamma_1 = 1$ indicating that she accepts the period 1 offer. Then the seller's period 1 and period 2 value functions can be written as:

$$
\begin{aligned}
V_{S,1}(\psi_1, \xi_1) &= \overset{\max}{\gamma_1}(1-\alpha)\{\psi_1, E[\psi_2|\tilde{f}(\psi_2|\psi_1, \xi_1)]\} \\
V_{S,2}(\psi_2, \xi_2) &= (1-\alpha)\psi_2
\end{aligned}
$$

The agent's value functions can be written:

$$
\begin{aligned}
V_{A,1}(\psi_1, z) = \overset{\max}{\xi_1} \{&\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, 0)\alpha\psi_1 + (1-\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, 0)))(E[V_{A,2}(\psi_2, z)]), \\
&\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, 1)\alpha\psi_1 + (1-\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, 1)))(E[V_{A,2}(\psi_2, z)])\} \\
V_{A,2}(\psi_2, z) = -c_a &+ \alpha\psi_2
\end{aligned}
$$

Define a Bayesian Nash equilibrium of the game between agents and sellers as a policy function $\xi_1(\psi_1, z)$ for the agent, a policy function $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$ for the seller, and a belief updating strategy $\tilde{f}(\psi_2|\psi_1, \xi_1)$ for the seller such that:

1. $\xi_1(\psi_1, z)$ maximizes the agent's value function for all (ψ_1, z), taking $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$ as given;

2. $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$ maximizes the seller's value function for all ψ_1, ξ_1 taking $\xi_1(\psi_1, z)$ as given; and

3. $\tilde{f}(\psi_2|\psi_1, \xi_1)$ is consistent with $\xi_1(\psi_1, z)$.

The agent's payoff in period 1 can be re-written as $\alpha(z+x_1)$, whereas his expected payoff if there is no sale in period 1 is $\alpha(z+\overline{x}) - c_a$. Therefore, in period 1, the agent would prefer the seller to accept any offer ψ_1 such that $\alpha x_1 \geq \alpha\overline{x} - c_a$, or equivalently, $x_1 \geq \overline{x} - \frac{c_a}{\alpha}$. Denote $\overline{x} - \frac{c_a}{\alpha}$ as $\hat{x}_1$, which represents the agent's cutoff value of x_1. The agent would like the seller to accept all offers with an idiosyncratic component x_1 above $\hat{x}_1$ and reject all others. Thus, $\frac{c_a}{\alpha}$ measures the degree of misalignment between the seller's and the agent's incentives. If $\frac{c_a}{\alpha}$ were zero, the agent's and seller's incentives would be perfectly aligned.

A Bayesian Nash equilibrium of the game between the agent and the seller is for the agent to recommend his preferences truthfully, and for the seller to update her beliefs taking the agent's recommendation as a truthful representation of his preferences.[6] Specifically, the agent recommends 'accept' ($\xi_1 = 1$) for any offer such that $x_1 \geq \hat{x}_1$ and 'reject' ($\xi_1 = 0$) for any offer such that $x_1 < \hat{x}_1$. Define $\tilde{x}_{L,1}$ as x_L if the agent recommends reject and $\hat{x}_1$ if the agent recommends accept, and define $\tilde{x}_{H,1}$ as $\hat{x}_1$ if the agent recommends reject and x_H if the agent recommends accept. Further, let $\tilde{z}_{L,1} = \max(z_L, \psi_1 - \tilde{x}_{H,1})$ and $\tilde{z}_{H,1} = \min(z_H, \psi_1 - \tilde{x}_{L,1})$. Then the seller's posterior belief is that $z \sim U[\tilde{z}_{L,1}, \tilde{z}_{H,1}]$.

Figure 4 displays the seller's expectation of ψ_2 as a function of the first period offer, ψ_1,

[6]This equilibrium is not unique. For instance, it would also be an equilibrium for the agent to choose his recommendation randomly and for the seller to ignore the recommendation in the decision-making process.

and the agent's recommendation, ξ_1. The red lines show the expectation when the agent recommends reject and the green lines show the expectation when the agent recommends accept.[7] As before, the two cases differ by whether z or x has the higher variance conditional on the agent's recommendation. Figure 4 illustrates that the seller's expectation of ψ_2 is always greater than ψ_1 when the agent recommends accept. Therefore, the seller will always reject the first offer when the realtor recommends it. However, for low values of ψ_1, the seller's expectation of ψ_2 is greater than ψ_1 even when the agent recommends accept. In those cases, the seller will reject the first period offer despite the agent's recommendation. Figure 4 illustrates those cases as values of ψ_1 for which the green line is above the dashed blue line.

To verify that reporting his own preference truthfully is a best response for the agent, note that in equilibrium, the agent's recommendation weakly increases the chance that the seller will take the agent's preferred action. For very high and very low values of ψ_1, the realtor's recommendation will not affect the seller's decision, so any policy the agent follows is a best response. However, for medium values of ψ_1, the expected value of ψ_2 when the agent recommends 'accept' is below ψ_1, but the expected value of ψ_2 when the agent recommends 'reject' is above ψ_1. The seller will follow the realtor's recommendation in those cases. Therefore, for this range of offers it must also be a best response for the agent to report his or her own preference truthfully. This result leads to Proposition 2.

Proposition 2: In the equilibrium in which the agent reports his preferences truthfully, the expected time on market is weakly decreasing in the state of aggregate demand, z, while

[7]The dashed portions of the red and green lines show off-equilibrium path recommendations.

the expected sales price is strictly increasing in z.

Figure 5 illustrates the expected time on market and sales price as a function z. The expected sales price is a strictly increasing function of z, but the expected time on market is weakly decreasing with z. When the variance of z is large, there will be a range in which the expected time on market is flat with respect to z.

It is difficult to know how large the incentive misalignment between the seller and agent is likely to be in practice. Levitt and Syverson (2008) present an example in which the seller's agent splits a 6 percent commission evenly with the buyer's agent, before splitting the remaining 3 percent with a broker. In that scenario, the agent's final commission would be just 1.5 percent of the sales price. Levitt and Syverson point out that an agent with an opportunity cost of \$200 per week of marketing a home would then be indifferent between selling a house immediately and waiting one week to sell the house for \$13,333 more.

That example suggests that the incentive misalignment is likely to be quite large, but it may overstate the average extent of misalignment. For instance, Hsieh and Moretti (2003) report that in a survey conducted by the National Association of Realtors, 59 percent of agents reported splitting commissions with their broker, while 32 percent reported keeping the full commission and paying the broker a fixed fee. The latter fee structure would mitigate the incentive misalignment relative to Levitt and Syverson's example. Furthermore, reputational concerns might serve to align agents' and sellers' incentives more closely than the analysis of a one-time transaction would suggest. Nonetheless, the typical contract between sellers and agents appears to leave room for a large misalignment between the parties' incentives.

3 Empirics

The model implies that the correlations between prices, time on market, and sales volumes are driven by sellers' incomplete information regarding the true state of the housing market. To test that prediction, I construct a measure of homeowner perceptions of housing values using data from the American Community Survey. I compare those perceptions to several market indices of house prices to construct measures of homeowner misperceptions of housing values at the state and MSA levels. I then regress sales volumes on the misperceptions index to test whether homeowner misperceptions of the state of the housing market are associated with sales. I conclude by exploring the determinants of homeowner misperceptions about the housing market.

3.1 Constructing the Misperceptions Indices

I begin by constructing time series indices of perceived house values at the state and MSA levels. I use data from the American Community Survey one-percent national samples, in which homeowners report a rich set of physical characteristics of their homes, and also answer the question:

> About how much do you think this house and lot, apartment, or mobile home (and lot, if owned) would sell for if it were for sale? Amount - *Dollars*
>
> $ __________.00

The data is available yearly from 2000 to 2010 at the state level, but MSA-level identifiers are only available for the years 2005 to 2010. At the state level, there are approximately 6.5 million home value observations in the data set, just over 100,000 in year 2000, between

300,000 and 400,000 per year between 2001 and 2004, and about 840,000 per year thereafter. Approximately 680,000 observations per year are located in MSAs from 2005 to 2010.

Within each geographical area i, I run regressions of the form:

$$\ln(Price_{ijt}) = \alpha_i + \beta X_{ijt} + \delta_{it} + \epsilon_{ijt} \tag{2}$$

where $Price_{ijt}$ is the self-reported value of housing unit j in area i and year t, X_{ijt} is a vector of housing characteristics, and δ_{it} is a set of year dummies. The X_{ijt} comprise 9 indicators of building size, 9 indicators for number of rooms, 5 indicators for number of bedrooms, the number of rooms interacted with the number of bedrooms, 2 indicators for lot size, 13 indicators for when the structure was built, 2 indicators for complete plumbing and kitchen facilities, an indicator for commercial use, and an indicator for condominium status. I drop the most and least expensive 5% of observations in each jurisdiction-year to control for outliers.

The estimated coefficients on the year dummies, $\hat{\delta}_{it}$, form the perceived housing value time series indices for each jurisdiction. The dummy for the first year in the data set (2000 in the state-level regressions and 2005 in the MSA-level regressions) is omitted, so the house price perception indices should be interpreted as the percent change in perceived prices from the base year in an area.

I define the house price misperceptions index as the difference between the house price perception index and the market-based house price index, also expressed as a log change from the base year. At the state level, the primary market-based house price index is the Federal Housing Finance Administration's (FHFA) purchase only index. The geographical coverage of the FHFA's purchase only house price index is limited at the MSA level. Therefore, I take the FHFA all-transactions index, which includes data from appraisals used in re-financings, as the primary MSA-level house price index. Figure 6 shows the perceived and market-based price indices for the 50 states, while figure 7 shows those indices for 19 of the 20 cities in the Case-Shiller 20-city house price index.[8] The misperceptions index is the vertical distance between the solid black lines, which represent perceived price changes, and the dashed red lines, which represent actual price changes. A positive value for the misperceptions index implies that homeowners are "too bullish" regarding price changes in an area since the base year, while a negative value implies homeowners are "too bearish".

There is substantial variation in the accuracy of homeowners' perceptions of price movements across areas and time. The standard deviation of the state-level misperceptions index is 6.6 percent. The average value of the misperceptions index within a state ranges from a minimum of -16 percent in Hawaii to 8.9 percent in Michigan. The index reaches its minimum average value across states of -3.2 percent in 2004, and its maximum average value of 4.0 percent in 2010. The time series pattern of the misperceptions index suggests that homeowners underestimated both the house price appreciation in the early part of the 2000s and the declines in the later part of the decade.

As a robustness check, I use several alternative market-based house price indices to construct the misperceptions indices. At the state level, I use the FHFA all-transactions index, the FHFA median price index, and the Zillow Home Value Index (ZHVI). I employ quantile regression to construct the homeowners' perceptions index for comparison with the FHFA median house price index and the ZHVI, which is also a median price index.[9] At the

[8] Washington, D.C. is omitted from figure 6 to facilitate presentation. Charlotte, NC is omitted from figure 7 because a lack of sales volume data excludes it from the analysis.

[9] See http://www.zillow.com/wikipages/What's-the-Zillow-Home-Value-Index/ for a description of the Zillow Home Value Index.

MSA level, I use the FHFA purchase only index, the Case-Shiller 20-city repeat sales index, and the MSA-level ZHVI.

3.2 Misperceptions and Home Sales

Figures 8 and 9 show the misperceptions indices as black solid lines, alongside the log change in sales volumes from the base year in dashed red lines. To test the model's implication that homeowner misperceptions of housing market conditions should be negatively correlated with sales volumes, I regress sales volumes on house price misperceptions at the state and MSA levels. At the state level, the sales volume data is from the National Association of Realtors, while at the MSA level it is from Zillow.com.

The baseline specification at both levels is a fixed effects regression of the form

$$\ln(Sales_{it}) = \alpha + \beta_1 Misperceptions_{it} + \beta_2 Price_{it} + \gamma_i + \delta_t + \epsilon_{it} \tag{3}$$

where $Sales_{it}$ is the number of single family homes sold in area i during year t, $Misperceptions_{it}$ is the value of the misperceptions index, $Price_{it}$ is the value of the relevant house price index (expressed as the log change from the base year), γ_i is a set of location fixed effects, and δ_t is a set of year fixed effects. In alternative specifications, I omit the house price index variable and the year fixed effects. I also estimate the model in first differences as

$$\Delta Sales_{it} = \alpha + \beta_1 \Delta Misperceptions_{it} + \beta_2 \Delta Price_{it} + \gamma_i + \delta_t + \epsilon_{it} \tag{4}$$

The baseline regressions are weighted by the number of housing units in the house price perceptions regressions to account for the varying sizes of different geographic areas.

Table 1 shows fixed effects regressions at the state level over the years 2000 to 2010. Column 1 shows results from a specification without prices or year fixed effects. The coefficient on the misperceptions index is -1.4, implying that a one percent increase in homeowner perceptions of prices relative to actual prices decreases home sales by 1.4 percent. The R^2-within of this regression is 0.22, suggesting that homeowner misperceptions of market conditions alone can account for more than one-fifth of the time series variation in sales volumes within a state. Column 2 includes the house price index in the regression in order to account for other channels by which prices might influence sales volumes, such as loss aversion or equity constraints. The estimated coefficient on the misperceptions index is essentially unchanged, and the coefficient on the market price index is not statistically different than zero. Column 3 shows the results of the baseline specification described in equation (3), with year fixed effects added to the specification in column 2. The estimated coefficient on the misperceptions index is slightly smaller at -1.2, but remains highly statistically significant. Meanwhile, the estimated coefficient on the house price index becomes significantly negative, consistent with theories of loss aversion and downpayment constraints. This specification serves as the baseline for the sensitivity analyses that follow. Columns 4 through 6 display estimates of the three models described above in first difference form, as in equation (4). The estimated coefficients on the misperceptions index are smaller in these specifications, but remain highly statistically significant. In column 6, which corresponds to the baseline specification estimated in first differences, the elasticity of sales volumes to house price misperceptions is estimated to be -0.64.

Table 2 assesses the robustness of the results in table 1. All columns use the same specification as column 3 of table 1, described in equation (3). Those baseline results are duplicated in column 1 of table 2 for ease of comparison. Column 2 shows results using the FHFA's all-transactions house price index, used both to calculate misperceptions and as the house price index in equation (3). Using the all-transactions index does not change the results appreciably. Column 3 uses the FHFA median house price index.[10] Again, the results in column 3 are broadly similar to the results in the baseline specification of column 1. Column 4 presents results using the Zillow Home Value Index as the market price index. A drawback of this specification is that the ZHVI regressions include only 35 states. On the other hand, the ZHVI includes home sales regardless of price, whereas the FHFA indices include only transactions with conforming mortgages, limiting their coverage of high-price sales. The coefficient on the misperceptions index shrinks to -0.95 in coulmn 4 but remains highly statistically significant. Finally, column 5 presents results using the same specification as in column 1, but without weighting the observations by the number of housing units. The estimated coefficient on the misperceptions index is slightly smaller than in the baseline specification but the difference is not statistically significant. Table 2 suggests that the association between home sales and house price misperceptions reported in table 1 are robust to alternative price indices and weights.

