Price and Time to Sale Dynamics in the Housing Market

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Abstract

I show that a model of the house-selling process in which rational sellers possess incomplete information regarding the state of the housing market is consistent with the stylized facts regarding price and time to sale dynamics in the U.S. housing market. These results hold even when sellers employ realtors with complete information, provided the incentives of the seller and the realtor are not perfectly aligned.

1 Introduction

It is a stylized fact of the market for existing homes that there is a strong positive correlation between sales prices and sales volumes and that there is a strong negative correlation between sales prices and the average time houses stay on the market (hereafter time on market or time to sale). Figure 1 illustrates this pattern using monthly U.S. data on sales volumes of existing homes, median sales prices of existing homes, and months' supply of homes on the market for the period January 1999 to January 2009.[1] Months' supply of homes on the market is the number of homes listed as being for sale at the end of the month divided by the number of sales that month. It is used as a proxy for the average time on market because nationally representative data for time on market is unavailable. Consistent with the stylized fact, the contemporaneous correlation between sales volumes and median prices is 0.688, with a bootstrapped 95% confidence interval of 0.659 to 0.727. The contemporaneous correlation between months' supply of houses on the market and median prices is -0.644, with a bootstrapped 95% confidence interval of -0.574 to -0.669[2]. Although time on market data is not

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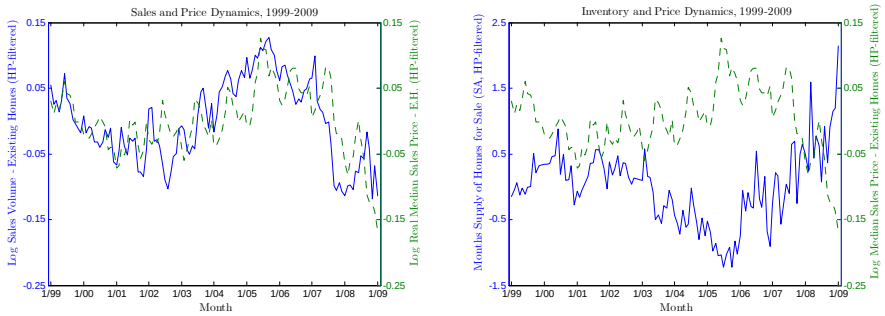
[1] Sales volume and price data are from the National Association of Realtors and inventory data is from the U.S. Census Bureau.

[2] I used a naive bootstrap with 5,000 repetitions. Prices were deflated with the seasonally adjusted CPI less shelter.

available at a national level, studies on the city level find a similar pattern when considering time on market directly. For instance, Genesove and Mayer (2001) document that in the Boston condominium market, fewer than 30 percent of listed units sold within 180 days during the trough year of 1992, while in 1997, after the market had recovered, more than 60 percent of new listings sold within 180 days. Miller and Sklarz (1986) document similar trends in Hawaii and Salt Lake City. Regarding the correlation between prices sales volumes, Stein (1995) shows that between 1986 and 1992, a 10 percent drop in prices reduces sales volumes by approximately 1.6 million units in a time span with average volume of three to four million units. Ortalo-Magne and Rady (1998) show the same qualitative pattern holds in the U.K. The robustness of this pattern leads Genesove and Mayer (1997) to describe it as "one of the most distinctive and puzzling macro features of the market for existing homes."

Figure 1: Sales Volumes, Home Prices, and Months' Supply 1999-2009



This paper offers a novel explanation for the stylized fact in which home sellers are not fully informed about conditions in the housing market. The assumption of incomplete information seems plausible in light of the infrequency with which most households participate in the real estate market. Anily et al. (1999) report that the expected total time for a household to reside in an owner-occupied unit is 13 years. Case and Shiller (2003) note the relative absence of professional traders in the housing market: "Buyers and sellers in the housing market are overwhelmingly amateurs, who have little experience with trading. High transactions costs, moral hazard problems, and government subsidization of owner-occupied homes have kept professional speculators out of the market." Therefore many sellers may begin the process of selling their homes with poor information regarding the state of real estate demand.

Another central assumption in the model is that sellers face idiosyncratic variation in the offers they receive. Such variation may come from differences in the match quality between the potential buyer and the house, or it may come from variations in buyers' eagerness to transact quickly. Unfortunately, there is not much data concerning the distribution of offers sellers receive. For instance, Haurin (1988) reports, "Information on the willingness to pay for a particular house is probably difficult to obtain. Data on the stream of formal offers for

a house do not exist, and many informal offers are not recorded." Merlo and Ortalo-Magne (2004) present complete histories of formal offers for 780 real estate transactions in the U.K. during the period 1995-1998. They claim this feature of their data is novel, and indeed I am unable to locate any other such data. The data show that within negotiations between a buyer and seller, buyers make an average of 1.6 offers, and that as a percentage of the property's list price, the second offer is on average 3.3% than the first offer and the third offer is on average 2.1% higher than the second. Furthermore, the first offer of the fourth potential buyer to bid on a property is on average 3.1% higher relative to list price than the first offer of the first buyer to bid on the property. Despite the lack of data, the existence of idiosyncratic variation in offers seems well-accepted in the theoretical literature. For instance, Haurin discusses the optimal decision rule for sellers when the idiosyncratic variation in offers is distributed uniformly and when it is distributed normally.

Finally, the model considers the interaction between sellers who have incomplete information about the state of the real estate market and realtors who have complete information the state of the market. Earlier research has emphasized the presence of realtors and brokers who have extensive experience in the marketplace. For instance, Haurin argues that the presence of informed agents justifies modeling sellers as if they know the distribution of offers. Recently, however, Levitt and Syverson (2002,2008) have shown that because realtors' typical compensation structure leads to misalignment between realtors' and sellers' incentives, realtors may not be able to convey their knowledge of the real estate market to sellers efficiently. They show that empirically, realtors seem to encourage their clients to sell their homes more quickly than would be optimal for a fully informed seller. The present paper considers how the interaction between realtors and sellers affects the relationship between housing prices and time to sale, and extends Levitt and Syverson's theoretical framework to allow for more than two periods. Expected time to sale remains negatively correlated with house prices even when fully informed realtors are included in the model.

2 Related Literature

There are three strands of related literature that attempt to explain the stylized fact. The first is the search literature, as in Wheaton (1990) and Diaz and Jerez (2009). Although explaining the correlations between prices, sales, and time on market is not a central aim of Wheaton's model, the model matches the observed correlations rather well. Exogenous shocks to the vacancy rate and to the flow benefit to owning a suitable home both generate negative co-movement between prices and time to sale in steady state. However, two related assumptions in Wheaton's model that drive these results, first that buyers' search intensity is endogenous and second that the trade surplus between buyers and sellers is divided according to a symmetric Nash bargain, seem to have fallen out of favor in the more recent housing search literature. Leung and Zhang (2007)

and Diaz and Jerez (2009) argue that the Nash bargaining assumption may be inappropriate because it implies the share of the surplus sellers receive does not depend on the ratio of buyers to sellers in the market. Furthermore, there is evidence (Merlo and Ortalo-Magne (2004), Genesove and Mayer (2001)) that the list price a seller posts is closely related to time to sale and eventual sales price. Therefore, these authors have argued that a competitive or directed search paradigm, in which sellers pre-commit to selling at the list price, is more appropriate than Nash bargaining for modeling the housing market.

Diaz and Jerez argue that their model of competitive search in the housing market matches the stylized facts without the assumptions of Nash bargaining and engogenous search intensity. Shocks to the vacancy rate and the rate at which households become mismatched from their homes generate a negative correlation between equilibrium prices and time to sale in their model, but shocks to the degree of search frictions in the market and to the flow benefit households receive from owning a well-matched home do not. Specifically, an increase in the value of owning a well-matched home increases prices but has no effect on time to sale in the Diaz and Jerez model. Therefore, to the extent that changes in the flow benefit parameter capture shocks to "housing demand" and these demand shocks are important in the housing market, it seems that the Diaz and Jerez model cannot fully explain the robust correlations between housing prices, sales volumes, and time on the market.

The second strand of related literature, as in Stein (1995) and Genesove and Mayer (1997), emphasizes the role of downpayment requirements and credit contstraints. Stein presents a model in which homeowners would like to buy a new house, but must pay a percentage of the purchase price as a downpayment. Because homeowners' chief asset is equity in their current homes, a drop in housing prices can leave them liquidity constrained and unable to make the down payment on a new home. Therefore, when prices fall, the percentage of households who purchase a new home falls, even though all households in the model would prefer to move if they were not liquidity constrained. Genesove and Mayer (1997) provide empirical evidence that a homeowner's equity position has a significant impact on his selling experience. In their data, a seller with a loan-to-value ratio of 100 percent (or equity of 0 percent) leaves his house on the market 15 percent longer than a seller with a loan-to-value ratio of 80 percent (or equity of 20 percent). However, in later research the same authors find that even after controlling for credit constraints, sellers who face an expected nominal loss on their homes set significantly higher asking prices and face a longer time to sale than sellers facing no loss. Thus, although credit constraints appear to explain the observed correlations partially, quantitatively it seems that they cannot explain the stylized facts entirely.