Table 3 shows the same regressions as table 1, but at the MSA level rather than at the state level. The regressions cover the period 2005 to 2010 in the 105 MSAs and PMSAs for which the necessary data are available. Column 1 shows estimates of equation (3) with prices and year fixed effects omitted. Column 2 adds house prices, and column 3 adds year fixed effects. The coefficient on the misperceptions index is negative and statistically significant in all three columns. The coefficients on the misperceptions index in columns 1 and 2 are

[10] Because this index is published only for the period 2000q1-2010q2, I take the average of the first and second quarters as the whole year average for 2010.

larger in absolute value than the corresponding coefficient in table 1, but the coefficient in the baseline specification, column 3, is of roughly the same magnitude. The R^2-within of the regression in column 1 is 0.19, again suggesting that approximately one-fifth of the time series variation in sales volumes in a geographical area can be explained by homeowner misperceptions of house prices. Columns 4 through 6 show the model estimated in first differences. The coefficient on the misperceptions index is statistically insignificant in these specifications. This difference seems to stem at least partly from the use of the FHFA all-transactions index, which, as illustrated in appendix figure A.4, shows much smaller price declines over the period 2005-2010 than other indices, such as the ZHVI and the Case-Shiller 20-city index. Unfortunately, both the Case-Shiller 20-city index and the metropolitan level FHFA purchase only index have limited geographic coverage. If the ZHVI is used as the house price index, the coefficients on house price misperceptions in the first difference specifications of the model (not shown) are highly statistically significant, and in columns 5 and 6 are larger in magnitude than the corresponding coefficients in table 1.

Table 4 assesses the robustness of the results in table 3. Again, column 1 reproduces the baseline specification in column 3 of table 3 for comparison. Column 2 presents results from the FHFA's purchase-only index, which is available in a limited set of metro areas.[11] The coefficient on house price misperceptions is larger in magnitude using the purchase only index, although consistent with the limited geographical coverage, the estimated standard error is larger as well. Column 3 shows results using the Case-Shiller 20-city house price index; the coefficient is of roughly the same magnitude as in column 1, but again the standard error is

[11]The FHFA publishes this index at the CBSA level, while the perceptions indices are constructed at the MSA and PMSA level. The MSA-level house price indices are constructed from the FHFA data using population-weighted allocation factors from the Missouri Census Data Center's geocorr engine.

larger. Column 4 shows the results using the ZHVI as the house price index, with the same geographical sample as in column 1. The magnitude of the coefficient on misperceptions is more than twice as large as in column 1. Finally, column 5 presents results from an unweighted regression. The coefficient on the misperceptions index is slightly smaller in magnitude in the unweighted regression. Overall, however, the MSA-level sensitivity analysis suggests that the results of the baseline specification may understate the association of house price misperceptions with sales volumes at the MSA level, possibly because the FHFA all-transactions index understates the magnitude of house price declines during the housing crash.

3.3 Determinants of House Price Misperceptions

Table 5 briefly explores the empirical determinants of homeowner misperceptions of housing market conditions. The dependent variable in each column is the standard deviation of the misperceptions index within each geographical area, which reflects the extent to which perceptions of house price movements in that area differed from actual movements. An advantage of using the standard deviation versus alternatives such as the mean absolute value of the misperceptions index is the potential ambiguity created by normalizing the misperceptions index to zero in the base year. For instance, in Hawaii the misperceptions index is negative every year from 2001 to 2010. That pattern could reflect either that Hawaii homeowners were unaware of the extent of Hawaiian price appreciation after the year 2000, or that they had unduly low perceptions of prices in the year 2000 and more accurate perceptions in later years.[12]

[12]Because the sales volumes regressions in the previous section all include state- or MSA-level fixed effects, this ambiguity should not affect the results in those regressions.

Columns 1 through 3 explore misperceptions at the state level, while columns 4 and 5 examine the MSA level. In column 1, the standard deviation of misperceptions is regressed on the standard deviation of house prices within the state over time. A one percentage point increase in the standard deviation of prices is associated with a 0.29 percentage point increase in the standard deviation of misperceptions. The $\overline{R}^2$ of the regression is 0.57. Column 2 includes the persistence of house price changes, as measured by a simple autoregression of the change in prices on their own lag. More persistent prices are associated with slightly more accurate perceptions. Column 3 includes the median percentage point absolute error of Zillow's home value estimate for homes in a state. A higher value for this number suggests that it is difficult to infer home values from observable house characteristics. A one percentage point increase in Zillow's median error is associated with a 0.55 point increase in the standard deviation of misperceptions. In other words, homeowners perceive movements in house prices less accurately in areas where Zillow's estimates are also less accurate. Columns 4 and 5 replicate the results from columns 1 and 2, respectively, at the MSA level rather than the state level. As in the state-level regressions, greater variability in house prices is associated with greater variability in misperceptions and greater persistence of house price changes is associated with less variability in misperceptions. However, the coefficients on these variables are smaller in magnitude at the MSA level, and explain less of the variation in misperceptions.[13]

[13] I do not consider the median error of the ZHVI at the MSA level because Zillow only publishes it for a very small number of large MSAs.

4 Conclusion

This paper demonstrates that a stylized model of the house-selling process in which sellers have incomplete information regarding the state of the housing market can generate the correlations between prices, sales volumes, and time on market observed in the data. This result can persist in the presence of real estate agents with complete information regarding the state of the market if sellers' and agents' incentives are not perfectly aligned. Empirically, an increase in homeowners' perceptions of house prices relative to true market conditions predicts a decrease in sales volumes. Homeowner misperceptions of housing market conditions can account for more than one-fifth of the time series variation in within-state sales volumes from 2000 to 2010.

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TABLE 1: HOUSE PRICE MISPERCEPTIONS AND HOME SALES AT THE STATE LEVEL

Dependent Variable	Log Homes Sold	Log Homes Sold	Log Homes Sold	Log Homes Sold	Log Homes Sold	Log Homes Sold
Specification	Levels	Levels	Levels	First Differences	First Differences	First Differences
	(1)	(2)	(3)	(4)	(5)	(6)
Misperceptions Index	-1.396	-1.394	-1.193	-0.878	-0.807	-0.639
	(0.195)	(0.195)	(0.169)	(0.277)	(0.326)	(0.192)
FHFA House Price Index		0.024	-0.730		0.094	-0.873
		(0.072)	(0.127)		(0.216)	(0.095)
Year Fixed Effects?	No	No	Yes	No	No	Yes
Number of States	51	51	51	51	51	51
Number of Observations	556	556	556	503	503	503
R-squared within	0.216	0.217	0.716	0.077	0.080	0.593

Notes: Fixed Effects estimates using yearly state-level data from 2000-2010. Standard errors clustered at state level in parentheses. All specifications include a constant term. Homes sold data from the National Association of Realtors. The misperceptions index is the log change in housing values constructed from American Community Survey data minus the log change in the FHFA purchase-only house price index. All regressions weighted by the number of housing units in ACS price regressions.

TABLE 2: HOUSE PRICE MISPERCEPTIONS AND HOME SALES AT THE STATE LEVEL - ALTERNATIVE INDICES

Dependent Variable	Log Homes Sold	Log Homes Sold	Log Homes Sold	Log Homes Sold	Log Homes Sold
House Price Index	FHFA Purchase Only Index	FHFA All Transactions Index	FHFA Median Index	Zillow Home Value Index	FHFA Purchase Only Index
	(1)	(2)	(3)	(4)	(5)
Misperceptions Index	-1.193	-1.268	-1.163	-0.949	-1.055
	(0.169)	(0.257)	(0.181)	(0.149)	(0.134)
House Price Index	-0.730	-0.694	-0.611	-0.643	-0.609
	(0.127)	(0.137)	(0.101)	(0.132)	(0.097)
Year Fixed Effects?	Yes	Yes	Yes	Yes	Yes
Weighted by Housing Units?	Yes	Yes	Yes	Yes	No
Number of States	51	51	51	35	51
Number of Observations	556	556	556	381	556
R-squared within	0.716	0.711	0.725	0.725	0.697

Notes: Fixed Effects estimates using yearly state-level data from 2000-2010. Standard errors clustered at state level in parentheses. All specifications include a constant term. Homes sold data from the National Association of Realtors.

TABLE 3: HOUSE PRICE MISPERCEPTIONS AND HOME SALES AT THE MSA LEVEL

Dependent Variable	Log Homes Sold	Log Homes Sold	Log Homes Sold	Log Homes Sold	Log Homes Sold	Log Homes Sold
Specification	Levels	Levels	Levels	First Differences	First Differences	First Differences
	(1)	(2)	(3)	(4)	(5)	(6)
Misperceptions Index	-2.627	-2.868	-1.093	-0.113	-0.075	-0.122
	(0.472)	(0.472)	(0.320)	(0.280)	(0.281)	(0.271)
FHFA House Price Index		0.024	-0.730		-1.021	-1.737
		(0.037)	(0.047)		(0.097)	(0.145)
Year Fixed Effects?	No	No	Yes	No	No	Yes
Number of MSAs	105	105	105	105	105	105
Number of Observations	630	630	630	525	525	525
R-squared within	0.193	0.207	0.734	0.001	0.258	0.474

Notes: Fixed Effects estimates using yearly MSA-level data from 2005-2010. Standard errors clustered at MSA level in parentheses. All specifications include a constant term. Homes sold data from Zillow.com. The misperceptions index is the log change in housing values constructed from American Community Survey data minus the log change in the FHFA all-transactions house price index. All regressions weighted by the number of housing units in ACS price regressions.

TABLE 4: HOUSE PRICE MISPERCEPTIONS AND HOME SALES AT THE MSA LEVEL - ALTERNATIVE INDICES

Dependent Variable	Log Homes Sold	Log Homes Sold	Log Homes Sold	Log Homes Sold	Log Homes Sold
House Price Index	FHFA All Transactions Index	FHFA Purchase Only Index	Case-Shiller Index	Zillow Home Value Index	FHFA All Transactions Index
	(1)	(2)	(3)	(4)	(5)
Misperceptions Index	-1.093	-1.638	-1.032	-2.255	-0.917
	(0.320)	(0.476)	(0.546)	(0.318)	(0.260)
House Price Index	-1.134	-1.539	-1.437	-1.264	-0.994
	(0.125)	(0.213)	(0.291)	(0.116)	(0.087)
Year Fixed Effects?	Yes	Yes	Yes	Yes	Yes
Weighted by Housing Units?	Yes	Yes	Yes	Yes	No
Number of MSAs	105	20	19	105	105
Number of Observations	630	120	114	630	630
R-squared within	0.734	0.778	0.756	0.758	0.695

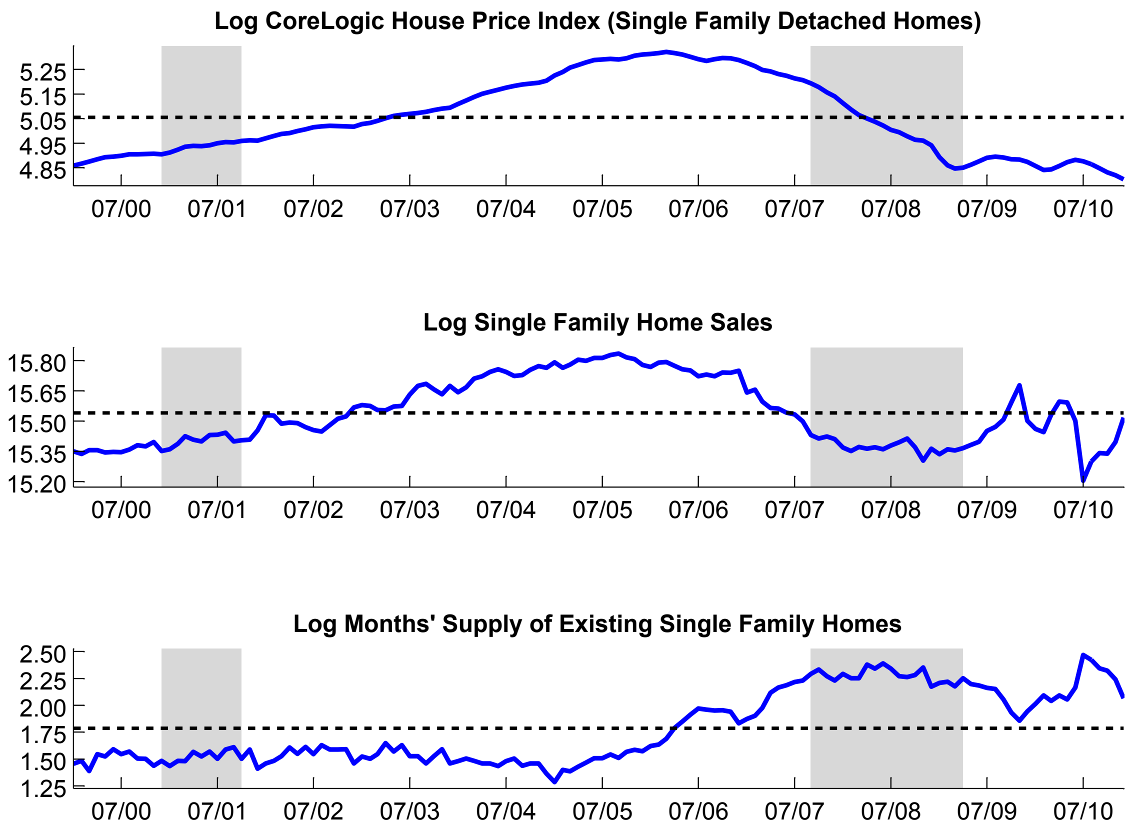
Notes: Fixed Effects estimates using yearly MSA-level data from 2005-2010. Standard errors clustered at state level in parentheses. All specifications include a constant term. Homes sold data from Zillow.com.

TABLE 5: CAUSES OF HOUSE PRICE MISPERCEPTIONS

Dependent Variable	Std. Dev. Of Mispercep-tions Index	Std. Dev. Of Mispercep-tions Index	Std. Dev. Of Mispercep-tions Index	Std. Dev. Of Mispercep-tions Index	Std. Dev. Of Mispercep-tions Index
	States	States	States	MSAs	MSAs
	(1)	(2)	(3)	(4)	(5)
Std. Dev. Of House Price Index	0.288	0.276	0.224	0.146	0.120
	(0.035)	(0.035)	(0.036)	(0.025)	(0.029)
House Price Persistence		-0.040	-0.090		-0.011
		(0.022)	(0.024)		(0.007)
Zillow Estimate Median Error			0.554		
			(0.220)		
Number of Observations	51	51	36	105	105
Adjusted R-squared	0.571	0.590	0.697	0.240	0.252

Notes: All specifications include a constant term and are weighted by the number of housing units in ACS price regressions. House price persistence is the autoregressive parameter from regressing the change in house prices on the lagged change within an area. Zillow estimate median error is the median error of Zillow's house price estimate within an area.

Figure 1: Housing Market Time Series January 2000 - December 2010



Note: All series have had a linear time trend removed.

Figure 2: Expected Values of ψ_2 as a function of ψ_1

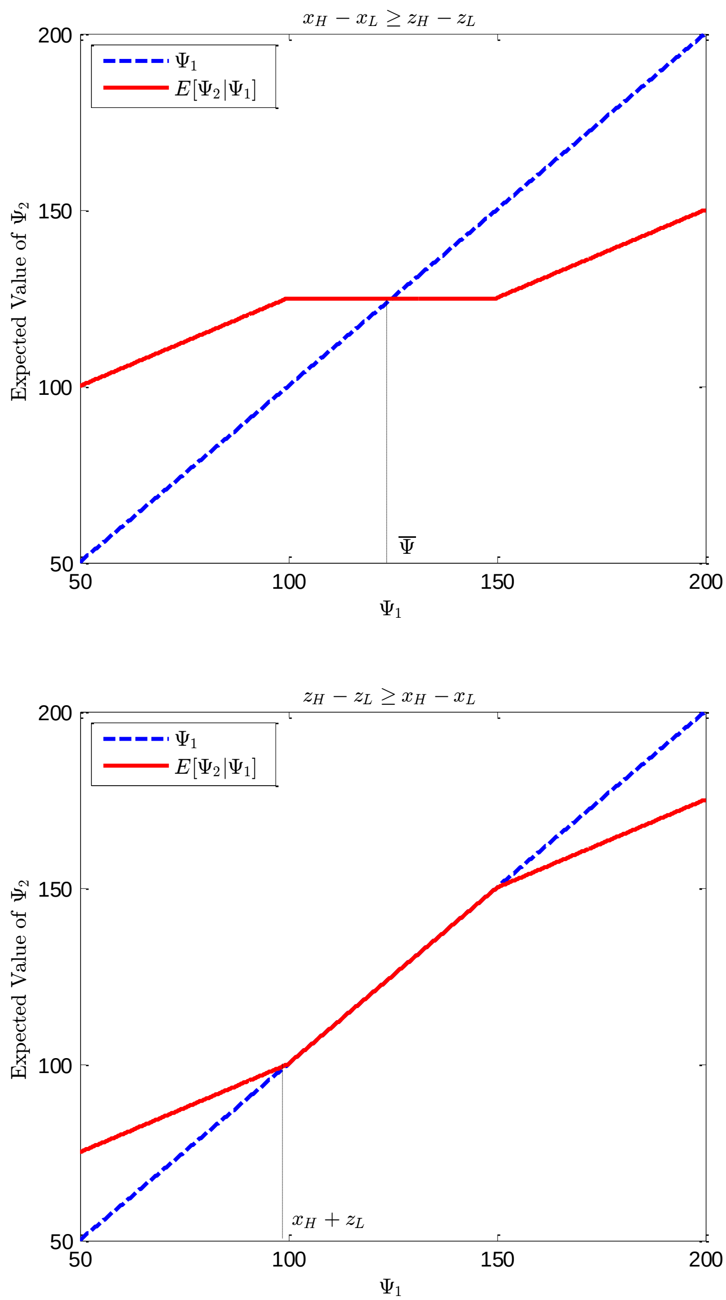


Figure 3: Expected Time to Sale and Sales Price without Realtor

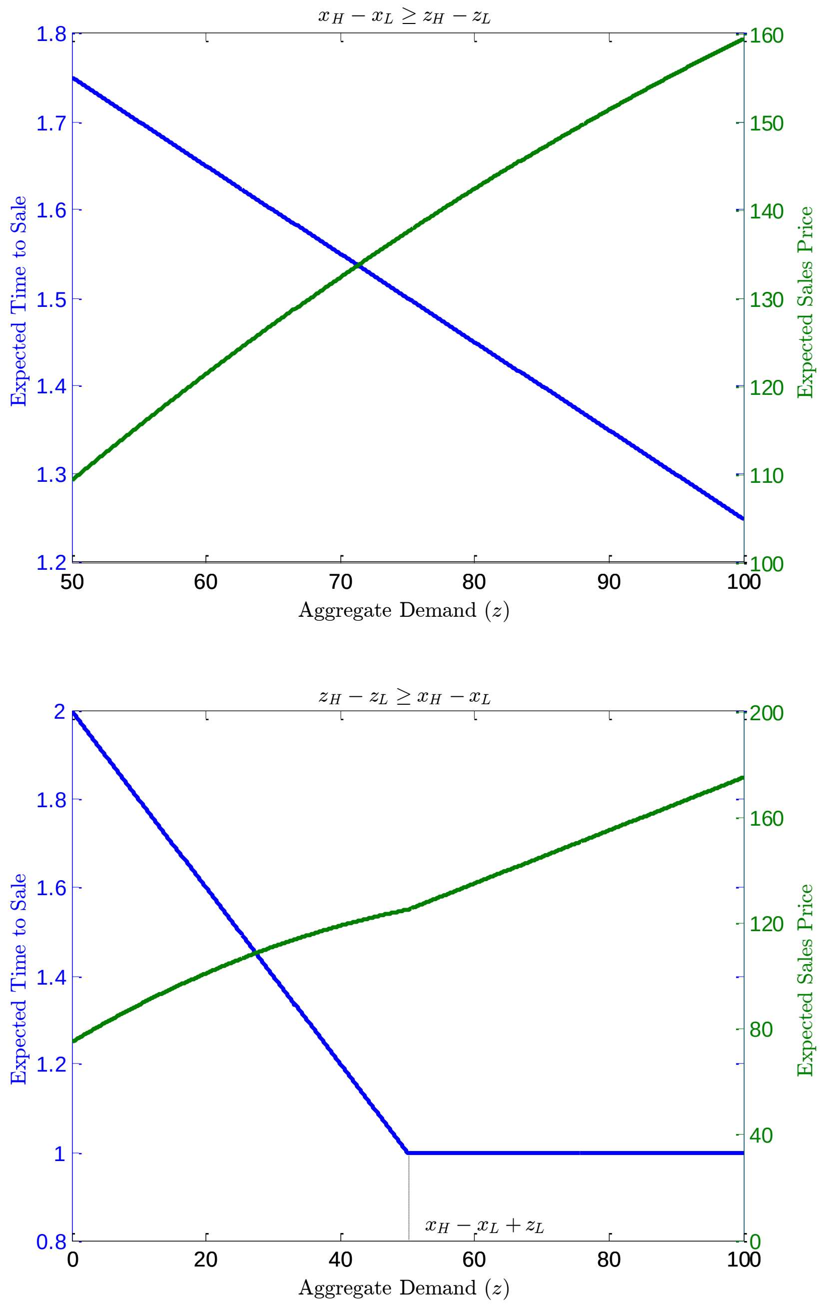


Figure 4: Expected Values of ψ_2 as a function of ψ_1

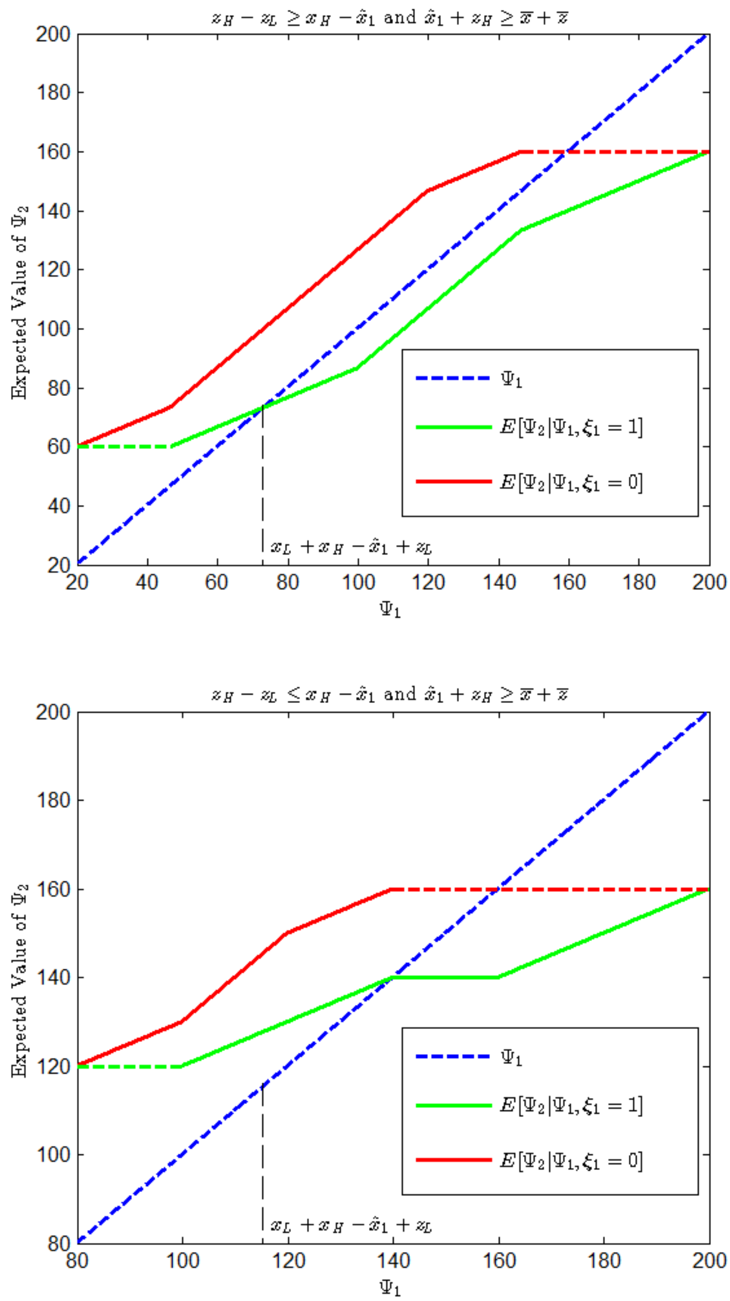
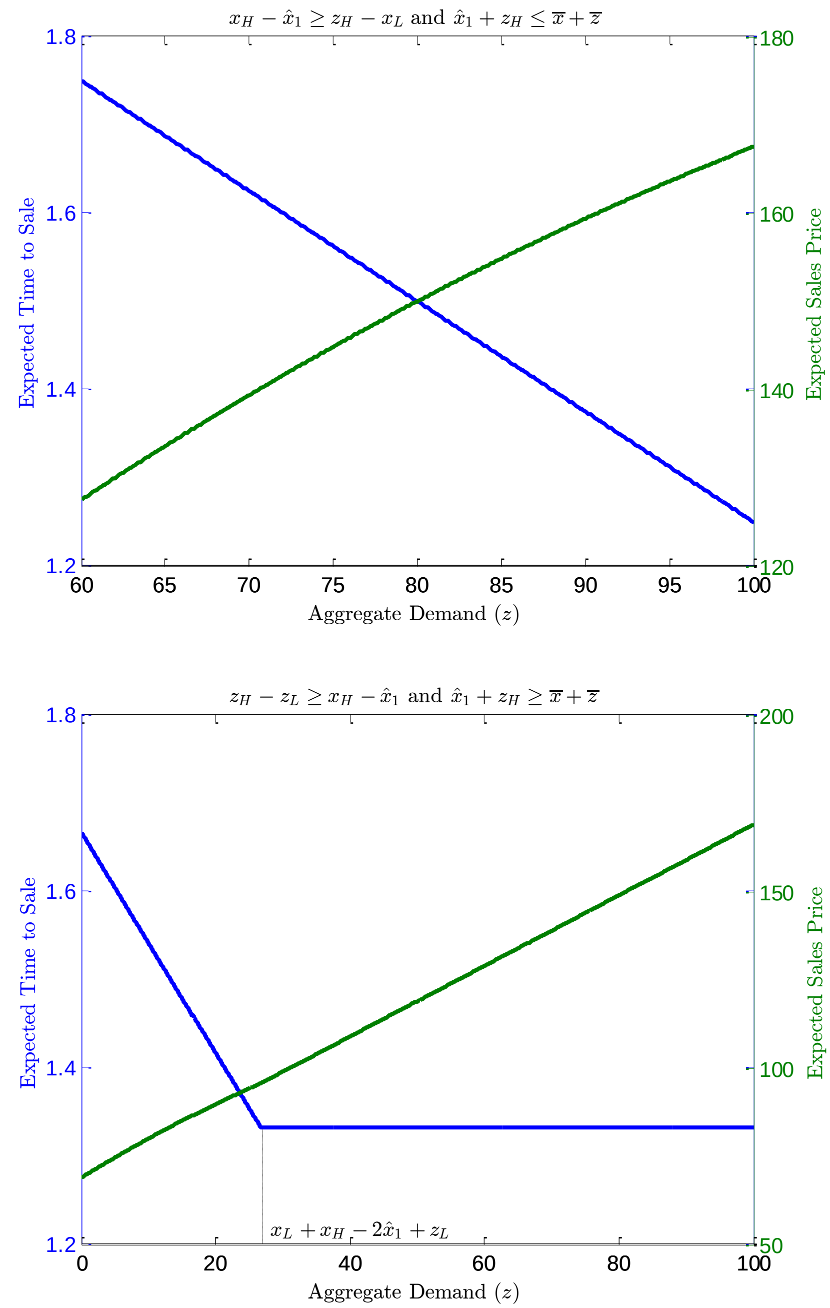


Figure 5: Expected Time to Sale and Sales Price with Realtor



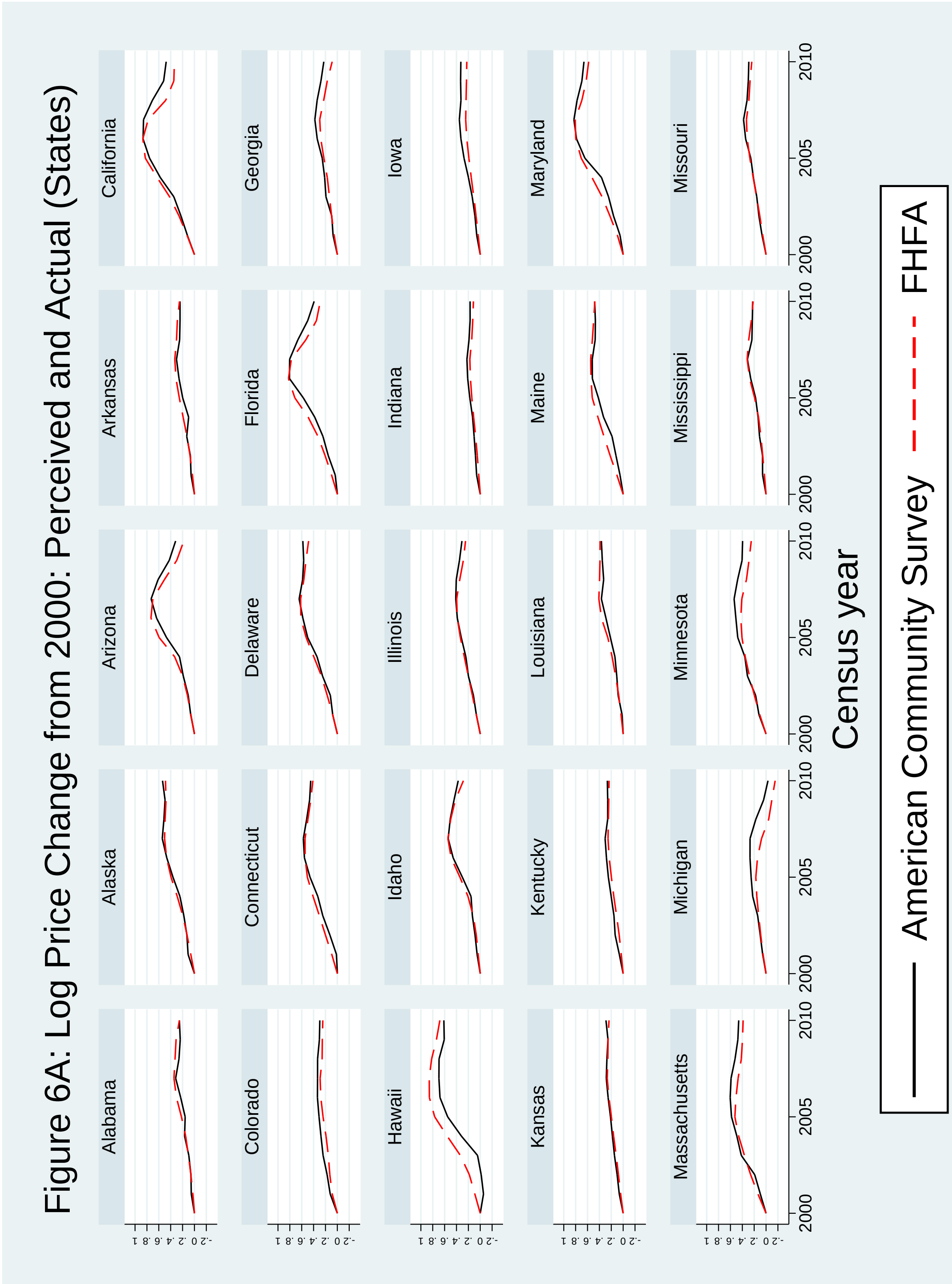


Figure 6A: Log Price Change from 2000: Perceived and Actual (States)
Alabama
Alaska
Arizona
Arkansas
California
Colorado
Connecticut
Delaware
Florida
Georgia
Hawaii
Idaho
Illinois
Indiana
Iowa
Kansas
Kentucky
Louisiana
Maine
Maryland
Massachusetts
Michigan
Minnesota
Mississippi
Missouri
-.2 0 .2 .4 .6 .8 1
2000 2005 2010
Census year
American Community Survey
FHFA