The final strand of related literature is behavioral, as in Genesove and Mayer (2001) and Albrecht et al. (2007). Genesover and Mayer argue that according to prospect theory, home sellers use the price they paid for their house as a reference point to evaluate offers. Compared to selling for the same price that they paid, sellers lose more utility by selling for a nominal loss than they gain by selling for an equally-sized gain. The resulting loss aversion causes sellers who

face an expected loss on their home to set above average list prices, resulting in a higher eventual sale price but a longer time to sale. The aggregate result of this behavior is that sales volume is lower and time to sale is higher in markets when prices have been falling, consistent with the stylized fact. Albrecht et al. model buyers and sellers as having time-inconsistent preferences: they enter the market in a "relaxed" state in which they do not much mind searching, but eventually they transition into a "desperate" state in which they have a stronger incentive to transact quickly. In this model as a house takes longer to sell, the more likely it is that the seller has become desperate and is thus willing to accept a lower price. Therefore the expected sales price falls with time to sale. Behavioral explanations can therefore offer plausible accounts of price and time to sale dynamics in the housing market, but they come at the cost of abandoning the traditional fully rational maximizing agent framework of economics.

3 The Model

3.1 The Offer Distribution

The model in this paper is a partial equilibrium model: the distribution of offers that sellers receive is taken to be exogenous.Sellers receive one offer per period, and there is no recall. Offers are the sum of an 'aggregate demand' component, z, which is constant in every period, and an idiosyncratic component, x_t, which is distributed i.i.d. each period. z is distributed $U[z_L, z_H]$, and x_t is distributed $U[x_L, x_H]$. z and x_t are independently distributed. Denoting the period t offer as ψ_t, we have $\psi_t = z + x_t$. Let $\psi_L = x_L + z_L$ and $\psi_H = x_H + z_H$. Then the p.d.f. of ψ is:

$$f(\psi) = \begin{cases} \frac{\psi - \psi_L}{(x_H - x_L)(z_H - z_L)} & \text{if } \psi_L \leq \psi \leq \psi_L + \min(x_H - x_L, z_H - z_L) \\ \frac{1}{\max(x_H - x_L, z_H - z_L)} & \text{if } \psi_L + \min(x_H - x_L, z_H - z_L) \leq \psi \\ & \leq \psi_L + \max(x_H - x_L, z_H - z_L) \\ \frac{\psi_H - \psi}{(x_H - x_L)(z_H - z_L)} & \text{if } \psi_L + \max(x_H - x_L, z_H - z_L) \leq \psi \leq \psi_H \end{cases}$$

3.2 The Model with no Realtor

First I will consider a two-period model with no realtor. Sellers receive one offer each period. If they do not accept the first offer they must accept the second one. Sellers are risk-neutral, perfectly patient, and do not bear any flow cost of leaving their homes on the market. Therefore, a seller's goal is simply to sell for the highest price possible. Accordingly, a seller will accept the period 1 offer, ψ_1, if and only if it is greater than or equal to the expectation of the period 2 offer, $E[\psi_2]$. However, sellers cannot observe the state of market demand z or the idiosyncratic component of the offer x_t. Therefore, sellers must infer the state of z using Bayes' Theorem:

$$f_Z(z|\Psi = \psi) = \frac{f_{Z,\Psi}(z, \psi)}{f_\Psi(\psi)} = \frac{f_\Psi(\psi|z) f_Z(z)}{f_\Psi(\psi)}$$

Proposition 1: Define $\tilde{z}_{L,1} = \max(z_L, \psi_1 - x_H)$ and $\tilde{z}_{H,1} = \min(z_H, \psi_1 - x_L)$. Then the seller's posterior belief about the distribution of z conditional on ψ_1 is $z \sim U[\tilde{z}_{L,1}, \tilde{z}_{H,1}]$.

Proof: See appendix. ∎

Let $\overline{x} = \frac{x_L + x_H}{2}$ and $\tilde{\overline{z}}_1 = \frac{\tilde{z}_{L,1} + \tilde{z}_{H,1}}{2}$. Then $E[\psi_2|\psi_1] = \overline{x} + \tilde{\overline{z}}_1$. If we further define $\overline{z} = \frac{z_L + z_H}{2}$, we can write the unconditional expectation of ψ as $E[\psi] = \overline{\psi} = \overline{x} + \overline{z}$. If $x_H - x_L \geq z_H - z_L$, we can write:

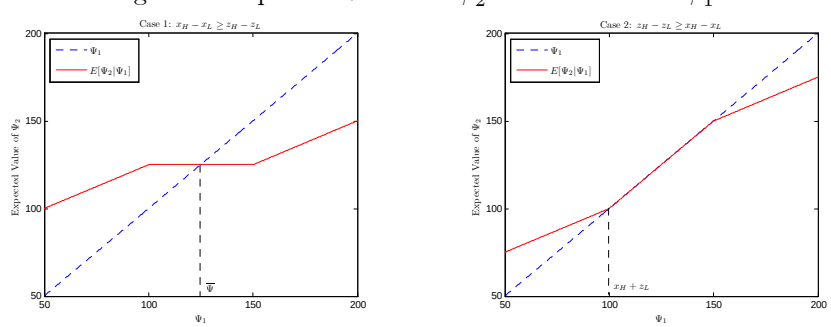
$$E[\psi_2|\psi_1] = \begin{cases} \frac{\psi_1 + x_H + z_L}{2} & \text{if } \psi_L \leq \psi_1 < x_L + z_H \\ \overline{\psi} & \text{if } x_L + z_H \leq \psi_1 \leq x_H + z_L \\ \frac{\psi_1 + x_L + z_H}{2} & \text{if } x_H + z_L < \psi_1 \leq \psi_H \end{cases}$$

Then for all $\psi_1 < \overline{\psi}$, $\psi_1 < E[\psi_2|\psi_1]$, while for all $\psi_1 \geq \overline{\psi}$, $\psi_1 \geq E[\psi_2|\psi_1]$.
If $z_H - z_L > x_H - x_L$:

$$E[\psi_2|\psi_1] = \begin{cases} \frac{\psi_1 + x_H + z_L}{2} & \text{if } \psi_L \leq \psi_1 < x_H + z_L \\ \psi_1 & \text{if } x_H + z_L \leq \psi_1 \leq x_L + z_H \\ \frac{\psi_1 + x_L + z_H}{2} & \text{if } x_L + z_H < \psi_1 \leq \psi_H \end{cases}$$

Then for all $\psi_1 < x_H + z_L$, $\psi_1 < E[\psi_2|\psi_1]$, while for all $\psi_1 \geq x_H + z_L$, $\psi_1 \geq E[\psi_2|\psi_1]$. Figure 1 illustrates the seller's expectation of ψ_2 as a function of ψ_1 in both cases.

Figure 2: Expected Values of ψ_2 as a function of ψ_1

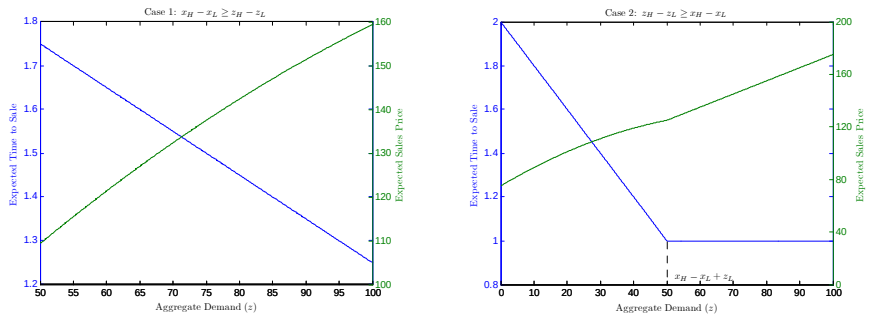


This allows us to state the main result of the model with no realtor.

Proposition 2: When the seller follows the optimal policy, the expected time to sale in the model with no realtor is weakly decreasing in the state of aggregate demand, z, while the expected sales price is strictly increasing in z.

Proof: See appendix. ∎

Figure 3: Expected Time to Sale and Expected Sales Price as a function of z



If the variance of the idiosyncratic component of the offer is greater than the variance of aggregate demand, then the expected time to sale will be falling for all values of z. If the variance of aggregate demand is greater, however, the expected time to sale will be constant for very low and very high values of aggregate demand, while for intermediate values expected time to sale will be falling in z. This section, although it presents an extremely simplified model of the home selling process, illustrates the essential mechanism by which the model generates a negative correlation between the strength of housing demand and the expected time to sale. When sellers are uncertain about the state of aggregate demand, they follow a reservation price strategy in period 1, whereas if they could observe z, they would ignore the aggregate demand component of the offer, which is stable over time, and make their decision based solely on the idiosyncratic component, x_1. In the incomplete information setup of this model, offers with middling values of x_1 will be above the seller's reservation price when z is high and below the reservation price when z is low. Therefore, the probability that the seller accepts the first period offer is an increasing function of z. It is by this mechanism that the model generates the negative correlation between the state of aggregate housing demand and the average time on the market.

3.3 A Two-Period Model with a Realtor

I will now introduce a realtor to the two-period model. It is assumed the realtor can observe the state of aggregate housing demand z directly. However, because of the incentive structure of the contract between the realtor and the seller, the realtor will not always be able to signal his knowledge of z to the seller in a credible way. A linear contract with parameter α is exogenously taken to be the only available contract between sellers and realtors in the model: upon the seller's acceptance of an offer ψ, the realtor receives a commission of $\alpha\psi$. It is assumed the seller cannot market his house without employing the realtor. If there is no sale in period 1, the realtor must pay flow cost c_R at the beginning of

period 2 in order to market the house[3]. The realtor can only communicate with the seller by advising him on whether or not to accept an offer after it has been received. As Levitt and Syverson (2002) argue, if the realtor is constrained to advising the seller on each offer only after it has been received, there is no way for the realtor to report the intensity of his preferences credibly. Therefore it is not overly restrictive to limit the realtor to recommending either 'accept' or 'reject' after each offer is received. Denote the realtor's recommendation about the time t offer as ξ_t, with $\xi_t = 0$ if the realtor recommends 'reject' and $\xi_t = 1$ if the realtor recommends 'accept'. Let $\tilde{f}(\psi_2|\psi_1, \xi_1)$ denote the seller's posterior belief about the distribution of ψ_2 conditional on the first period offer ψ_1 and the realtor's recommendation ξ_1. Denote the seller's period 1 policy function as $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$, with $\gamma_1 = 0$ indicating that the seller rejects the period 1 offer and $\gamma_1 = 1$ indicating that the seller accepts the period 1 offer. Then the seller's value function can be written as:

$$V_S(\psi_1, \xi_1, 1) = \overset{\max}{\gamma_1}(1-\alpha)\{\psi_1, E[\psi_2|\tilde{f}(\psi_2|\psi_1, \xi_1)]\}$$

$$V_S(\psi_2, \xi_2, 2) = (1-\alpha)\psi_2$$

The realtor's value function can be written:

$$V_R(\psi_1, z, 1) = \overset{\max}{\xi_1}\{\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, 0)\alpha\psi_1 + (1-\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, 0)))(E[V_R(\psi_2, z, 2)]),$$
$$\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, 1)\alpha\psi_1 + (1-\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, 1)))(E[V_R(\psi_2, z, 2)])\}$$

$$V_R(\psi_2, z, 2) = -c_R + \alpha\psi_2$$

Definition: a **Bayesian Nash equilibrium** of the game between realtors and sellers is a policy function $\xi_1(\psi_1, z)$ for the realtor, a policy function $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$ for the seller, and a belief updating strategy $\tilde{f}(\psi_2|\psi_1, \xi_1)$ for the seller such that:
1. $\xi_1(\psi_1, z)$ maximizes the realtor's value function for all (ψ_1, z), taking $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$ as given;
2. $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$ maximizes the seller's value function for all ψ_1, ξ_1 taking $\xi_1(\psi_1, z)$ as given; and
3. $\tilde{f}(\psi_2|\psi_1, \xi_1)$ is consistent with $\xi_1(\psi_1, z)$.

Consider the seller's optimal policy from the realtor's perspective (i.e. what the realtor would choose if he could decide which offers the seller would accept and which he would reject). The realtor's value function in period 1 can be re-written

$$\begin{aligned} V_r(\psi_1, z, 1) &= \max\{\alpha\psi_1, E[\alpha\psi_2 - c_r|z]\} \\ &= \max\{\alpha(z + x_1), E[\alpha(z + x_2) - c_r|z]\} \\ &= \alpha z + \max\{\alpha x_1, \alpha E[x_2] - c_r\} \\ &= \alpha z + \max\{\alpha x_1, \alpha\overline{x} - c_r\} \end{aligned}$$

[3] Implicitly, one could imagine that c_R is a flow cost the realtor must pay at the beginning of each period to market the house, but the cost paid at the beginning of period 1 is sunk and does not affect the realtor's maximization problem.

Therefore in period 1 the realtor would prefer that the seller accept any offer ψ_1 such that $\alpha x_1 \geq \alpha\overline{x} - c_r$, or equivalently, $x_1 \geq \overline{x} - \frac{c_r}{\alpha}$. Denote $\overline{x} - \frac{c_r}{\alpha}$ as $\hat{x}_1$ (I will assume that α and c_R are such that $\hat{x}_1 \geq x_L$).

Then the following is a Bayesian Nash equilibrium of the two-period game between the realtor and the seller. The realtor reports 'accept' ($\xi_1 = 1$) for any offer such that $x_1 \geq \hat{x}_1$ and 'reject' ($\xi_1 = 0$) for any offer such that $x_1 < \hat{x}_1$. Define $\tilde{x}_{L,1}$ and $\tilde{x}_{H,1}$ as follows:

$$\tilde{x}_{L,1} = \begin{cases} x_L & \text{if } \gamma_1 = 0 \\ \hat{x}_1 & \text{if } \gamma_1 = 1 \end{cases}$$

$$\tilde{x}_{H,1} = \begin{cases} \hat{x}_1 & \text{if } \gamma_1 = 0 \\ x_H & \text{if } \gamma_1 = 1 \end{cases}$$

Further define $\tilde{z}_{L,1} = \max(z_L, \psi_1 - \tilde{x}_{H,1})$ and $\tilde{z}_{H,1} = \min(z_H, \psi_1 - \tilde{x}_{L,1})$. Then the seller's belief updating strategy is

$$\tilde{f}(z|\psi_1, \xi_1) = \begin{cases} \frac{1}{\tilde{z}_{H,1} - \tilde{z}_{L,1}} & \text{if } \tilde{z}_{L,1} \leq z \leq \tilde{z}_{H,1} \\ 0 & \text{otherwise} \end{cases}$$

In other words, the seller's posterior belief is that $z \sim U[\tilde{z}_{L,1}, \tilde{z}_{H,1}]$.[4] Let $\tilde{\psi}_L = \tilde{z}_{L,1} + x_L$ and $\tilde{\psi}_L = \tilde{z}_{L,1} + x_L$. The seller's posterior belief about the distribution of ψ_2 is then

$$\tilde{f}(\psi_2|\psi_1, \xi_1) = \begin{cases} \frac{\psi - \tilde{\psi}_L}{(x_H - x_L)(\tilde{z}_{H,1} - \tilde{z}_{L,1})} & \text{if } \tilde{\psi}_L \leq \psi \leq \tilde{\psi}_L + \min(x_H - x_L, \tilde{z}_{H,1} - \tilde{z}_{L,1}) \\ \frac{1}{\max(x_H - x_L, \tilde{z}_{H,1} - \tilde{z}_{L,1})} & \text{if } \tilde{\psi}_L + \min(x_H - x_L, \tilde{z}_{H,1} - \tilde{z}_{L,1}) \leq \psi \\ & \leq \tilde{\psi}_L + \max(x_H - x_L, \tilde{z}_{H,1} - \tilde{z}_{L,1}) \\ \frac{\tilde{\psi}_H - \psi}{(x_H - x_L)(\tilde{z}_{H,1} - \tilde{z}_{L,1})} & \text{if } \tilde{\psi}_L + \max(x_H - x_L, \tilde{z}_{H,1} - \tilde{z}_{L,1}) \leq \psi \leq \tilde{\psi}_H \end{cases}$$

It is straightforward to see that when $\xi_1 = 0$, $E[\psi_2|\tilde{f}(\psi_2|\psi_1, \xi_1)] > \psi_1$: because $\hat{x}_1 < \overline{x}$, $\xi_1 = 0$ implies that $x_1 < \overline{x} = E[x_2]$. Then $z + x_1 < z + E[x_2]$, or $\psi_1 < E[\psi_2]$. Therefore, the seller will always reject the period 1 offer when the realtor recommends 'reject'. However, the opposite is not true: the seller will sometimes reject an offer when the realtor recommends 'accept'. Consider $E[\psi_2|\psi_1, \xi_1 = 1]$. If $x_H - \hat{x}_1 \geq z_H - z_L$ (call this case 1),

$$E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)] = \begin{cases} \overline{x} + \frac{\psi_1 - \hat{x}_1 + z_L}{2} & \text{if } \hat{x}_1 + z_L \leq \psi_1 \leq \hat{x}_1 + z_H \\ \overline{x} + \overline{z} & \text{if } \hat{x}_1 + z_H \leq \psi_1 \leq x_H + z_L \\ \overline{x} + \overline{z} + \frac{\psi_1 - x_H - z_L}{2} & \text{if } x_H + z_L \leq \psi_1 \leq \psi_H \end{cases}$$

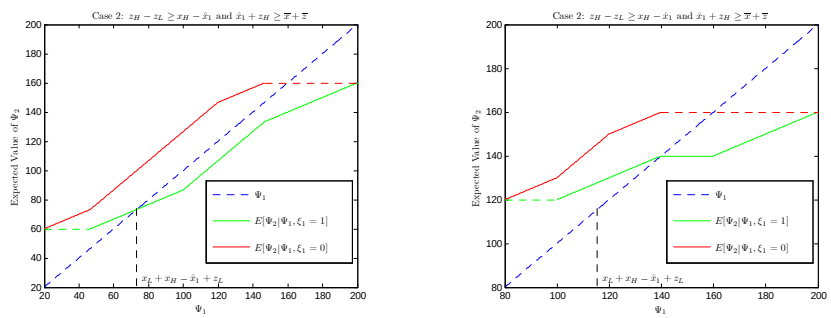
If $z_H - z_L > x_H - \hat{x}_1$ (case 2),

$$E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)] = \begin{cases} \overline{x} + \frac{\psi_1 - \hat{x}_1 + z_L}{2} & \text{if } \hat{x}_1 + z_L \leq \psi_1 \leq x_H + z_L \\ \psi_1 - \frac{\hat{x}_1 - x_L}{2} & \text{if } x_H + z_L \leq \psi_1 \leq \hat{x}_1 + z_H \\ \frac{\psi_1 + x_L + z_H}{2} & \text{if } \hat{x}_1 + z_H \leq \psi_1 \leq \psi_H \end{cases}$$

[4] I do not currently offer a proof but it would follow the proof of Proposition 1 very closely.

In case 1, if $\hat{x}_1 + z_H \leq \overline{x} + \overline{z}$ (call this case 1a), $\psi_1 \geq E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ when $\psi_1 \geq \overline{x} + \overline{z}$ and $\psi_1 < E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ otherwise. If $\hat{x}_1 + z_H > \overline{x} + \overline{z}$ (case 1b), $\psi_1 \geq E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ when $\psi_1 \geq x_L + x_H - \hat{x}_1 + z_L$ and $\psi_1 < E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ otherwise. In case 2 $\psi_1 \geq E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ when $\psi_1 \geq x_L + x_H - \hat{x}_1 + z_L$ and $\psi_1 < E[\psi_2|\tilde{f}(\psi_2|\psi_1, 1)]$ otherwise. Figure 4 illustrates the seller's belief about $E[\psi_2]$ as a function of ψ_1 and ξ_1 for the three cases.