Figure 6B: Log Price Change from 2000: Perceived and Actual (States)

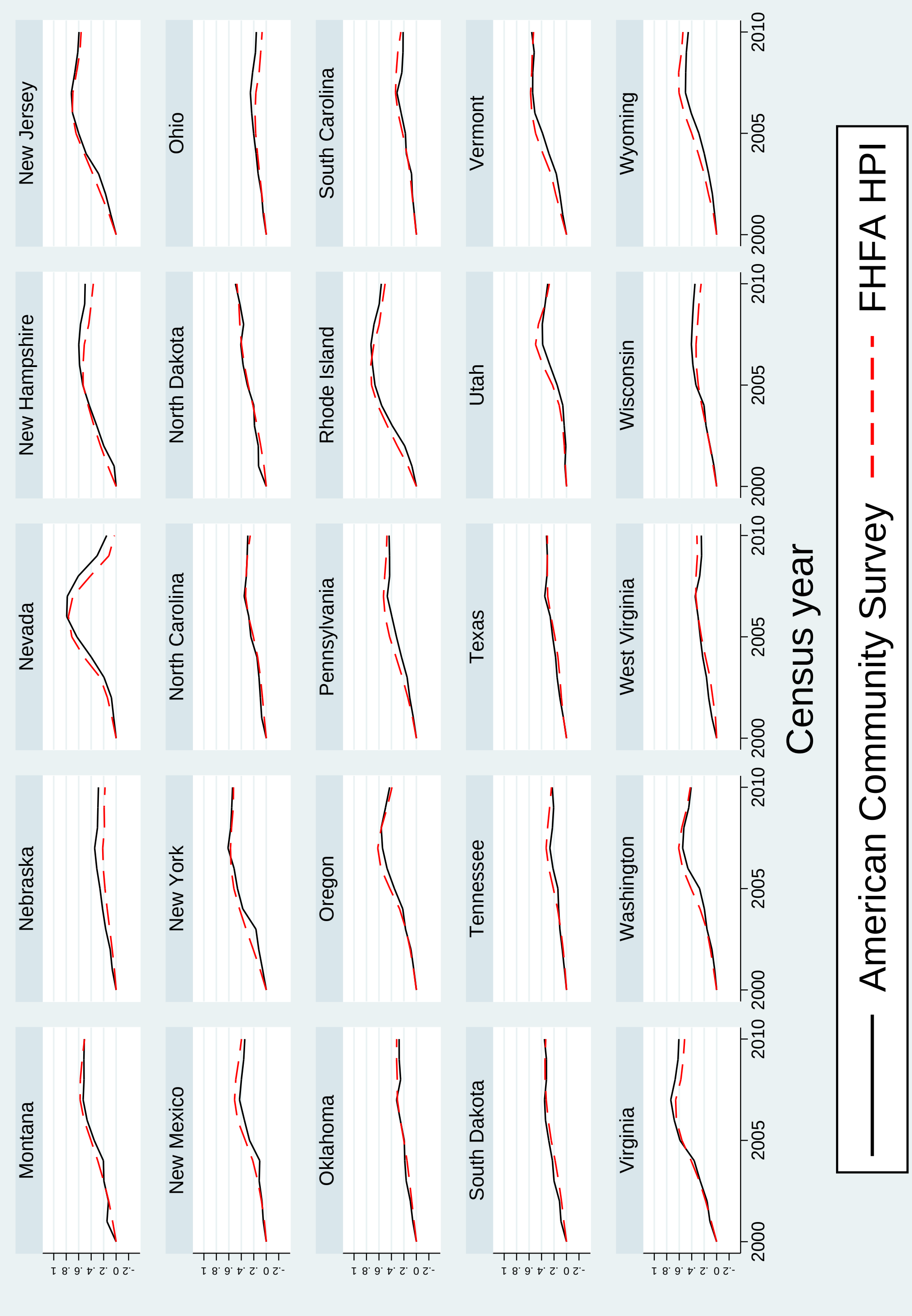
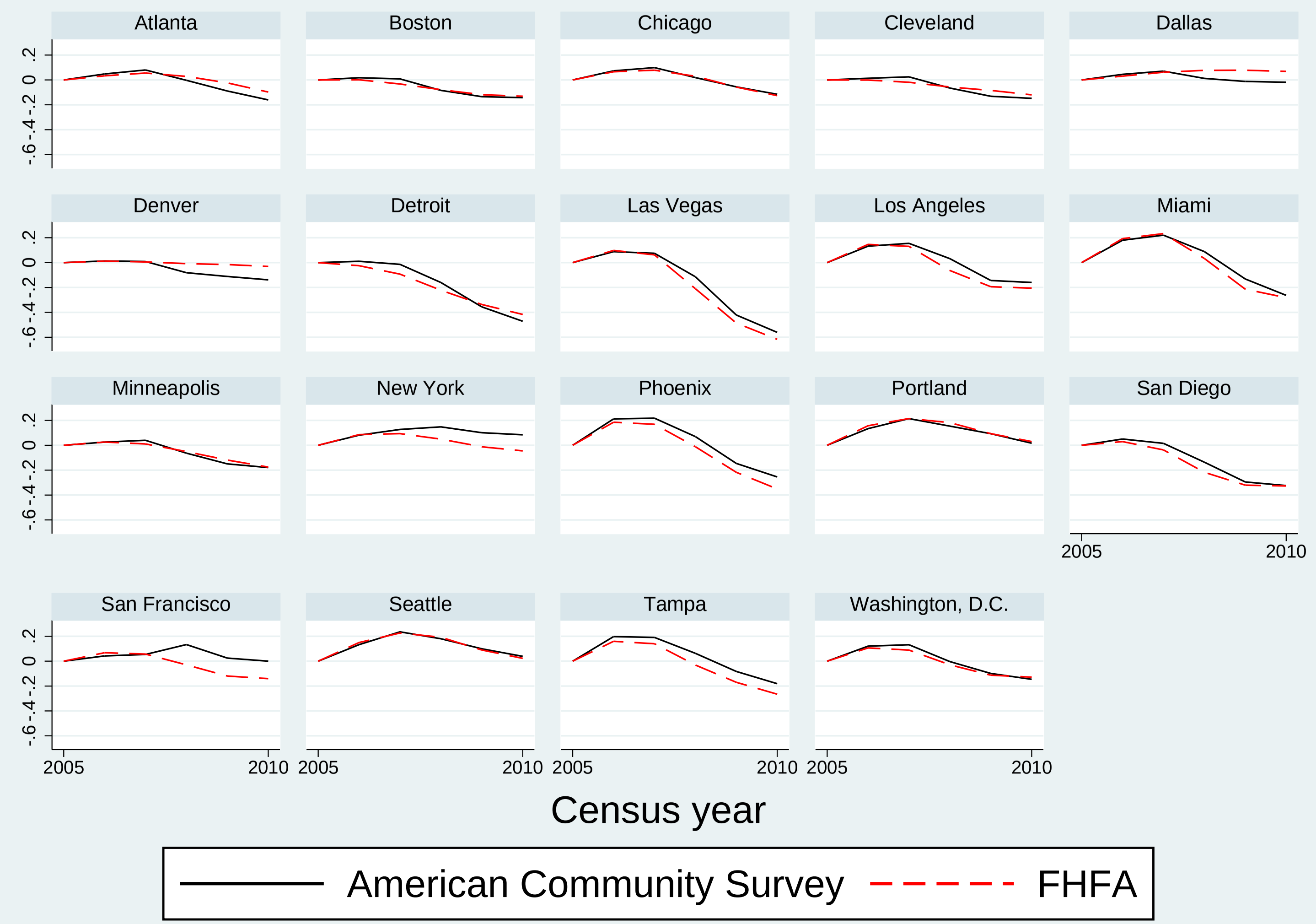


Figure 7: Percent Price change from 2005: Perceived and Actual (MSAs)



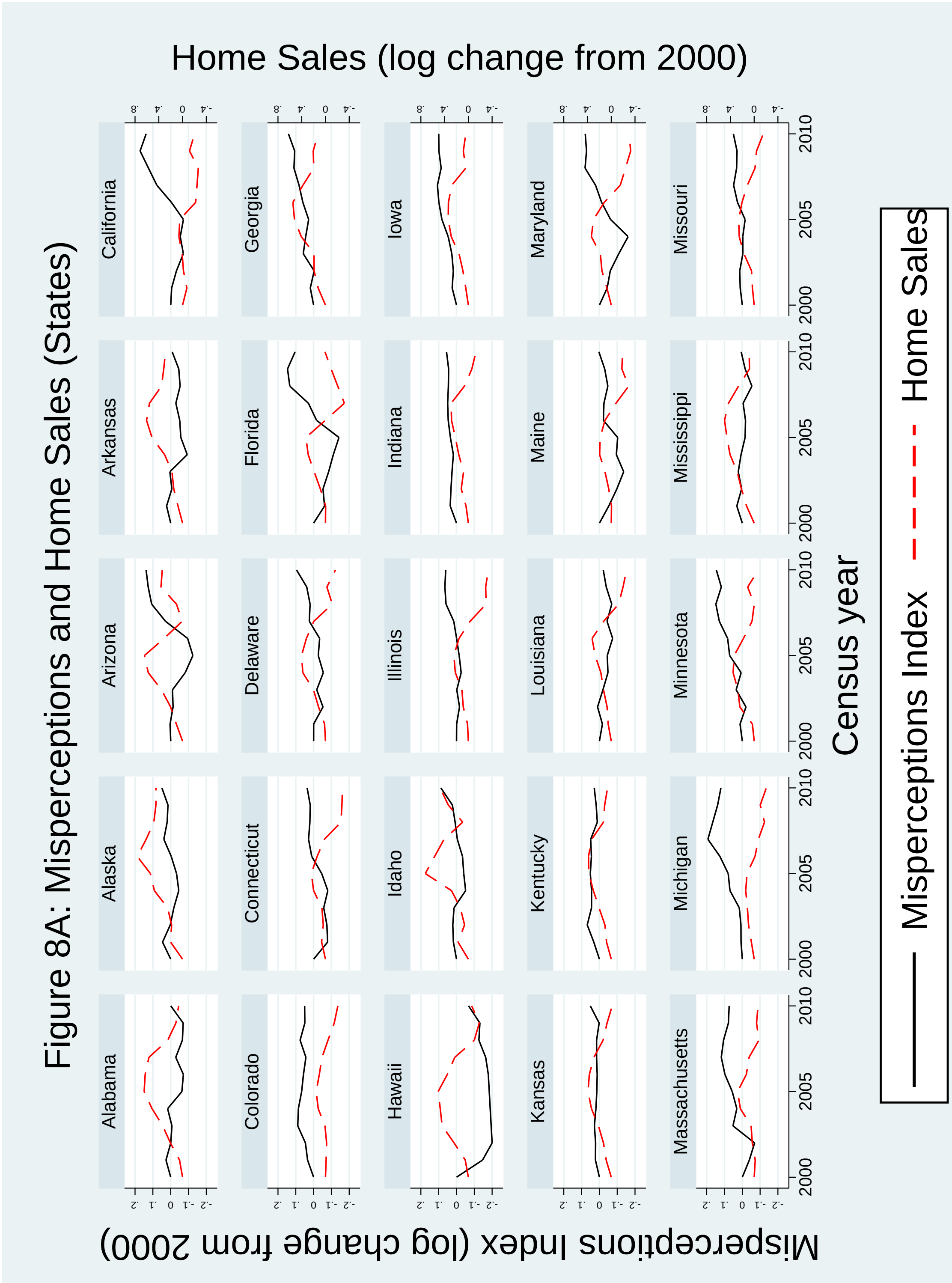


Figure 8A: Misperceptions and Home Sales (States)
Home Sales (log change from 2000)
Misperceptions Index (log change from 2000)
Census year
Alabama
Alaska
Arizona
Arkansas
California
Colorado
Connecticut
Delaware
Florida
Georgia
Hawaii
Idaho
Illinois
Indiana
Iowa
Kansas
Kentucky
Louisiana
Maine
Maryland
Massachusetts
Michigan
Minnesota
Mississippi
Missouri
2000
2005
2010
.2
.1
0
-.1
-.2
.8
.4
0
-.4
Misperceptions Index
Home Sales