Figure 4: Expected Values of ψ_2 as a function of ψ_1



To verify that reporting his own preference truthfully is a best response for the realtor, note that in equilibrium, the realtor's recommendation weakly increases the chance that the seller will take the realtor's preferred action. In case 1a, the realtor's recommendation will never change the seller's action, so any policy the realtor follows is a best response (note that where the expected value of ψ_2 lines are dashed in Figure 4, such a recommendation for the given value of ψ_1 is an off-equilibrium path event). In cases 1b and 2, for very low and very high values of ψ_1, the realtor's recommendation will also not affect the seller's action, so the realtor's policy must be a best response in these cases. However, for medium values of ψ_1, the expected value of ψ_2 when the realtor recommends 'accept' is below ψ_1, but the expected value of ψ_2 when the realtor recommends 'reject' is above ψ_1. For these offers, the realtor's recommendation will be decisive - the seller will accept those offers the realtor recommends accepting and reject those offers the realtor recommends rejecting. Therefore, for this range of offers it must also be a best response for the realtor to report his own preference truthfully. This allows us to state Proposition 3, which shows that the main result from the model with no realtor continues to hold when the realtor is added to the model.

Proposition 3*:* In the equilibrium I consider, the expected time to sale in the model with a realtor is weakly decreasing in the state of aggregate demand, z, while the expected sales price is strictly increasing in z.

Proof*:* See appendix. ∎

Figure 5: Expected Time to Sale and Expected Sales Price as a function of z

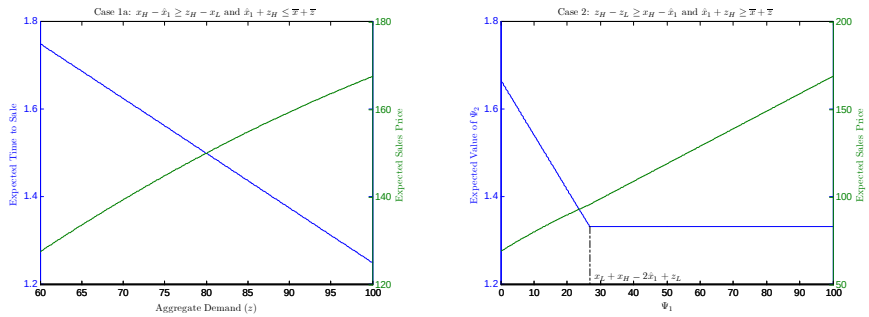


Figure 5 illustrates the expected time to sale and sales price as a function z. Cases 1b and 2 are identical in their implications for these measures so they are presented together. Because the realtor's incentives are not perfectly aligned with the buyer's, the realtor's recommendation to the seller is not entirely informative about whether $x_1 \geq \overline{x}$, in which case the seller would prefer to accept ψ_1, or $x_1 < \overline{x}$, in which case he would not. The seller knows it is always in his interest to reject an offer when the realtor recommends 'reject', but because the realtor's bias is for the seller to accept some offers that a fully informed seller would not, a recommendation of 'accept' is not fully informative. When the realtor recommends 'accept', the seller must use a reservation price strategy, thus generating the same correlation between housing demand and average time on the market as in the model with no realtor.

A special case in this section is when $c_R = 0$. In this case, the realtor's and the seller's preferences are perfectly aligned. We have already seen that for any value of c_R, the seller will reject any offer that the realtor tells him to reject. When $c_R = 0$, $\hat{x}_1 = \overline{x} - \frac{c_R}{\alpha} = \overline{x}$. Case 1a is not possible when $\hat{x}_1 = \overline{x}$. Both in case 1b and in case 2, when ψ_1 takes its lowest possible value consistent with the realtor recommending "accept", $z_L + \hat{x}_1$, $E[\psi_2|\tilde{f}(\psi_2|\psi_1, \xi_1)] = \overline{x} + \frac{\psi_1 - \hat{x}_1 + z_L}{2} = \overline{x} + \frac{\psi_1 - \overline{x} + z_L}{2} = \frac{\psi_1 + \overline{x} + z_L}{2} = \frac{\psi_1 + \psi_1}{2} = \psi_1$. Inspection shows that for higher values of ψ_1, $E[\psi_2|\tilde{f}(\psi_2|\psi_1, \xi_1)] > \psi_1$. Therefore, when $c_R = 0$, the seller will accept every offer that the realtor recommends accepting and reject every offer the realtor recommends rejecting. The expected time to sale can then be written:

$$\begin{aligned}
\bar{t}(z) &= 2Pr(\xi_1 = 0) + Pr(\xi_1 = 1) \\
&= 2\frac{\hat{x}_1 - x_L}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L}) = 1 + \frac{\hat{x}_1 - x_L}{x_H - x_L} \\
&= 1 + \frac{\overline{x} - x_L}{x_H - x_L} = 1.5
\end{aligned}$$

Thus when $\hat{x}_1 = \overline{x}$, the expected time to sale does not depend on z. This case illustrates that the distortion between the realtor's and the seller's incentives is what drives the negative correlation between aggregate demand and time to

sale in the model. When there is no distortion between the parties' incentives, the realtor's recommendation, which does not depend on z, is a credible signal to the seller as to his own best interests. When there is a distortion between the parties' incentives, the realtor's signal is not always credible: the seller knows the realtor will advise accepting some offers it would be in his best interest to reject. In this case, the seller must again resort to a reservation price strategy, giving rise to an inverse relationship between aggregate housing demand and time to sale.

3.4 Extending the Model to Multiple Periods

This section of the paper will relax one of the more restrictive assumptions in the previous sections, that the seller must accept the second period offer if he rejects the first period offer. Although the present paper extends the model only to allow for a third period, hopefully this extension will illustrate the key differences between a two period and multiple period model and present an equilibrium concept that is compatible with any finite number of periods. There are two main differences between the equilibria of the two period model and the three period model. First, in the first period of the three period model, the realtor's cutoff rule for recommending 'accept' will be a function of z. Second, in equilibrium there will be ranges of first-period offers for which the realtor will "babble": his recommendation will contain no information regarding the state of demand, and consequently the seller will ignore it when updating his beliefs. Without this feature of equilibrium, the realtor will sometimes have an incentive to misreport his own preference regarding the seller's decision in period 1 in order to manipulate the seller's beliefs regarding the state of demand.

The notation of the three-period model will follow the notation of the two-period model closely. The realtor's recommendation in period t shall be denoted ξ_t and the seller's decision in period t shall be denoted γ_t, where in both cases a value of 1 indicates 'accept' and a value of 0 indicates 'reject'. Let $\widehat{x}_1(z)$ and $\widehat{x}_2$ denote the realtor's cutoff value for reporting 'accept' in periods 1 and 2, respectively. As discussed, in equilibrium there will be some values for ψ_1 such that the realtor will babble; for all other values of ψ_1 the realtor will employ a cutoff rule in x_1 to determine his recommendation, but the cutoff will be a function of z. Let $\widetilde{z}_{L,t}$ and $\widetilde{z}_{H,t}$ denote the seller's beliefs about the lowest and highest possible values of z after receiving the period t offer and recommendation. Then the realtor's value function can be written:

$$V_R(\psi_1, z, 1) = \overset{\max}{\xi_1} \{\gamma_1(\psi_1, \tilde{f}(\psi|\psi_1, 0)\alpha\psi_1 + (1 - \gamma_1(\psi_1, \tilde{f}(\psi|\psi_1, 0)))(E[V_R(\psi_2, z, \widetilde{z}_{L,1}, \widetilde{z}_{H,1}, 2)|\xi_1 = 0]),$$
$$\gamma_2(\psi_1, \tilde{f}(\psi|\psi_1, 1)\alpha\psi_1 + (1 - \gamma_1(\psi_1, \tilde{f}(\psi|\psi_1, 1)))(E[V_R(\psi_2, z, \widetilde{z}_{L,1}, \widetilde{z}_{H,1}, 2)|\xi_1 = 1])\}$$

$$V_R(\psi_2, z, \widetilde{z}_{L,1}, \widetilde{z}_{H,1}, 2) = \overset{\max}{\xi_2} \{\gamma_2(\psi_2, \tilde{f}(\psi|\psi_2, 0, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})\alpha\psi_2 +$$
$$(1 - \gamma_2(\psi_2, \tilde{f}(\psi_2|\psi_2, 0, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})))(E[V_R(\psi_3, z, \widetilde{z}_{L,2}, \widetilde{z}_{H,2}, 3)|\xi_2 = 0]),$$
$$\gamma_2(\psi_2, \tilde{f}(\psi|\psi_1, 1, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})\alpha\psi_2 +$$
$$(1 - \gamma_2(\psi_2, \tilde{f}(\psi_2|\psi_1, 1, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})))(E[V_R(\psi_2, z, \widetilde{z}_{L,2}, \widetilde{z}_{H,2}, 2)|\xi_2 = 1])\}$$

$$V_R(\psi_3, z, \widetilde{z}_{L,2}, \widetilde{z}_{H,2}, 3) = -c_R + \alpha\psi_3$$

The seller's value function can be written:

$$V_S(\psi_1, \xi_1, 1) = \overset{\max}{\gamma_1}(1 - \alpha)\{\psi_1, E[V_S(\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1}, 2)|\tilde{f}(\psi_2|\psi_1, \xi_1)]\}$$

$$V_S(\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1}, 2) = \overset{\max}{\gamma_2}(1 - \alpha)\{\psi_2, E[\psi_3|\tilde{f}(\psi_3|\psi_1, \xi_2,)]\}$$
$$V_S(\psi_3, \xi_3, \widetilde{z}_{L,2}, \widetilde{z}_{H,2}, 3) = (1 - \alpha)\psi_3$$

Then we can update our equilibrium definition as follows.

Definition: a **Bayesian Nash equilibrium** of the game between realtors and sellers is a set of policy functions $\xi_1(\psi_1, z)$ and $\xi_2(\psi 2, z, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})$ for the realtor, a set of policy functions $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$ and $\gamma_2(\psi_2, \tilde{f}(\psi_3|\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1}))$ for the seller, and a set of belief updating strategies $\tilde{f}(\psi_2|\psi_1, \xi_1)$ and $\tilde{f}(\psi_3|\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})$ for the seller such that:
1. $\xi_1(\psi_1, z)$ and $\xi_2(\psi 2, z, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})$ solve the realtor's problem taking the seller's policy functions and belief updating strategies as given;
2. $\gamma_1(\psi_1, \tilde{f}(\psi_2|\psi_1, \xi_1))$ and $\gamma_2(\psi_2, \tilde{f}(\psi_3|\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1}))$ solve the seller's problem taking the realtor's policy functions as given; and
3. $\tilde{f}(\psi_2|\psi_1, \xi_1)$ and $\tilde{f}(\psi_3|\psi_2, \xi_2, \widetilde{z}_{L,1}, \widetilde{z}_{H,1})$ are consistent with the realtor's policy functions.

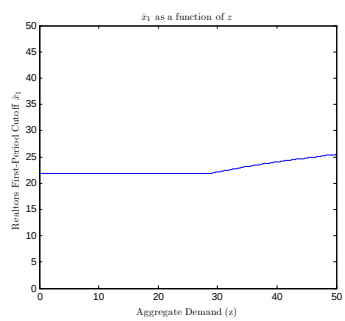
As discussed, the first main difference between the equilibria of three-period model and the two-period model is that in the first period of the three-period model, the realtor's optimal cutoff strategy is a function of z. This is because when z is low, it is more likely that ψ_2 will be so low enough that the seller rejects the offer regardless of the realtor's recommendation, and this loss of control lowers the realtor's payoff. To determine the realtor's optimal cutoff rule in period 1, consider the realtor's payoff minus αz. If the seller accepts an offer in period t, this value will be $\alpha(x_t - c_R(t-1))$. Call this value the idiosyncratic component of the realtor's payoff. The realtor will receive the αz portion of his payoff no matter which offer the seller accepts, but the idiosyncratic portion of his payoff depends on the seller's decisions and on the realizations of x_t. If the seller accepts the first period offer, the idiosyncratic portion of the realtor's payoff is αx_1. Therefore the realtor would prefer that

the seller accept any period 1 offers such that $\alpha x_1 \geq E[\alpha(x_t - c_R(t-1))]$ and to reject all others. Recall that the realtor's preference in the penultimate period is for the seller to accept any offers such that $x_2 \geq \overline{x} - \frac{c_R}{\alpha}$ and reject all others. Call this value $\widehat{x}_2$. Then the realtor's optimal cutoff rule in period 1 is defined implicitly by the following mapping (where $\widehat{x}_1(z)$ is written $\widehat{x}_1$ on the right-hand side for simplicity):

$$\widehat{x}_1(z) = \begin{cases} \frac{\frac{1}{2}(x_H^2 - \overline{x}^2 - \widetilde{\overline{z}}_1(\widehat{x}_1)^2 - z^2) + (\overline{x} - \widehat{x}_2 + \widetilde{\overline{z}}_1(\widehat{x}_1))z + (\overline{x} - x_L + \widetilde{\overline{z}}_1(\widehat{x}_1))\widehat{x}_2 - \widetilde{\overline{z}}_1(\widehat{x}_1)\overline{x}}{x_H - x_L} & \text{if } \widehat{x}_2 + \widetilde{z}_{H,1}(\widehat{x}_1) \leq \overline{x} + \widetilde{\overline{z}}_1(\widehat{x}_1) \\ \frac{\widehat{x}_2 + x_H}{2} + \left(\frac{\widehat{x}_2 - x_L}{x_H - x_L}\right)\left(\frac{\widehat{x}_2 - x_H}{2}\right) & \text{if } \widehat{x}_2 + \widetilde{z}_{H,1}(\widehat{x}_1) \geq \overline{x} + \widetilde{\overline{z}}_1(\widehat{x}_1) \\ & \text{and } z \geq x_L + x_H - 2\widehat{x}_2 + \widetilde{z}_{L,1}(\widehat{x}_1) \\ \frac{(x_H - \widehat{x}_2 + \widetilde{z}_{L,1}(\widehat{x}_1) - z)\widehat{x}_2 + \frac{1}{2}(\widehat{x}_2 - x_L - \widetilde{z}_{L,1}(\widehat{x}_1) + z)(x_L + 2x_H - \widehat{x}_2 + \widetilde{z}_{L,1}(\widehat{x}_1) - z)}{(x_H - x_L)} & \text{if } \widehat{x}_2 + \widetilde{z}_{H,1}(\widehat{x}_1) \geq \overline{x} + \widetilde{\overline{z}}_1(\widehat{x}_1) \\ & \text{and } z < x_L + x_H - 2\widehat{x}_2 + \widetilde{z}_{L,1}(\widehat{x}_1) \end{cases}$$

Because $\widehat{x}_1(z)$ is the fixed point of this functional equation, I have estimated it numerically. A description of the estimation algorithm is included in the appendix. Figure 6 illustrates the realtor's optimal period 1 cutoff rule as a function of z.

Figure 6: Realtor's Optimal Cutoff Rule as a Function of Aggregate Demand



Given the realtor's cutoff rule, the seller will update his beliefs concerning z as follows. Define $\widetilde{\overline{z}}$ as the value of z such that $\psi_1 - z = \widehat{x}_1(z)$. Further define

$$\widetilde{z}_{L,1} = \begin{cases} \max(z_L, \widetilde{\overline{z}}) & \text{if } \gamma_1 = 0 \\ \max(z_L, \psi_1 - x_H) & \text{if } \gamma_1 = 1 \end{cases} \quad \text{and } \widetilde{z}_{H,1} = \begin{cases} \min(z_H, \psi_1 - x_L) & \text{if } \gamma_1 = 0 \\ \min(\widetilde{\overline{z}}, \psi_1 - x_L) & \text{if } \gamma_1 = 1 \end{cases}$$

Then the seller's posterior belief about the distribution of z will again be that $z \sim U[\widetilde{z}_{L,1}, \widetilde{z}_{H,1}]$.[5] In period 2, the seller's belief updating strategy will be the same as the one outlined in the two-period model.

The second main difference between the equilibria of the two and three period models is the presence of 'babbling regions' in period 1 of the three period model. To see that these regions are a necessary feature of equilibrium,

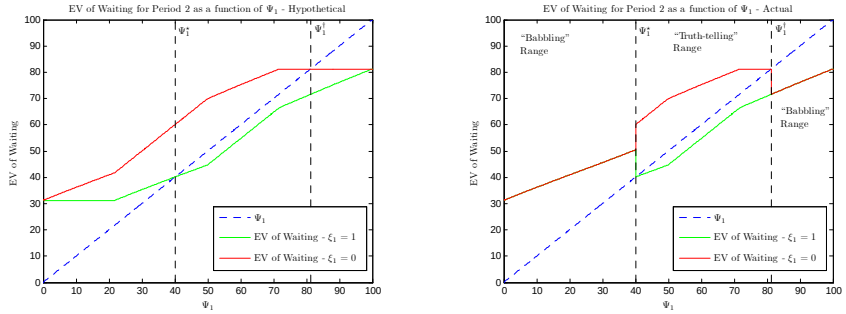
[5] Again I omit the proof, but it will follow the proof of Proposition 1 closely.

consider a hypothetical equilibrium in which the realtor always reports his own preference truthfully to the seller in period 1, and in which the realtor's and seller's behavior in the second period is the same as their behavior in the first period of the two-period model. Then the seller's expected value of rejecting the first offer would look as in the left panel of figure 7. Define $\psi_1^\star$ as the fixed point of the seller's expected value of waiting for period 2 conditional on the realtor recommending 'accept', and $\psi_1^\dagger$ as the fixed point of the seller's expected value of waiting for period 2 conditional on the realtor recommending 'reject'. Then for $\psi_1 < \psi_1^\star$, the expected value of waiting for period 2 is higher than ψ_1 whether the the realtor recommends 'accept' or 'reject'–the seller's action will not depend on the realtor's recommendation in this range. Similarly, when $\psi_1 > \psi_1^\dagger$, the expected value of waiting for period 2 will be below ψ_1 no matter what the realtor recommends. Therefore, the seller will always reject offers $\psi_1 < \psi_1^\star$ and will always accept offers $\psi_1 > \psi_1^\dagger$.

Consider the realtor's best response when $\psi_1 < \psi_1^\star$ and $x_1 > \widehat{x}_1(z)$ in the hypothetical equilibrium. No matter what the realtor recommends, the seller will reject the offer. However, because the seller expects the realtor to report his own preference regarding the offer truthfully, the seller will have a higher expectation of future offers if the realtor recommends 'reject' (thus indicating that z is high) than if the realtor recommends 'accept' (indicating that z is low). Intuitively, this state of affairs cannot be optimal for the realtor: we have shown in the two-period model that in the second-to-last period, the seller will always reject an offer that the realtor recommends rejecting. However, there are second-period offers the realtor recommends accepting that the seller will not accept. Therefore, the realtor will always prefer that the seller be more pessimistic (have a lower expectation of ψ) in the second period: a pessimistic seller is more likely to accept offers the realtor would like him to accept, but will always reject offers the realtor would like him to reject. Therefore, the realtor has a unilateral incentive to deviate from his proposed strategy in the hypothetical equilibrium. For any offer $\psi_1 < \psi_1^\star$, the realtor should recommend 'reject'.