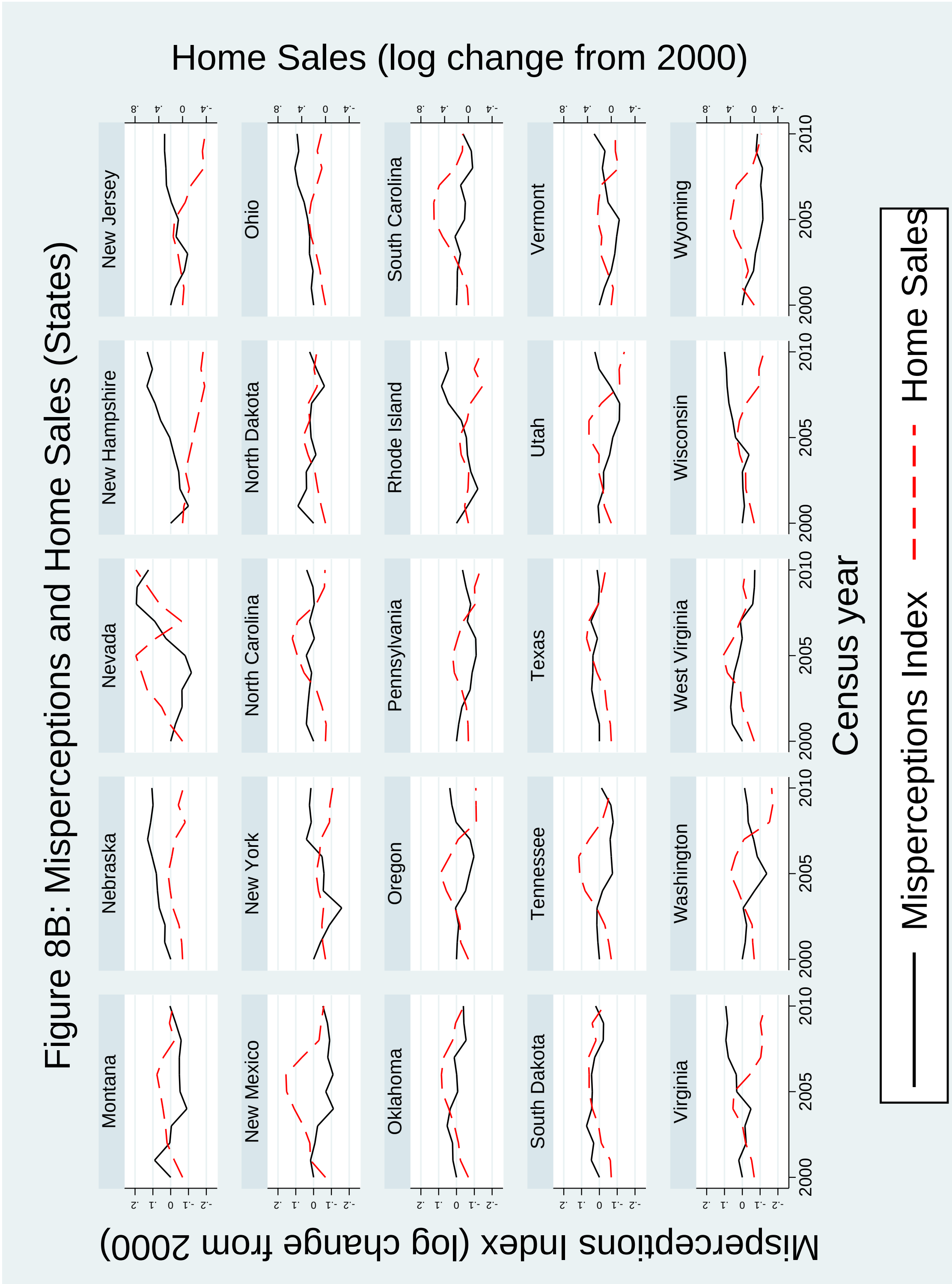
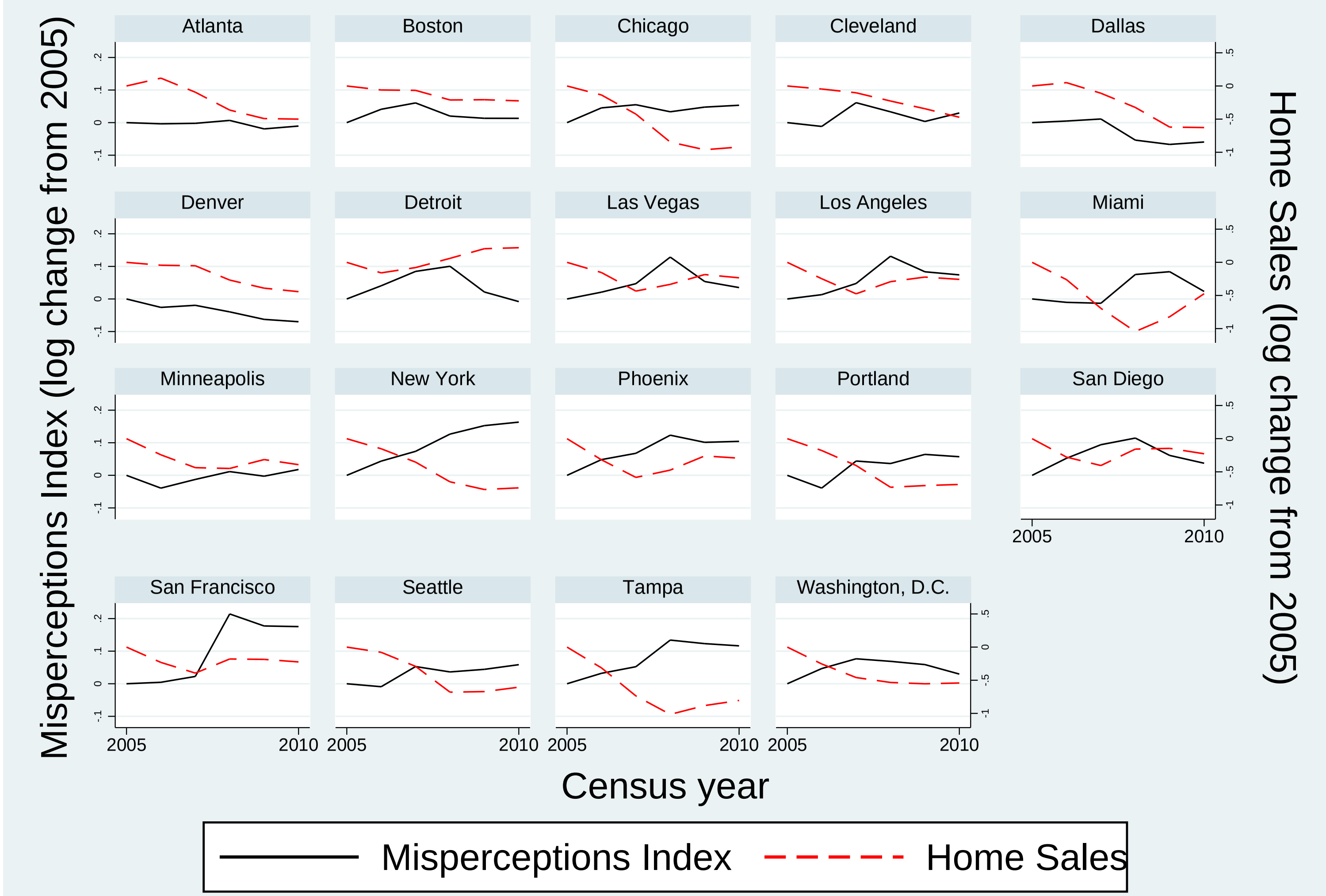


Figure 8B: Misperceptions and Home Sales (States)
Home Sales (log change from 2000)
Misperceptions Index (log change from 2000)
Census year
Montana
Nebraska
Nevada
New Hampshire
New Jersey
New Mexico
New York
North Carolina
North Dakota
Ohio
Oklahoma
Oregon
Pennsylvania
Rhode Island
South Carolina
South Dakota
Tennessee
Texas
Utah
Vermont
Virginia
Washington
West Virginia
Wisconsin
Wyoming
Misperceptions Index
Home Sales

Figure 9: Misperceptions Index and Home Sales (MSAs)



Appendix

A Extending the Model to Multiple Periods

This section extends the model to three periods. This extension illustrates the key differences between a two period model and a multiple period model and presents an equilibrium concept that is compatible with any finite number of periods. There are two main differences between the equilibria of the two period model and the three period model. First, in the first period of the three period model, the agent's cutoff rule for recommending 'accept' will be a function of z. Second, in equilibrium there will be ranges of first-period offers for which the agent will 'babble': his recommendation will contain no information regarding the state of demand, and consequently the seller will ignore it when updating her beliefs. Otherwise, the agent would sometimes have an incentive to misreport his own preference in order to manipulate the seller's beliefs regarding the state of demand.

Denote the agent's period t recommendation ξ_t and the seller's decision in period t γ_t, where in both cases a value of 1 indicates 'accept' and a value of 0 indicates 'reject'. Let $\widehat{x}_1(z)$ and $\widehat{x}_2$ represent the agent's cutoff value for reporting 'accept' in periods 1 and 2, respectively. As discussed, in equilibrium there will be some values for ψ_1 such that the agent will babble. For all other values of ψ_1 the agent's recommendation will follow a cutoff rule in x_1, which will be a function of z. Let $\widetilde{z}_{L,t}$ and $\widetilde{z}_{H,t}$ denote the seller's beliefs about the lowest and highest possible values of z after receiving the period t offer and recommendation. Then the agent's value function can be written:

$$\begin{aligned} V_{A,1}(\psi_1, z) = \overset{\max}{\xi_1} \{ & \gamma_1(\psi_1, \tilde{f}(\psi|\psi_1, 0)\alpha\psi_1 + (1-\gamma_1(\psi_1, \tilde{f}(\psi|\psi_1, 0)))(E[V_{A,2}(\psi_2, z, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})|\xi_1 = 0]), \\ & \gamma_2(\psi_1, \tilde{f}(\psi|\psi_1, 1)\alpha\psi_1 + (1-\gamma_1(\psi_1, \tilde{f}(\psi|\psi_1, 1)))(E[V_{A,2}(\psi_2, z, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})|\xi_1 = 1])\} \end{aligned}$$

$$\begin{aligned} V_{A,2}(\psi_2, z, \widetilde{z}_{L,1}, \widetilde{z}_{H,1}) = -c_a + \overset{\max}{\xi_2} \{ & \gamma_2(\psi_2, \tilde{f}(\psi|\psi_2, 0, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})\alpha\psi_2 + \\ & (1-\gamma_2(\psi_2, \tilde{f}(\psi_2|\psi_2, 0, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})))(E[V_{A,3}(\psi_3, z, \widetilde{z}_{L,2}, \widetilde{z}_{H,2})|\xi_2 = 0]), \\ & \gamma_2(\psi_2, \tilde{f}(\psi|\psi_1, 1, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})\alpha\psi_2 + \\ & (1-\gamma_2(\psi_2, \tilde{f}(\psi_2|\psi_1, 1, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})))(E[V_{A,3}(\psi_2, z, \widetilde{z}_{L,2}, \widetilde{z}_{H,2})|\xi_2 = 1])\} \end{aligned}$$

$$V_{A,3}(\psi_3, z, \widetilde{z}_{L,2}, \widetilde{z}_{H,2}) = -2c_a + \alpha\psi_3$$

The seller's value functions can be written:

$$V_{S,1}(\psi_1, \xi_1) = \overset{\max}{\gamma_1}(1-\alpha)\{\psi_1, E[V_{S,2}(\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})|\tilde{f}(\psi_2|\psi_1, \xi_1)]\}$$

$$V_{S,2}(\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1}, 2) = \overset{\max}{\gamma_2}(1-\alpha)\{\psi_2, E[\psi_3|\tilde{f}(\psi_3|\psi_1, \xi_2)]\}$$

$$V_{S,3}(\psi_3, \xi_3, \widetilde{z}_{L,2}, \widetilde{z}_{H,2}) = (1-\alpha)\psi_3$$

The definition of a Bayesian Nash equilibrium of the game between the seller and the agent can be updated as a set of policy functions $\xi_1(\psi_1, z)$ and $\xi_2(\psi 2, z, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})$ for the agent, a set of policy functions $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$ and $\gamma_2(\psi_2, \tilde{f}(\psi_3|\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1}))$ for the seller, and a set of belief updating strategies $\tilde{f}(\psi_2|\psi_1, \xi_1)$ and $\tilde{f}(\psi_3|\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})$ for the seller such that:

1. $\xi_1(\psi_1, z)$ and $\xi_2(\psi 2, z, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})$ solve the agent's problem taking the seller's policy functions and belief updating strategies as given;
2. $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$ and $\gamma_2(\psi_2, \tilde{f}(\psi_3|\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1}))$ solve the seller's problem taking the agent's policy functions as given; and
3. $\tilde{f}(\psi_2|\psi_1, \xi_1)$ and $\tilde{f}(\psi_3|\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})$ are consistent with the agent's policy functions.

The agent's optimal cutoff strategy is a function of z because when z is low, it is more likely that ψ_2 will be low enough that the seller rejects the offer regardless of the agent's recommendation in the second period. This loss of control lowers the agent's payoff. The agent's cutoff rule in period 1 can be solved by backwards iteration. In the penultimate period, the agent's preference is for the seller to accept any offers such that $x_2 \geq \overline{x} - \frac{c_a}{\alpha}$ and reject all others. Call this value $\widehat{x}_2$.

It is tedious but straightforward to show that the optimal cutoff rule is implicitly defined by the following equation, in which $\widehat{x}_1(z)$ is written $\widehat{x}_1$ on the right-hand side for simplicity:

$$\widehat{x}_1(z) = \begin{cases} \frac{\frac{1}{2}(x_H^2 - \overline{x}^2 - \widetilde{\widetilde{z}}_1(\widehat{x}_1)^2 - z^2) + (\overline{x} - \widehat{x}_2 + \widetilde{\widetilde{z}}_1(\widehat{x}_1))z + (\overline{x} - x_L + \widetilde{\widetilde{z}}_1(\widehat{x}_1))\widehat{x}_2 - \widetilde{\widetilde{z}}_1(\widehat{x}_1)\overline{x}}{x_H - x_L} & \text{if } \widehat{x}_2 + \widetilde{z}_{H,1}(\widehat{x}_1) \leq \overline{x} + \widetilde{\widetilde{z}}_1(\widehat{x}_1) \\ \frac{\widehat{x}_2 + x_H}{2} + (\frac{\widehat{x}_2 - x_L}{x_H - x_L})(\frac{\widehat{x}_2 - x_H}{2}) & \text{if } \widehat{x}_2 + \widetilde{z}_{H,1}(\widehat{x}_1) \geq \overline{x} + \widetilde{\widetilde{z}}_1(\widehat{x}_1) \\ & \text{and } z \geq x_L + x_H - 2\widehat{x}_2 + \widetilde{z}_{L,1}(\widehat{x}_1) \\ \frac{(x_H - \widehat{x}_2 + \widetilde{z}_{L,1}(\widehat{x}_1) - z)\widehat{x}_2 + \frac{1}{2}(\widehat{x}_2 - x_L - \widetilde{z}_{L,1}(\widehat{x}_1) + z)(x_L + 2x_H - \widehat{x}_2 + \widetilde{z}_{L,1}(\widehat{x}_1) - z)}{(x_H - x_L)} & \text{if } \widehat{x}_2 + \widetilde{z}_{H,1}(\widehat{x}_1) \geq \overline{x} + \widetilde{\widetilde{z}}_1(\widehat{x}_1) \\ & \text{and } z < x_L + x_H - 2\widehat{x}_2 + \widetilde{z}_{L,1}(\widehat{x}_1) \end{cases}$$

$\widehat{x}_1(z)$ can be estimated numerically as the fixed point of this functional equation. A description of the estimation algorithm is included at the end of this section. Figure A.1 illustrates the agent's optimal period 1 cutoff rule as a function of z.

Given the agent's cutoff rule, the seller will update her beliefs concerning z as follows. Define $\widetilde{\widetilde{z}}$ as the value of z such that $\psi_1 - z = \widehat{x}_1(z)$. Further define

$$\widetilde{z}_{L,1} = \begin{cases} \max(z_L, \widetilde{\widetilde{z}}) & \text{if } \gamma_1 = 0 \\ \max(z_L, \psi_1 - x_H) & \text{if } \gamma_1 = 1 \end{cases} \text{ and } \widetilde{z}_{H,1} = \begin{cases} \min(z_H, \psi_1 - x_L) & \text{if } \gamma_1 = 0 \\ \min(\widetilde{\widetilde{z}}, \psi_1 - x_L) & \text{if } \gamma_1 = 1 \end{cases}$$

Then the seller's posterior belief about the distribution of z will again be that $z \sim U[\widetilde{z}_{L,1}, \widetilde{z}_{H,1}]$. The seller's belief updating strategy in period 2 will be the same as in the two-period model.

To see that 'babbling regions' in period 1 are a necessary feature of equilibrium, consider a hypothetical equilibrium in which the agent always reports his own preference truthfully in period 1, and the agent's and seller's behavior in the second period is the same as their behavior in the first period of the two-period model. Then the seller's expected value of rejecting the first offer would look as in the top panel of figure A.2. Define $\psi_1^\star$ as the fixed point of the seller's expected value of waiting for period 2 conditional on the agent recommending 'accept', and $\psi_1^\dagger$ as the fixed point of the seller's expected value of waiting for period 2 conditional on the agent recommending 'reject'. For $\psi_1 < \psi_1^\star$, the expected value of waiting for period 2 is higher than ψ_1 whether the the agent recommends 'accept' or 'reject'–the seller's action will not depend on the agent's recommendation in this range. Similarly, when $\psi_1 > \psi_1^\dagger$, the seller's expected value of waiting for period 2 will be below ψ_1 no matter what the agent recommends. Therefore, the seller will always reject offers $\psi_1 < \psi_1^\star$ and will always accept offers $\psi_1 > \psi_1^\dagger$.

Consider the agent's best response when $\psi_1 < \psi_1^\star$ and $x_1 > \widehat{x}_1(z)$ in the hypothetical equilibrium. The seller will reject the offer regardless of the agent's recommendation. However, because the seller expects the agent to report his own preference truthfully, the seller will have a higher expectation of future offers if the agent recommends 'reject' (indicating that z is high) than if the agent recommends 'accept' (indicating that z is low).

This situation cannot be optimal for the agent, who intuitively would always prefer that the seller be more pessimistic (have a lower expectation of ψ) in the second period. A pessimistic seller is more likely to accept offers the agent would like her to accept, but will always reject offers the agent would like her to reject. Therefore, the agent has a unilateral incentive to deviate from his proposed strategy in the hypothetical equilibrium. For any offer $\psi_1 < \psi_1^\star$, the agent should recommend 'reject'.

Babbling regions solve this problem. If the agent's recommendation in period 1 changes the seller's expectation of future offers without changing the seller's action, the agent will always choose to send the message that will make the seller more pessimistic. Then in equilibrium, the agent's first period recommendation cannot change the seller's beliefs without changing her first period action. Therefore, in equilibrium, the agent will babble in the first period when his recommendation will not change the seller's action, and will report his own preference truthfully when his recommendation is decisive. The bottom panel of figure A.2 illustrates the period 1 equilibrium, assuming that in the second period the seller and the realtor play the same strategies as they did in the first period of the two-period model. The second period equilibrium will then look like it does in Figure 4.

Because $\widehat{x}_1(z)$ must be estimated numerically, the expected sales price and expected time to sale must be simulated as well. Figure A.3 shows the results from such a simulation. The general pattern from the two-period model persists: as aggregate demand rises, the expected sales price rises and the expected time on market mostly falls. However, there is a slight bump in the expected time to sale for high levels of z. This stems from the upward-sloping portion of $\widehat{x}_1(z)$, which causes the agent to recommend rejecting a higher percentage of offers when z is high. Overall, however, the correlation between expected sales price and expected time to sale is negative, and the simulated results of the three period model are consistent with the stylized facts observed in the data.

The following algorithm solves for the agent's cutoff rule $\widehat{x}_1(z)$ in the three period model:

1. Pick a candidate schedule for $\widehat{x}_1(z)$. In practice I chose $\widehat{x}_1(z) = \overline{x}$ for all z.

2. On a fine grid of points for z:

   (a) Go through a fine grid of points for all values of x to create a grid of all values of ψ consistent with each value of z.

   (b) Calculate $\widetilde{z}_{L,1}$ and $\widetilde{z}_{H,1}$ for each value of ψ conditional on the agent recommending 'reject'.

   (c) For each value of ψ, find the expected value of the idiosyncratic component of the agent's payoff if the seller rejects the first period offer. Denote this value $\overline{x}_A$. For a fixed z, this gives $\overline{x}_R$ as a function of x.

   (d) Find the fixed point of $\overline{x}_A(x)$; use this value as the new candidate for $\widehat{x}_1(z)$.

3. Repeat this procedure using the new schedule for $\widehat{x}_1(z)$ until the maximum distance between the old and new schedules is below a specified tolerance level.

Figure A.1: Realtor's Optimal Cutoff Rule

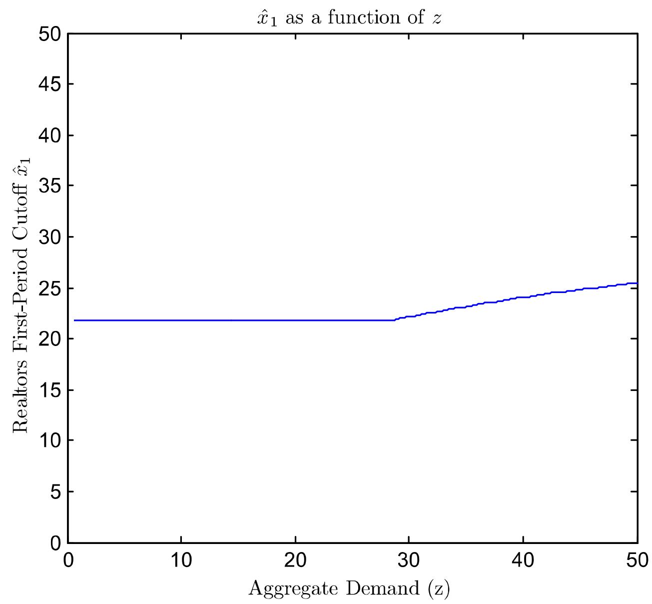


Figure A.2: Seller's Expected Value of waiting for Period 2 in 3-period Model

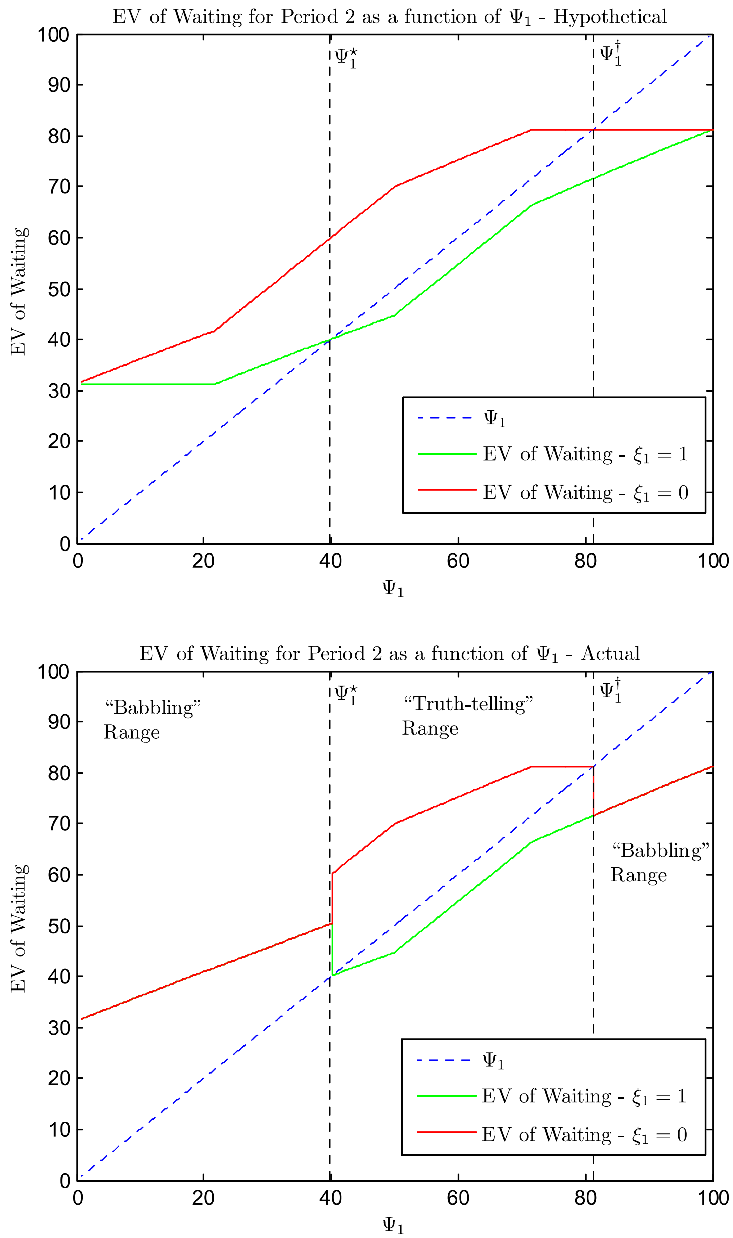


Figure A.3: Expected Time to Sale and Sales Price in 3-period Model

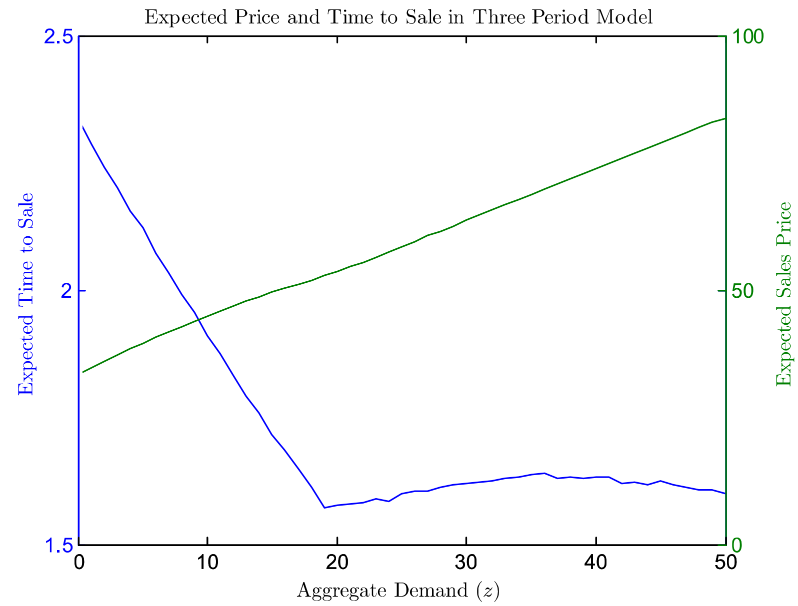
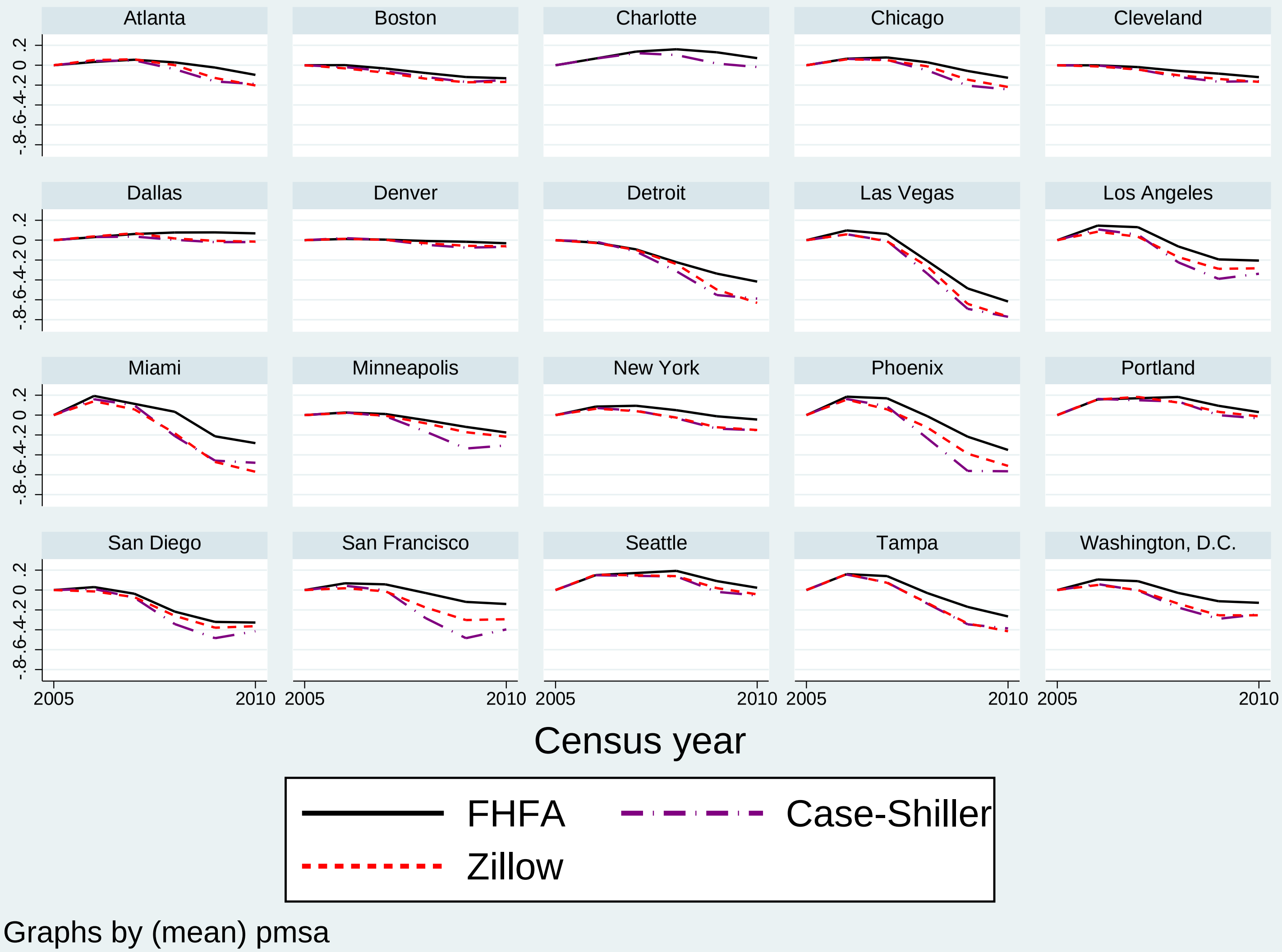


Figure A.4: Percent Price change from 2005: FHFA, Case-Shiller, and Zillow



B Derivations and Proofs

B.1 The p.d.f. of ψ

Let $\psi_L = x_L + z_L$ and $\psi_H = x_H + z_H$. Then the p.d.f. of ψ is:

$$f(\psi) = \begin{cases} \frac{\psi - \psi_L}{(x_H - x_L)(z_H - z_L)} & \text{if } \psi_L \leq \psi \leq \psi_L + \min(x_H - x_L, z_H - z_L) \\ \frac{1}{\max(x_H - x_L, z_H - z_L)} & \text{if } \psi_L + \min(x_H - x_L, z_H - z_L) \leq \psi \\ & \leq \psi_L + \max(x_H - x_L, z_H - z_L) \\ \frac{\psi_H - \psi}{(x_H - x_L)(z_H - z_L)} & \text{if } \psi_L + \max(x_H - x_L, z_H - z_L) \leq \psi \leq \psi_H \end{cases}$$

B.2 Seller's posterior belief about z

First note

$$f_\Psi(\psi|z) = \begin{cases} \frac{1}{(x_H - x_L)} & \text{if } z + x_L \leq \psi \leq z + x_H \\ 0 & \text{otherwise} \end{cases}$$

There are multiple cases to consider to verify that the seller's posterior distribution for z is $z \sim U[\tilde{z}_L, \tilde{z}_H]$.