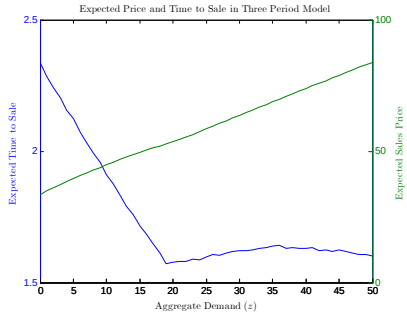
Babbling regions solve the problem of the realtor's incentive to misreport his preferences in the first period. As we have seen, if the realtor's recommendation in period 1 changes the seller's expectation of future offers but does not change the seller's action, the realtor will always choose to send the message that will make the seller more pessimistic. Then in equilibrium, it cannot be the case that the realtor's first period recommendation changes the seller's beliefs when it does not change the seller's first period action. Therefore, in the equilibrium I will consider, the realtor will babble in the first period when his recommendation will not change the seller's action, and will report his own preference truthfully when his recommendation is decisive for the seller's action. The right panel of figure 7 illustrates the period 1 equilibrium, assuming that in the second period the seller and the realtor play the same strategies as they did in the first period of the two-period model. The second period equilibrium will then look like it does in Figure 4.

Figure 7: Seller's Expected Value of waiting for Period 2 in Hypothetical and Actual Equilibria



Because $\widehat{x}_1(z)$ must be estimated numerically, the expected sales price and expected time to sale must be simulated as well. Figure 8 shows the results from such a simulation. The general pattern from the two-period model remains intact: as aggregate demand rises, the expected sales price rises and the expected time on market mostly falls. However, there is a slight bump in the expected time to sale for high levels of z. This is due to the upward-sloping portion of $\widehat{x}_1(z)$, which causes the realtor to recommend rejecting a higher percentage of offers when z is high. Overall, however, the correlation between expected sales price and expected time to sale is negative, and the simulated results of the three period model are consistent with the stylized facts observed in the data.

Figure 8: Expected Sales Price and Time to Sale as a Function of Aggregate Demand in Three-Period Model



4 Conclusion

This paper has shown that a model of the housing market featuring fully rational sellers with incomplete information regarding the state of housing demand predicts the negative correlation between prices and time on the market observed in the data. Importantly, this prediction is robust to the inclusion of

realtors with complete information about the state of housing demand as long as the preferences of realtors and sellers are not perfectly aligned. Therefore, the model in this paper offers a novel explanation for the stylized facts concerning price and time to sale dynamics in the housing market.

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A Appendix

A.1 Proofs of Propositions

Proof of Proposition 1: First note

$$f_\Psi(\psi|z) = \begin{cases} \frac{1}{(x_H - x_L)} & \text{if } z + x_L \leq \psi \leq z + x_H \\ 0 & \text{otherwise} \end{cases}$$

There are multiple cases to consider to verify that the seller's posterior distribution for z is $z \sim U[\tilde{z}_L, \tilde{z}_H]$.

Case 1: $x_H - x_L \geq z_H - z_L$

Case 1a: $\psi \leq x_L + z_H$. In this range, $f(\psi) = \frac{\psi - \psi_L}{(x_H - x_L)(z_H - z_L)}$. If $z < z_L$ or $z > z_H$, $f(z) = 0$. If $z < \psi - x_H$ or $z > \psi - x_L$, $f(\psi|z) = 0$. Therefore, $f(z|\psi) = 0$ if $z < \max(z_L, \psi - x_H) \equiv \tilde{z}_L$ or if $z > \min(z_H, \psi - x_L) \equiv \tilde{z}_H$. In the range $[\tilde{z}_L, \tilde{z}_H]$,

$$f_Z(z|\Psi = \psi) = \frac{f_\Psi(\psi|z) f_Z(z)}{f_\Psi(\psi)} = \frac{\frac{1}{x_H - x_L} \cdot \frac{1}{z_H - z_L}}{\frac{\psi - \psi_L}{(x_H - x_L)(z_H - z_L)}} = \frac{1}{\psi - \psi_L}$$

To see that $f_Z(z|\Psi = \psi)$ is a proper density, note that in this case $\tilde{z}_L = z_L$ and $\tilde{z}_H = \psi - x_L$, so that $\tilde{z}_H - \tilde{z}_L = \psi - x_L - z_L = \psi - \psi_L$.

Case 1b: $x_L + z_H \leq \psi \leq x_H + z_L$. In this case, $f(\psi) = \frac{1}{x_H - x_L}$, $f(z) = \frac{1}{z_H - z_L}$, $f(\psi|z) = \frac{1}{x_H - x_L}$, $\tilde{z}_L = z_L$, and $\tilde{z}_H = z_H$. Then in the range $[\tilde{z}_L, \tilde{z}_H]$,

$$f_Z(z|\Psi = \psi) = \frac{f_\Psi(\psi|z) f_Z(z)}{f_\Psi(\psi)} = \frac{\frac{1}{x_H - x_L} \cdot \frac{1}{z_H - z_L}}{\frac{1}{(x_H - x_L)}} = \frac{1}{z_H - z_L}$$

and $f_Z(z|\Psi = \psi) = 0$ elsewhere.

Case 1c: $x_H + z_L < \psi$. In this case $f(\psi) = \frac{\psi_H - \psi}{(x_H - x_L)(z_H - z_L)}$. $f(\psi|z) = \frac{1}{x_H - x_L}$, $f(z) = \frac{1}{z_H - z_L}$, $\tilde{z}_L = \psi - x_H$, and $\tilde{z}_H = z_H$. Then in the range $[\tilde{z}_L, \tilde{z}_H]$,

$$f_Z(z|\Psi = \psi) = \frac{f_\Psi(\psi|z) f_Z(z)}{f_\Psi(\psi)} = \frac{\frac{1}{x_H - x_L} \cdot \frac{1}{z_H - z_L}}{\frac{\psi_H - \psi}{(x_H - x_L)(z_H - z_L}} = \frac{1}{\psi_H - \psi}$$

and $f_Z(z|\Psi = \psi) = 0$ elsewhere. Because $\tilde{z}_H - \tilde{z}_L = x_H + z_H - \psi = \psi_H - \psi$, the posterior distribution is a proper density.

Case 2: $z_H - z_L > x_H - x_L$.

Case 2a: $\psi < x_H + z_L$. In this case the proof is the same as in case 1a.

Case 2b: $x_H + z_L \leq \psi \leq x_L + z_H$. In this case $f(\psi) = \frac{1}{z_H - z_L}$, $f(z) = \frac{1}{z_H - z_L}$, $f(\psi|z) = \frac{1}{x_H - x_L}$, $\tilde{z}_L = \psi - x_H$, and $\tilde{z}_H = \psi - x_L$. Then in the range $[\tilde{z}_L, \tilde{z}_H]$,

$$f_Z(z|\Psi = \psi) = \frac{f_\Psi(\psi|z) f_Z(z)}{f_\Psi(\psi)} = \frac{\frac{1}{x_H - x_L} \cdot \frac{1}{z_H - z_L}}{\frac{1}{(z_H - z_L)}} = \frac{1}{x_H - x_L}$$

and $f_Z(z|\Psi = \psi) = 0$ elsewhere. Because $\tilde{z}_H - \tilde{z}_L = \psi - x_L - (\psi - x_H) = x_H - x_L$, the posterior distribution is a proper density.

Case 2c: $x_L + z_H < \psi$. In this case the proof is the same as in case 1c. ∎

Proof of Proposition 2: We assume the seller will accept any offer $\psi_1 \geq E[\psi_2|\psi_1]$. Let $\bar{t}(z)$ denote the expected number of periods the seller leaves his house on the market. Then if $x_H - x_L \geq z_H - z_L$:

$$\begin{aligned}
\bar{t}(z) &= Pr(\psi_1 \geq \overline{\psi}) + 2Pr(\psi_1 < \overline{\psi}) \\
&= Pr(x_1 \geq \overline{\psi} - z) + 2Pr(x_1 < \overline{\psi} - z) \\
&= 1 - \frac{\overline{\psi} - x_L - z}{x_H - x_L} + 2\frac{\overline{\psi} - x_L - z}{x_H - x_L} \\
&= 1 + \frac{\overline{\psi} - x_L - z}{x_H - x_L}
\end{aligned}$$

Then

$$\frac{\partial \bar{t}}{\partial z} = \frac{-1}{x_H - x_L} < 0$$

Now consider the case in which $z_H - z_L > x_H - x_L$:

$$\begin{aligned}
\bar{t}(z) &= Pr(\psi_1 \geq x_H + z_L) + 2Pr(\psi_1 < x_H + z_L) \\
&= Pr(x_1 \geq x_H + z_L - z) + 2Pr(x_1 < x_H + z_L - z)
\end{aligned}$$

If $z > x_H - x_L + z_L$, $Pr(x_1 \geq x_H + z_L - z) = 1$, so $\bar{t} = 1 + 0 = 1$. If

$z \leq x_H - x_L + z_L$,

$$\begin{aligned}\bar{t}(z) &= 1 - \frac{x_H - x_L + z_L - z}{x_H - x_L} + 2\frac{x_H - x_L + z_L - z}{x_H - x_L} \\ &= 1 + \frac{x_H - x_L + z_L - z}{x_H - x_L} \\ &= 2 - \frac{z - z_L}{x_H - x_L}\end{aligned}$$