Case 1: $x_H - x_L \geq z_H - z_L$

Case 1a: $\psi \leq x_L + z_H$. In this range, $f(\psi) = \frac{\psi - \psi_L}{(x_H - x_L)(z_H - z_L)}$. If $z < z_L$ or $z > z_H$, $f(z) = 0$. If $z < \psi - x_H$ or $z > \psi - x_L$, $f(\psi|z) = 0$. Therefore, $f(z|\psi) = 0$ if $z < \max(z_L, \psi - x_H) \equiv \tilde{z}_L$ or if $z > \min(z_H, \psi - x_L) \equiv \tilde{z}_H$. In the range $[\tilde{z}_L, \tilde{z}_H]$,

$$f_Z(z|\Psi = \psi) = \frac{f_\Psi(\psi|z) f_Z(z)}{f_\Psi(\psi)} = \frac{\frac{1}{x_H - x_L} \cdot \frac{1}{z_H - z_L}}{\frac{\psi - \psi_L}{(x_H - x_L)(z_H - z_L)}} = \frac{1}{\psi - \psi_L}$$

To see that $f_Z(z|\Psi = \psi)$ is a proper density, note that in this case $\tilde{z}_L = z_L$ and $\tilde{z}_H = \psi - x_L$, so that $\tilde{z}_H - \tilde{z}_L = \psi - x_L - z_L = \psi - \psi_L$.

Case 1b: $x_L + z_H \leq \psi \leq x_H + z_L$. In this case, $f(\psi) = \frac{1}{x_H - x_L}$, $f(z) = \frac{1}{z_H - z_L}$, $f(\psi|z) = \frac{1}{x_H - x_L}$, $\tilde{z}_L = z_L$, and $\tilde{z}_H = z_H$. Then in the range $[\tilde{z}_L, \tilde{z}_H]$,

$$f_Z(z|\Psi = \psi) = \frac{f_\Psi(\psi|z) f_Z(z)}{f_\Psi(\psi)} = \frac{\frac{1}{x_H - x_L} \cdot \frac{1}{z_H - z_L}}{\frac{1}{(x_H - x_L)}} = \frac{1}{z_H - z_L}$$

and $f_Z(z|\Psi = \psi) = 0$ elsewhere.

Case 1c: $x_H + z_L < \psi$. In this case $f(\psi) = \frac{\psi_H - \psi}{(x_H - x_L)(z_H - z_L)}$. $f(\psi|z) = \frac{1}{x_H - x_L}$,

$f(z) = \frac{1}{z_H - z_L}$, $\tilde{z}_L = \psi - x_H$, and $\tilde{z}_H = z_H$. Then in the range $[\tilde{z}_L, \tilde{z}_H]$,

$$f_Z(z|\Psi = \psi) = \frac{f_\Psi(\psi|z) f_Z(z)}{f_\Psi(\psi)} = \frac{\frac{1}{x_H - x_L} \cdot \frac{1}{z_H - z_L}}{\frac{\psi_H - \psi}{(x_H - x_L)(z_H - z_L}} = \frac{1}{\psi_H - \psi}$$

and $f_Z(z|\Psi = \psi) = 0$ elsewhere. Because $\tilde{z}_H - \tilde{z}_L = x_H + z_H - \psi = \psi_H - \psi$, the posterior distribution is a proper density.

Case 2: $z_H - z_L > x_H - x_L$.

Case 2a: $\psi < x_H + z_L$. In this case the proof is the same as in case 1a.

Case 2b: $x_H + z_L \leq \psi \leq x_L + z_H$. In this case $f(\psi) = \frac{1}{z_H - z_L}$, $f(z) = \frac{1}{z_H - z_L}$, $f(\psi|z) = \frac{1}{x_H - x_L}$, $\tilde{z}_L = \psi - x_H$, and $\tilde{z}_H = \psi - x_L$. Then in the range $[\tilde{z}_L, \tilde{z}_H]$,

$$f_Z(z|\Psi = \psi) = \frac{f_\Psi(\psi|z) f_Z(z)}{f_\Psi(\psi)} = \frac{\frac{1}{x_H - x_L} \cdot \frac{1}{z_H - z_L}}{\frac{1}{(z_H - z_L)}} = \frac{1}{x_H - x_L}$$

and $f_Z(z|\Psi = \psi) = 0$ elsewhere. Because $\tilde{z}_H - \tilde{z}_L = \psi - x_L - (\psi - x_H) = x_H - x_L$, the posterior distribution is a proper density.

Case 2c: $x_L + z_H < \psi$. In this case the proof is the same as in case 1c.

B.3 The seller's expectation of ψ_2 conditional on ψ_1

Let $\overline{x} = \frac{x_L + x_H}{2}$ and $\overline{\tilde{z}}_1 = \frac{\tilde{z}_{L,1} + \tilde{z}_{H,1}}{2}$. Then $E[\psi_2|\psi_1] = \overline{x} + \overline{\tilde{z}}_1$. If we further define $\overline{z} = \frac{z_L + z_H}{2}$, we can write the unconditional expectation of ψ as $E[\psi] = \overline{\psi} = \overline{x} + \overline{z}$. If $x_H - x_L \geq z_H - z_L$, we can write:

$$E[\psi_2|\psi_1] = \begin{cases} \frac{\psi_1 + x_H + z_L}{2} & \text{if } \psi_L \leq \psi_1 < x_L + z_H \\ \overline{\psi} & \text{if } x_L + z_H \leq \psi_1 \leq x_H + z_L \\ \frac{\psi_1 + x_L + z_H}{2} & \text{if } x_H + z_L < \psi_1 \leq \psi_H \end{cases}$$

Then for all $\psi_1 < \overline{\psi}$, $\psi_1 < E[\psi_2|\psi_1]$, while for all $\psi_1 \geq \overline{\psi}$, $\psi_1 \geq E[\psi_2|\psi_1]$.
If $z_H - z_L > x_H - x_L$:

$$E[\psi_2|\psi_1] = \begin{cases} \frac{\psi_1 + x_H + z_L}{2} & \text{if } \psi_L \leq \psi_1 < x_H + z_L \\ \psi_1 & \text{if } x_H + z_L \leq \psi_1 \leq x_L + z_H \\ \frac{\psi_1 + x_L + z_H}{2} & \text{if } x_L + z_H < \psi_1 \leq \psi_H \end{cases}$$

Then for all $\psi_1 < x_H + z_L$, $\psi_1 < E[\psi_2|\psi_1]$, while for all $\psi_1 \geq x_H + z_L$, $\psi_1 \geq E[\psi_2|\psi_1]$.

B.4 Proof of Proposition 1

We assume the seller will accept any offer $\psi_1 \geq E[\psi_2|\psi_1]$. Let $\bar{t}(z)$ denote the expected number of periods the seller leaves his house on the market. Then if $x_H - x_L \geq z_H - z_L$:

$$\begin{aligned}
\bar{t}(z) &= Pr(\psi_1 \geq \overline{\psi}) + 2Pr(\psi_1 < \overline{\psi}) \\
&= Pr(x_1 \geq \overline{\psi} - z) + 2Pr(x_1 < \overline{\psi} - z) \\
&= 1 - \frac{\overline{\psi} - x_L - z}{x_H - x_L} + 2\frac{\overline{\psi} - x_L - z}{x_H - x_L} \\
&= 1 + \frac{\overline{\psi} - x_L - z}{x_H - x_L}
\end{aligned}$$

Then

$$\frac{\partial \bar{t}}{\partial z} = \frac{-1}{x_H - x_L} < 0$$

Now consider the case in which $z_H - z_L > x_H - x_L$:

$$\begin{aligned}
\bar{t}(z) &= Pr(\psi_1 \geq x_H + z_L) + 2Pr(\psi_1 < x_H + z_L) \\
&= Pr(x_1 \geq x_H + z_L - z) + 2Pr(x_1 < x_H + z_L - z)
\end{aligned}$$

If $z > x_H - x_L + z_L$, $Pr(x_1 \geq x_H + z_L - z) = 1$, so $\bar{t} = 1 + 0 = 1$. If $z \leq x_H - x_L + z_L$,

$$\begin{aligned}
\bar{t}(z) &= 1 - \frac{x_H - x_L + z_L - z}{x_H - x_L} + 2\frac{x_H - x_L + z_L - z}{x_H - x_L} \\
&= 1 + \frac{x_H - x_L + z_L - z}{x_H - x_L} \\
&= 2 - \frac{z - z_L}{x_H - x_L}
\end{aligned}$$

Then when $z_H - z_L > x_H - x_L$,

$$\bar{t}(z) = \begin{cases} 2 - \frac{z - z_L}{x_H - x_L} & \text{if } z_L \leq z \leq x_H - x_L + z_L \\ 1 & \text{if } z > x_H - x_L + z_L \end{cases}$$

and

$$\frac{\partial \bar{t}}{\partial z} = \begin{cases} \frac{-1}{x_H - x_L} & \text{if } z_L \leq z \leq x_H - x_L + z_L \\ 0 & \text{if } x_H - x_L + z_L \leq z \leq z_H \end{cases}$$

Therefore, $\frac{\partial \bar{t}}{\partial z}$ must always be weakly negative.
Let $\overline{p}(z)$ denote the expected sales price for the house. When $x_H - x_L \geq z_H - z_L$,

$$\begin{aligned}
\overline{p}(z) &= Pr(\psi_1 \geq \overline{x} + \overline{z})E[\psi_1|\psi_1 \geq \overline{x} + \overline{z}] + Pr(\psi_1 < \overline{x} + \overline{z})E[\psi_2] \\
&= Pr(x_1 \geq \overline{x} + \overline{z} - z)E[\psi_1|\psi_1 \geq \overline{x} + \overline{z}] + Pr(x_1 < \overline{x} + \overline{z} - z)E[\psi_2] \\
&= (1 - \frac{\overline{x} - x_L + \overline{z} - z}{x_H - x_L})(\frac{\overline{x} + x_H + \overline{z} + z}{2}) + (\frac{\overline{x} - x_L + \overline{z} - z}{x_H - x_L})(\overline{x} + z) \\
&= \frac{\overline{x} + x_H + \overline{z} + z}{2} + (\frac{\overline{x} - x_L + \overline{z} - z}{x_H - x_L})(\frac{\overline{x} - x_H - \overline{z} + z}{2})
\end{aligned}$$

This implies $\frac{\partial \overline{p}}{\partial z} = 1 + \frac{\overline{z} - z}{x_H - x_L}$. To see that this must always be positive, consider the case $z = z_H$. Then $\frac{\partial \overline{p}}{\partial z} = 1 + \frac{\overline{z} - z_H}{x_H - x_L} = \frac{x_H - x_L - (z_H - \overline{z})}{x_H - x_L}$. $\overline{z} > z_L$, so $z_H - \overline{z} < z_H - z_L$. By assumption, $x_H - x_L \geq z_H - z_L$. Therefore the numerator of this expression is positive, and $\frac{\partial \overline{p}}{\partial z} > 0$ when $x_H - x_L \geq z_H - z_L$.
When $z_H - z_L > x_H - x_L$,

$$\begin{aligned}
\overline{p}(z) &= Pr(\psi_1 \geq x_H + z_L)E[\psi_1|\psi_1 \geq x_H + z_L] + Pr(\psi_1 < x_H + z_L)E[\psi_2] \\
&= Pr(x_1 \geq x_H + z_L - z)E[\psi_1|\psi_1 \geq x_H + z_L] + Pr(x_1 < x_H + z_L - z)E[\psi_2]
\end{aligned}$$

If $z \geq x_H - x_L + z_L$, $Pr(\psi_1 \geq x_H + z_L) = 1$, and $E[\psi_1|\psi_1 \geq x_H + z_L] = E[\psi_1]$. Then $\overline{p} = \overline{x} + z$, so $\frac{\partial \overline{p}}{\partial z} = 1$. If $z < x_H - x_L + z_L$, $Pr(\psi_1 \geq x_H + z_L) = \frac{z - z_L}{x_H - x_L}$. Then

$$\begin{aligned}
\overline{p} &= (\frac{z - z_L}{x_H - x_L})(x_H + \frac{z_L + z}{2}) + (1 - \frac{z - z_L}{x_H - x_L})(\overline{x} + z) \\
&= \overline{x} + z + (\frac{z - z_L}{x_H - x_L})(\frac{x_H - x_L + z_L - z}{2})
\end{aligned}$$

implying

$$\begin{aligned}
\frac{\partial \overline{p}}{\partial z} &= 1 + \frac{x_H - x_L + z_L - 2z + z_L}{2(x_H - x_L)} \\
&= \frac{3}{2} - \frac{z - z_L}{x_H - x_L}
\end{aligned}$$

Therefore when $z_H - z_L > x_H - x_L$,

$$\overline{p}(z) = \begin{cases} \overline{x} + z + (\frac{z - z_L}{x_H - x_L})(\frac{x_H - x_L + z_L - z}{2}) & \text{if } z_L \leq z \leq x_H - x_L + z_L \\ \overline{x} + z & \text{if } z > x_H - x_L + z_L \end{cases}$$

and

$$\frac{\partial \overline{p}}{\partial z} = \begin{cases} \frac{3}{2} - \frac{z-z_L}{x_H-x_L} & \text{if } z_L \leq z \leq x_H - x_L + z_L \\ 1 & \text{if } x_H - x_L + z_L \leq z \leq z_H \end{cases}$$

By assumption $z < x_H - x_L + z_L$, so $\frac{z-z_L}{x_H-x_L} < 1$. Therefore $\frac{\partial \overline{p}}{\partial z} > 0$ when $z_H - z_L > x_H - x_L$ and $z < x_H - x_L + z_L$, implying that $\frac{\partial \overline{p}}{\partial z} > 0$ in all cases. ■

B.5 Proof of Proposition 2

Let $\tilde{\psi}_L = \tilde{z}_{L,1} + x_L$ and $\tilde{\psi}_L = \tilde{z}_{L,1} + x_L$. The seller's posterior belief about the distribution of ψ_2 is then

$$\tilde{f}(\psi_2|\psi_1, \xi_1) = \begin{cases} \frac{\psi - \tilde{\psi}_L}{(x_H-x_L)(\tilde{z}_{H,1}-\tilde{z}_{L,1})} & \text{if } \tilde{\psi}_L \leq \psi \leq \tilde{\psi}_L + \min(x_H - x_L, \tilde{z}_{H,1} - \tilde{z}_{L,1}) \\ \frac{1}{\max(x_H-x_L, \tilde{z}_{H,1}-\tilde{z}_{L,1})} & \text{if } \tilde{\psi}_L + \min(x_H - x_L, \tilde{z}_{H,1} - \tilde{z}_{L,1}) \leq \psi \\ & \leq \tilde{\psi}_L + \max(x_H - x_L, \tilde{z}_{H,1} - \tilde{z}_{L,1}) \\ \frac{\tilde{\psi}_H - \psi}{(x_H-x_L)(\tilde{z}_{H,1}-\tilde{z}_{L,1})} & \text{if } \tilde{\psi}_L + \max(x_H - x_L, \tilde{z}_{H,1} - \tilde{z}_{L,1}) \leq \psi \leq \tilde{\psi}_H \end{cases}$$

Consider $E[\psi_2|\psi_1, \xi_1 = 1]$. If $x_H - \hat{x}_1 \geq z_H - z_L$ (call this case 1),

$$E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)] = \begin{cases} \overline{x} + \frac{\psi_1 - \hat{x}_1 + z_L}{2} & \text{if } \hat{x}_1 + z_L \leq \psi_1 \leq \hat{x}_1 + z_H \\ \overline{x} + \overline{z} & \text{if } \hat{x}_1 + z_H \leq \psi_1 \leq x_H + z_L \\ \overline{x} + \overline{z} + \frac{\psi_1 - x_H - z_L}{2} & \text{if } x_H + z_L \leq \psi_1 \leq \psi_H \end{cases}$$

If $z_H - z_L > x_H - \hat{x}_1$ (case 2),

$$E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)] = \begin{cases} \overline{x} + \frac{\psi_1 - \hat{x}_1 + z_L}{2} & \text{if } \hat{x}_1 + z_L \leq \psi_1 \leq x_H + z_L \\ \psi_1 - \frac{\hat{x}_1 - x_L}{2} & \text{if } x_H + z_L \leq \psi_1 \leq \hat{x}_1 + z_H \\ \frac{\psi_1 + x_L + z_H}{2} & \text{if } \hat{x}_1 + z_H \leq \psi_1 \leq \psi_H \end{cases}$$

In case 1, if $\hat{x}_1 + z_H \leq \overline{x} + \overline{z}$ (call this case 1a), $\psi_1 \geq E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ when $\psi_1 \geq \overline{x} + \overline{z}$ and $\psi_1 < E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ otherwise. If $\hat{x}_1 + z_H > \overline{x} + \overline{z}$ (case 1b), $\psi_1 \geq E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ when $\psi_1 \geq x_L + x_H - \hat{x}_1 + z_L$ and $\psi_1 < E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ otherwise. In case 2 $\psi_1 \geq E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ when $\psi_1 \geq x_L + x_H - \hat{x}_1 + z_L$ and $\psi_1 < E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ otherwise.

Again, we assume the seller accepts any offer $\psi_1 \geq E[\psi_2|\psi_1, \xi_1]$. Recall that the seller always rejects an offer when the realtor recommends 'reject'. Then in case 1a ($x_H - \hat{x}_1 \geq$

$z_H - z_L$ and $\hat{x}_1 + z_H \leq \overline{x} + \overline{z}$),

$$\begin{aligned}
\overline{t}(z) &= 2Pr(\xi_1 = 0) + Pr(\psi_1 \geq \overline{x} + \overline{z} \cap \xi_1 = 1) + 2Pr(\psi_1 < \overline{x} + \overline{z} \cap \xi_1 = 1) \\
&= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(Pr(x_1 \geq \overline{x} + \overline{z} - z|\xi_1 = 1) + 2Pr(x_1 < \overline{x} + \overline{z} - z|\xi_1 = 1)) \\
&= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(1 - \frac{\overline{x} - \hat{x}_1 + \overline{z} - z}{x_H - \hat{x}_1} + 2(\frac{\overline{x} - \hat{x}_1 + \overline{z} - z}{x_H - \hat{x}_1})) \\
&= 1 + \frac{\hat{x}_1 - x_L}{x_H - x_L} + (\frac{x_H - \hat{x}_1}{x_H - x_L})(\frac{\overline{x} - \hat{x}_1 + \overline{z} - z}{x_H - \hat{x}_1}) \\
&= 1 + \frac{\overline{x} - x_L + \overline{z} - z}{x_H - x_L}
\end{aligned}$$

Therefore in case 1a,

$$\frac{\partial \overline{t}}{\partial z} = \frac{-1}{x_H - x_L} < 0$$

In cases 1b and 2 ($\hat{x}_1 + z_H > \overline{x} + \overline{z}$),

$$\begin{aligned}
\overline{t}(z) &= 2Pr(\xi_1 = 0) + Pr(\psi_1 \geq x_L + x_H - \hat{x}_1 + z_L \cap \xi_1 = 1) + 2Pr(\psi_1 < x_L + x_H - \hat{x}_1 + z_L \cap \xi_1 = 1) \\
&= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(Pr(x_1 \geq x_L + x_H - \hat{x}_1 + z_L - z|\xi_1 = 1) \\
&+ 2Pr(x_1 < x_L + x_H - \hat{x}_1 + z_L - z|\xi_1 = 1))
\end{aligned}$$

If $z > x_L + x_H - 2\hat{x}_1 + z_L$, $Pr(x_1 \geq x_L + x_H - \hat{x}_1 + z_L - z|\xi_1 = 1) = 1$. In that case $\overline{t} = 2Pr(\xi_1 = 0) + Pr(\xi_1 = 1) = 2\frac{(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L}) = 1 + \frac{\hat{x}_1 - x_L}{x_H - x_L}$. If $z < x_L + x_H - 2\hat{x}_1 + z_L$,

$$\begin{aligned}
\overline{t}(z) &= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(1 - \frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1} + 2(\frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1})) \\
&= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(1 + \frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1}) \\
&= 1 + \frac{\hat{x}_1 - x_L}{x_H - x_L} + (\frac{x_H - \hat{x}_1}{x_H - x_L})(\frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1}) \\
&= 1 + \frac{x_H - \hat{x}_1 + z_L - z}{x_H - x_L}
\end{aligned}$$

Then in cases 1b and 2,

$$\bar{t}(z) = \begin{cases} 1 + \frac{x_H - \hat{x}_1 + z_L - z}{x_H - x_L} & \text{if } z_L \leq z < x_L + x_H - 2\hat{x}_1 + z_L \\ 1 + \frac{\hat{x}_1 - x_L}{x_H - x_L} & \text{if } x_L + x_H - 2\hat{x}_1 + z_L \leq z \leq z_H \end{cases}$$

and

$$\frac{\partial \bar{t}}{\partial z} = \begin{cases} \frac{-1}{x_H - x_L} & \text{if } z_L \leq z < x_L + x_H - 2\hat{x}_1 + z_L \\ 0 & \text{if } x_L + x_H - 2\hat{x}_1 + z_L \leq z \leq z_H \end{cases}$$

In case 2 ($z_H - z_L > x_H - \hat{x}_1$),

$$\begin{aligned} \bar{t}(z) &= 2Pr(\xi_1 = 0) + Pr(\psi_1 \geq x_L + x_H - \hat{x}_1 + z_L \cap \xi_1 = 1) + 2Pr(\psi_1 < x_L + x_H - \hat{x}_1 + z_L \cap \xi_1 = 1) \\ &= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(Pr(x_1 \geq x_L + x_H - \hat{x}_1 + z_L - z | \xi_1 = 1) \\ &+ 2Pr(x_1 < x_L + x_H - \hat{x}_1 + z_L - z | \xi_1 = 1)) \end{aligned}$$

If $z > x_L + x_H - 2\hat{x}_1 + z_L$, $Pr(x_1 \geq x_L + x_H - \hat{x}_1 + z_L - z | \xi_1 = 1) = 1$. In that case $\bar{t} = 2Pr(\xi_1 = 0) + Pr(\xi_1 = 1) = 2\frac{(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L}) = 1 + \frac{\hat{x}_1 - x_L}{x_H - x_L}$. Therefore $\frac{\partial \bar{t}}{\partial z} = 0$. If $z < x_L + x_H - 2\hat{x}_1 + z_L$,

$$\begin{aligned} \bar{t}(z) &= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(1 - \frac{x_L + x_H - \hat{x}_1 + z_L - z}{x_H - \hat{x}_1} + 2(\frac{x_L + x_H - \hat{x}_1 + z_L - z}{x_H - \hat{x}_1})) \\ &= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(1 + \frac{x_L + x_H - \hat{x}_1 + z_L - z}{x_H - \hat{x}_1}) \end{aligned}$$

In this case,

$$\frac{\partial \bar{t}}{\partial z} = (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})\frac{-1}{x_H - \hat{x}_1} < 0$$

Therefore, in case 2

$$\frac{\partial \bar{t}}{\partial z} = \begin{cases} (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})\frac{-1}{x_H - \hat{x}_1} & \text{if } z_L \leq z < x_L + x_H - 2\hat{x}_1 + z_L \\ 0 & \text{if } x_L + x_H - 2\hat{x}_1 + z_L \leq z \leq z_H \end{cases}$$

Thus, $\frac{\partial \bar{t}}{\partial z}$ must always be weakly negative.

In case 1a,

$$\begin{aligned}
\overline{p}(z) &= Pr(\xi_1 = 0)E[\psi_2] + Pr(\xi_1 = 1)\left[Pr(\psi_1 \geq \overline{x} + \overline{z}|\xi_1 = 1)E[\psi_1|\psi_1 \geq \overline{x} + \overline{z}, \xi_1 = 1]\right. \\
&\quad \left. + Pr(\psi_1 < \overline{x} + \overline{z}|\xi_1 = 1)E[\psi_2])\right] \\
&= (\frac{\hat{x}_1 - x_L}{x_H - x_L})(\overline{x} + z) + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})\left[(1 - \frac{\overline{x} - \hat{x}_1 + \overline{z} - z}{x_H - \hat{x}_1})(z + \frac{\overline{x} + x_H + \overline{z} - z}{2})\right. \\
&\quad \left. + (\frac{\overline{x} - \hat{x}_1 + \overline{z} - z}{x_H - \hat{x}_1})(\overline{x} + z))\right] \\
&= \frac{\frac{1}{2}(\overline{x}^2 + x_H^2 - \overline{z}^2 - z^2) - x_L\overline{x} + (x_H - x_L + \overline{z})z}{x_H - x_L}
\end{aligned}$$

Therefore in case 1a,

$$\frac{\partial \overline{p}}{\partial z} = \frac{x_H - x_L + \overline{z} - z}{x_H - x_L}$$

To see that this must be positive, consider the case in which $z = z_H$. Then

$$\frac{\partial \overline{p}}{\partial z} = \frac{x_H - x_L + \overline{z} - z_H}{x_H - x_L} = \frac{x_H - x_L - \frac{z_H - z_L}{2}}{x_H - x_L}$$

By assumption $x_H - x_L \geq x_H - \hat{x}_1 \geq z_H - z_L$, so $\frac{\partial \overline{p}}{\partial z} \geq 0$ in case 1a. In cases 1b and 2, if $z > x_L + x_H - 2\hat{x}_1 + z_L$ (so that the seller will always accept the period 1 offer if the realtor recommends accept),

$$\begin{aligned}
\overline{p} &= Pr(\xi_1 = 0)E[\psi_2] + Pr(\xi_1 = 1)E[\psi_1|x_1 \geq \hat{x}_1] \\
&= (\frac{\hat{x}_1 - x_L}{x_H - x_L})(\overline{x} + z) + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(z + \frac{\hat{x}_1 + x_H}{2}) \\
&= \frac{(\hat{x}_1 - x_L)(x_L + x_H) + (x_H - \hat{x}_1)(\hat{x}_1 + x_H)}{2(x_H - x_L)} + z
\end{aligned}$$

If $z < x_L + x_H - 2\hat{x}_1 + z_L$,

$$\begin{aligned}
\overline{p}(z) &= Pr(\xi_1 = 0)E[\psi_2] + Pr(\xi_1 = 1)\left(Pr(\psi_1 \geq x_L + x_H - \hat{x}_1 + z_L | x_1 \geq \hat{x}_1)\right.\\
&\quad \times E[\psi_1 | \psi_1 \geq x_L + x_H - \hat{x}_1 + z_L, x_1 \geq \hat{x}_1] + Pr(\psi_1 < x_L + x_H - \hat{x}_1 + z_L | x_1 \geq \hat{x}_1)E[\psi_2]\left.\right)\\
&= (\frac{\hat{x}_1 - x_L}{x_H - x_L})(\overline{x} + z) + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})\Big[(1 - \frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1})\\
&\quad \times (z + \frac{x_L + x_H - \hat{x}_1 + z_L - z + x_H}{2}) + (\frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1})(\overline{x} + z)\Big]\\
&= (\frac{1}{x_H - x_L})\Big(- \frac{1}{2}(x_L^2 + \hat{x}_1^2 + z_L^2 + z^2) + (\overline{x} + z_L)\hat{x}_1 + (x_H + z_L)\overline{x}\\
&\quad - (x_H + z_L + z_H)x_L + (2x_H - \overline{x} - \hat{x}_1 - x_L + z_L)z\Big)
\end{aligned}$$

Then in case 1b,

$$\overline{p}(z) = \begin{cases} \frac{-\frac{1}{2}(x_L^2+\hat{x}_1^2+z_L^2+z^2)+(\overline{x}+z_L)\hat{x}_1+(x_H+z_L)\overline{x}-(x_H+z_L+z_H)x_L+(2x_H-\overline{x}-\hat{x}_1-x_L+z_L)z}{x_H-x_L} & \text{if } z_L \leq z \\ & < x_L + x_H - 2\hat{x}_1 + z_L \\ \frac{(\hat{x}_1-x_L)(x_L+x_H)+(x_H-\hat{x}_1)(\hat{x}_1+x_H)}{2(x_H-x_L)} + z & \text{if } x_L + x_H - 2\hat{x}_1 + z_L \leq z \\ & \leq z_H \end{cases}$$

and

$$\frac{\partial \overline{p}}{\partial z} = \begin{cases} \frac{2x_H-\overline{x}-\hat{x}_1-x_L+z_L-z}{x_H-x_L} & \text{if } z_L \leq z < x_L + x_H - 2\hat{x}_1 + z_L \\ 1 & \text{if } x_L + x_H - 2\hat{x}_1 + z_L \leq z \leq z_H \end{cases}$$

To see that $\frac{\partial \overline{p}}{\partial z} > 0$ when $z \leq x_L + x_H - 2\hat{x}_1 + z_L$, note that $2x_H - \overline{x} - \hat{x}_1 + z_L \geq x_L + x_H - 2\hat{x}_1 + z_L$ is equivalent to $x_H - x_L \geq \overline{x} - \hat{x}_1$, which must be true because $x_H \geq \overline{x}$ and $\hat{x}_1 \geq x_L$. Therefore $\frac{\partial \overline{p}}{\partial z} > 0$ in cases 1b and 2. ■