Then when $z_H - z_L > x_H - x_L$,

$$\bar{t}(z) = \begin{cases} 2 - \frac{z - z_L}{x_H - x_L} & \text{if } z_L \leq z \leq x_H - x_L + z_L \\ 1 & \text{if } z > x_H - x_L + z_L \end{cases}$$

and

$$\frac{\partial \bar{t}}{\partial z} = \begin{cases} \frac{-1}{x_H - x_L} & \text{if } z_L \leq z \leq x_H - x_L + z_L \\ 0 & \text{if } x_H - x_L + z_L \leq z \leq z_H \end{cases}$$

Therefore, $\frac{\partial \bar{t}}{\partial z}$ must always be weakly negative.
Let $\overline{p}(z)$ denote the expected sales price for the house. When $x_H - x_L \geq z_H - z_L$,

$$\begin{aligned}\overline{p}(z) &= Pr(\psi_1 \geq \overline{x} + \overline{z})E[\psi_1 | \psi_1 \geq \overline{x} + \overline{z}] + Pr(\psi_1 < \overline{x} + \overline{z})E[\psi_2] \\ &= Pr(x_1 \geq \overline{x} + \overline{z} - z)E[\psi_1 | \psi_1 \geq \overline{x} + \overline{z}] + Pr(x_1 < \overline{x} + \overline{z} - z)E[\psi_2] \\ &= (1 - \frac{\overline{x} - x_L + \overline{z} - z}{x_H - x_L})(\frac{\overline{x} + x_H + \overline{z} + z}{2}) + (\frac{\overline{x} - x_L + \overline{z} - z}{x_H - x_L})(\overline{x} + z) \\ &= \frac{\overline{x} + x_H + \overline{z} + z}{2} + (\frac{\overline{x} - x_L + \overline{z} - z}{x_H - x_L})(\frac{\overline{x} - x_H - \overline{z} + z}{2})\end{aligned}$$

This implies $\frac{\partial \overline{p}}{\partial z} = 1 + \frac{\overline{z} - z}{x_H - x_L}$. To see that this must always be positive, consider the case $z = z_H$. Then $\frac{\partial \overline{p}}{\partial z} = 1 + \frac{\overline{z} - z_H}{x_H - x_L} = \frac{x_H - x_L - (z_H - \overline{z})}{x_H - x_L}$. $\overline{z} > z_L$, so $z_H - \overline{z} < z_H - z_L$. By assumption, $x_H - x_L \geq z_H - z_L$. Therefore the numerator of this expression is positive, and $\frac{\partial \overline{p}}{\partial z} > 0$ when $x_H - x_L \geq z_H - z_L$.
When $z_H - z_L > x_H - x_L$,

$$\begin{aligned}\overline{p}(z) &= Pr(\psi_1 \geq x_H + z_L)E[\psi_1 | \psi_1 \geq x_H + z_L] + Pr(\psi_1 < x_H + z_L)E[\psi_2] \\ &= Pr(x_1 \geq x_H + z_L - z)E[\psi_1 | \psi_1 \geq x_H + z_L] + Pr(x_1 < x_H + z_L - z)E[\psi_2]\end{aligned}$$

If $z \geq x_H - x_L + z_L$, $Pr(\psi_1 \geq x_H + z_L) = 1$, and $E[\psi_1 | \psi_1 \geq x_H + z_L] = E[\psi_1]$. Then $\overline{p} = \overline{x} + z$, so $\frac{\partial \overline{p}}{\partial z} = 1$. If $z < x_H - x_L + z_L$, $Pr(\psi_1 \geq x_H + z_L) = \frac{z - z_L}{x_H - x_L}$. Then

$$\begin{aligned}\overline{p} &= (\frac{z - z_L}{x_H - x_L})(x_H + \frac{z_L + z}{2}) + (1 - \frac{z - z_L}{x_H - x_L})(\overline{x} + z) \\ &= \overline{x} + z + (\frac{z - z_L}{x_H - x_L})(\frac{x_H - x_L + z_L - z}{2})\end{aligned}$$

implying

$$\begin{aligned}\frac{\partial \overline{p}}{\partial z} &= 1 + \frac{x_H - x_L + z_L - 2z + z_L}{2(x_H - x_L)} \\ &= \frac{3}{2} - \frac{z - z_L}{x_H - x_L}\end{aligned}$$

Therefore when $z_H - z_L > x_H - x_L$,

$$\overline{p}(z) = \begin{cases} \overline{x} + z + (\frac{z-z_L}{x_H-x_L})(\frac{x_H-x_L+z_L-z}{2}) & \text{if } z_L \leq z \leq x_H - x_L + z_L \\ \overline{x} + z & \text{if } z > x_H - x_L + z_L \end{cases}$$

and

$$\frac{\partial \overline{p}}{\partial z} = \begin{cases} \frac{3}{2} - \frac{z-z_L}{x_H-x_L} & \text{if } z_L \leq z \leq x_H - x_L + z_L \\ 1 & \text{if } x_H - x_L + z_L \leq z \leq z_H \end{cases}$$

By assumption $z < x_H - x_L + z_L$, so $\frac{z-z_L}{x_H-x_L} < 1$. Therefore $\frac{\partial \overline{p}}{\partial z} > 0$ when $z_H - z_L > x_H - x_L$ and $z < x_H - x_L + z_L$, implying that $\frac{\partial \overline{p}}{\partial z} > 0$ in all cases. ∎

Proof of Proposition 3: Again, we assume the seller accepts any offer $\psi_1 \geq E[\psi_2|\psi_1, \xi_1]$. Recall that the seller always rejects an offer when the realtor recommends 'reject'. Then in case 1a ($x_H - \hat{x}_1 \geq z_H - z_L$ and $\hat{x}_1 + z_H \leq \overline{x} + \overline{z}$),

$$\begin{aligned}
\overline{t}(z) &= 2Pr(\xi_1 = 0) + Pr(\psi_1 \geq \overline{x} + \overline{z} \cap \xi_1 = 1) + 2Pr(\psi_1 < \overline{x} + \overline{z} \cap \xi_1 = 1) \\
&= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(Pr(x_1 \geq \overline{x} + \overline{z} - z|\xi_1 = 1) + 2Pr(x_1 < \overline{x} + \overline{z} - z|\xi_1 = 1)) \\
&= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(1 - \frac{\overline{x} - \hat{x}_1 + \overline{z} - z}{x_H - \hat{x}_1} + 2(\frac{\overline{x} - \hat{x}_1 + \overline{z} - z}{x_H - \hat{x}_1})) \\
&= 1 + \frac{\hat{x}_1 - x_L}{x_H - x_L} + (\frac{x_H - \hat{x}_1}{x_H - x_L})(\frac{\overline{x} - \hat{x}_1 + \overline{z} - z}{x_H - \hat{x}_1}) \\
&= 1 + \frac{\overline{x} - x_L + \overline{z} - z}{x_H - x_L}
\end{aligned}$$

Therefore in case 1a,

$$\frac{\partial \overline{t}}{\partial z} = \frac{-1}{x_H - x_L} < 0$$

In cases 1b and 2 ($\hat{x}_1 + z_H > \overline{x} + \overline{z}$),

$$\begin{aligned}
\overline{t}(z) &= 2Pr(\xi_1 = 0) + Pr(\psi_1 \geq x_L + x_H - \hat{x}_1 + z_L \cap \xi_1 = 1) + 2Pr(\psi_1 < x_L + x_H - \hat{x}_1 + z_L \cap \xi_1 = 1) \\
&= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(Pr(x_1 \geq x_L + x_H - \hat{x}_1 + z_L - z|\xi_1 = 1) \\
&+ 2Pr(x_1 < x_L + x_H - \hat{x}_1 + z_L - z|\xi_1 = 1))
\end{aligned}$$

If $z > x_L + x_H - 2\hat{x}_1 + z_L$, $Pr(x_1 \geq x_L + x_H - \hat{x}_1 + z_L - z|\xi_1 = 1) = 1$. In that case $\overline{t} = 2Pr(\xi_1 = 0) + Pr(\xi_1 = 1) = 2\frac{(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L}) = 1 + \frac{\hat{x}_1 - x_L}{x_H - x_L}$. If $z < x_L + x_H - 2\hat{x}_1 + z_L$,

$$\begin{aligned}
\overline{t}(z) &= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(1 - \frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1} + 2(\frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1})) \\
&= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(1 + \frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1}) \\
&= 1 + \frac{\hat{x}_1 - x_L}{x_H - x_L} + (\frac{x_H - \hat{x}_1}{x_H - x_L})(\frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1}) \\
&= 1 + \frac{x_H - \hat{x}_1 + z_L - z}{x_H - x_L}
\end{aligned}$$

Then in cases 1b and 2,

$$\bar{t}(z) = \begin{cases} 1 + \frac{x_H - \hat{x}_1 + z_L - z}{x_H - x_L} & \text{if } z_L \leq z < x_L + x_H - 2\hat{x}_1 + z_L \\ 1 + \frac{\hat{x}_1 - x_L}{x_H - x_L} & \text{if } x_L + x_H - 2\hat{x}_1 + z_L \leq z \leq z_H \end{cases}$$

and

$$\frac{\partial \bar{t}}{\partial z} = \begin{cases} \frac{-1}{x_H - x_L} & \text{if } z_L \leq z < x_L + x_H - 2\hat{x}_1 + z_L \\ 0 & \text{if } x_L + x_H - 2\hat{x}_1 + z_L \leq z \leq z_H \end{cases}$$

In case 2 ($z_H - z_L > x_H - \hat{x}_1$),

$$\begin{aligned} \bar{t}(z) &= 2Pr(\xi_1 = 0) + Pr(\psi_1 \geq x_L + x_H - \hat{x}_1 + z_L \cap \xi_1 = 1) + 2Pr(\psi_1 < x_L + x_H - \hat{x}_1 + z_L \cap \xi_1 = 1) \\ &= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(Pr(x_1 \geq x_L + x_H - \hat{x}_1 + z_L - z | \xi_1 = 1) \\ &+ 2Pr(x_1 < x_L + x_H - \hat{x}_1 + z_L - z | \xi_1 = 1)) \end{aligned}$$

If $z > x_L + x_H - 2\hat{x}_1 + z_L$, $Pr(x_1 \geq x_L + x_H - \hat{x}_1 + z_L - z | \xi_1 = 1) = 1$. In that case $\bar{t} = 2Pr(\xi_1 = 0) + Pr(\xi_1 = 1) = 2\frac{(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L}) = 1 + \frac{\hat{x}_1 - x_L}{x_H - x_L}$. Therefore $\frac{\partial \bar{t}}{\partial z} = 0$. If $z < x_L + x_H - 2\hat{x}_1 + z_L$,

$$\begin{aligned} \bar{t}(z) &= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(1 - \frac{x_L + x_H - \hat{x}_1 + z_L - z}{x_H - \hat{x}_1} + 2(\frac{x_L + x_H - \hat{x}_1 + z_L - z}{x_H - \hat{x}_1})) \\ &= \frac{2(\hat{x}_1 - x_L)}{x_H - x_L} + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(1 + \frac{x_L + x_H - \hat{x}_1 + z_L - z}{x_H - \hat{x}_1}) \end{aligned}$$

In this case,

$$\frac{\partial \bar{t}}{\partial z} = (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})\frac{-1}{x_H - \hat{x}_1} < 0$$

Therefore, in case 2

$$\frac{\partial \bar{t}}{\partial z} = \begin{cases} (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})\frac{-1}{x_H - \hat{x}_1} & \text{if } z_L \leq z < x_L + x_H - 2\hat{x}_1 + z_L \\ 0 & \text{if } x_L + x_H - 2\hat{x}_1 + z_L \leq z \leq z_H \end{cases}$$

Thus, $\frac{\partial \bar{t}}{\partial z}$ must always be weakly negative.
In case 1a,

$$\begin{aligned} \overline{p}(z) &= Pr(\xi_1 = 0)E[\psi_2] + Pr(\xi_1 = 1)\left[Pr(\psi_1 \geq \overline{x} + \overline{z} | \xi_1 = 1)E[\psi_1 | \psi_1 \geq \overline{x} + \overline{z}, \xi_1 = 1]\right. \\ &\left. + Pr(\psi_1 < \overline{x} + \overline{z} | \xi_1 = 1)E[\psi_2]\right)] \\ &= (\frac{\hat{x}_1 - x_L}{x_H - x_L})(\overline{x} + z) + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})\left[(1 - \frac{\overline{x} - \hat{x}_1 + \overline{z} - z}{x_H - \hat{x}_1})(z + \frac{\overline{x} + x_H + \overline{z} - z}{2})\right. \\ &\left. + (\frac{\overline{x} - \hat{x}_1 + \overline{z} - z}{x_H - \hat{x}_1})(\overline{x} + z))\right] \\ &= \frac{\frac{1}{2}(\overline{x}^2 + x_H^2 - \overline{z}^2 - z^2) - x_L\overline{x} + (x_H - x_L + \overline{z})z}{x_H - x_L} \end{aligned}$$

Therefore in case 1a,

$$\frac{\partial \overline{p}}{\partial z} = \frac{x_H - x_L + \overline{z} - z}{x_H - x_L}$$

To see that this must be positive, consider the case in which $z = z_H$. Then

$$\frac{\partial \overline{p}}{\partial z} = \frac{x_H - x_L + \overline{z} - z_H}{x_H - x_L} = \frac{x_H - x_L - \frac{z_H - z_L}{2}}{x_H - x_L}$$

By assumption $x_H - x_L \geq x_H - \hat{x}_1 \geq z_H - z_L$, so $\frac{\partial \overline{p}}{\partial z} \geq 0$ in case 1a. In cases 1b and 2, if $z > x_L + x_H - 2\hat{x}_1 + z_L$ (so that the seller will always accept the period 1 offer if the realtor recommends accept),

$$\begin{aligned}
\overline{p} &= Pr(\xi_1 = 0)E[\psi_2] + Pr(\xi_1 = 1)E[\psi_1 | x_1 \geq \hat{x}_1] \\
&= (\frac{\hat{x}_1 - x_L}{x_H - x_L})(\overline{x} + z) + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})(z + \frac{\hat{x}_1 + x_H}{2}) \\
&= \frac{(\hat{x}_1 - x_L)(x_L + x_H) + (x_H - \hat{x}_1)(\hat{x}_1 + x_H)}{2(x_H - x_L)} + z
\end{aligned}$$

If $z < x_L + x_H - 2\hat{x}_1 + z_L$,

$$\begin{aligned}
\overline{p}(z) &= Pr(\xi_1 = 0)E[\psi_2] + Pr(\xi_1 = 1)\left(Pr(\psi_1 \geq x_L + x_H - \hat{x}_1 + z_L | x_1 \geq \hat{x}_1)\right. \\
&\quad \times E[\psi_1 | \psi_1 \geq x_L + x_H - \hat{x}_1 + z_L, x_1 \geq \hat{x}_1] + Pr(\psi_1 < x_L + x_H - \hat{x}_1 + z_L | x_1 \geq \hat{x}_1)E[\psi_2]) \\
&= (\frac{\hat{x}_1 - x_L}{x_H - x_L})(\overline{x} + z) + (1 - \frac{\hat{x}_1 - x_L}{x_H - x_L})\left[(1 - \frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1})\right. \\
&\quad \left.\times (z + \frac{x_L + x_H - \hat{x}_1 + z_L - z + x_H}{2}) + (\frac{x_L + x_H - 2\hat{x}_1 + z_L - z}{x_H - \hat{x}_1})(\overline{x} + z)\right] \\
&= (\frac{1}{x_H - x_L})\Big(- \frac{1}{2}(x_L^2 + \hat{x}_1^2 + z_L^2 + z^2) + (\overline{x} + z_L)\hat{x}_1 + (x_H + z_L)\overline{x} \\
&\quad - (x_H + z_L + z_H)x_L + (2x_H - \overline{x} - \hat{x}_1 - x_L + z_L)z\Big)
\end{aligned}$$

Then in case 1b,

$$\overline{p}(z) = \begin{cases} \frac{-\frac{1}{2}(x_L^2 + \hat{x}_1^2 + z_L^2 + z^2) + (\overline{x} + z_L)\hat{x}_1 + (x_H + z_L)\overline{x} - (x_H + z_L + z_H)x_L + (2x_H - \overline{x} - \hat{x}_1 - x_L + z_L)z}{x_H - x_L} & \text{if } z_L \leq z < x_L + x_H - 2\hat{x}_1 + z_L \\ \frac{(\hat{x}_1 - x_L)(x_L + x_H) + (x_H - \hat{x}_1)(\hat{x}_1 + x_H)}{2(x_H - x_L)} + z & \text{if } x_L + x_H - 2\hat{x}_1 + z_L \leq z \leq z_H \end{cases}$$

and

$$\frac{\partial \overline{p}}{\partial z} = \begin{cases} \frac{2x_H - \overline{x} - \hat{x}_1 - x_L + z_L - z}{x_H - x_L} & \text{if } z_L \leq z < x_L + x_H - 2\hat{x}_1 + z_L \\ 1 & \text{if } x_L + x_H - 2\hat{x}_1 + z_L \leq z \leq z_H \end{cases}$$

To see that $\frac{\partial \overline{p}}{\partial z} > 0$ when $z \leq x_L + x_H - 2\hat{x}_1 + z_L$, note that $2x_H - \overline{x} - \hat{x}_1 + z_L \geq x_L + x_H - 2\hat{x}_1 + z_L$ is equivalent to $x_H - x_L \geq \overline{x} - \hat{x}_1$, which must be true because $x_H \geq \overline{x}$ and $\hat{x}_1 \geq x_L$. Therefore $\frac{\partial \overline{p}}{\partial z} > 0$ in cases 1b and 2. ∎

A.2 Description of Estimation Algorithm for the Realtor's Cutoff Rule in the Three Period Model

I used the following algorithm for finding the realtor's cutoff rule $\widehat{x}_1(z)$ in the three period model:

1. Pick a candidate schedule for $\widehat{x}_1(z)$. In practice I chose $\widehat{x}_1(z) = \overline{x}$ for all z.

2. On a fine grid of points for z:

   (a) Go through a fine grid of points for all values of x to create a grid of all values of ψ consistent with each value of z.

   (b) Calculate $\widetilde{z}_{L,1}$ and $\widetilde{z}_{H,1}$ for each value of ψ conditional on the realtor recommending 'reject'.

   (c) For each value of ψ, find the expected value of the idiosyncratic component of the realtor's payoff if the seller rejects the first period offer. Denote this value $\overline{x}_R$. For a fixed z, this gives $\overline{x}_R$ as a function of x.

   (d) Find the fixed point of $\overline{x}_R(x)$; use this value as the new candidate for $\widehat{x}_1(z)$.

3. Repeat this procedure using the new schedule for $\widehat{x}_1(z)$ until the maximum distance between the old and new schedules is below a specified tolerance level.